PLANE GEOMETRY TEST

1965

Scoring Formula: 4R - W

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in plane geometry. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

    (1) 2/3 ;   (2) 3 ;   (3) 6 ;

    (4) 3/2 ;   (5) none of these.

The correct answer for the sample problem is "3/2", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Three towns, Scottsbluff, Mitchell, and Torrington, are collinear but
not necessarily in that order. It is 9 miles from Scottsbluff to
Mitchell and 21 miles from Mitchell to Torrington. Which one of the
following statements is not correct?

   (1) it is possible to tell which town is between the other two.
   (2) Torrington is not between the other two towns.
   (3) the distance from Scottsbluff to Torrington could be 30 miles.
   (4) the distance from Scottsbluff to Torrington could be 12 miles.
   (5) none of the above answers.

2. Let the points L, M, and N on sides BC,
CA, and AB respectively of triangle ABC
be collinear. If a line through C and
parallel to AB cuts LM at D and if BC = 4,
CL = 2, BN = 4, NA = 3, and AM = 5, then
MC is

   (1) 1-1/3
   (2) 2-2/5
   (3) 2-2/9
   (4) 1-2/3
   (5) none of these
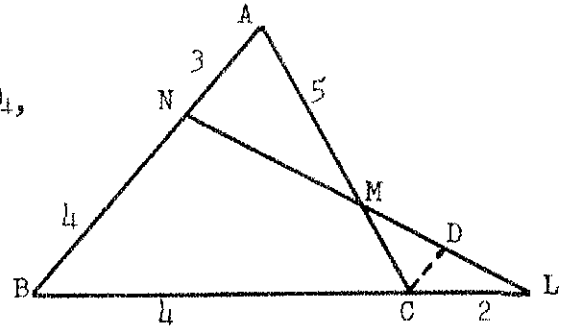
3. We are given the following theorem: If two angles of a triangle are equal,
the sides opposite these angles are equal. Now we are given a rewording of
the theorem: If two sides of a triangle are not equal, the angles opposite
these sides are not equal.

   The second statement has which of the following relationships to the
   theorem:

   (1) inverse
   (2) converse
   (3) contrapositive
   (4) positive
   (5) no relationship is apparent

4. The centroid of any triangle is determined by

   (1) the intersection of the medians of the triangle
   (2) the intersection of the altitudes of the triangle
   (3) the intersection of the angle bisectors of the triangle
   (4) the intersection of the bisectors of the exterior angles of the triangle
   (5) the intersection of the perpendicular bisectors of the sides of the
   triangle

3

5. Two circles with centers O and M intersect in two distinct points A and B such that circle M passes through the center of O. Let their line of centers intersect circle M at the point P. If the radius of circle O is 3 inches and AP is 5 inches, then the radius of circle M is
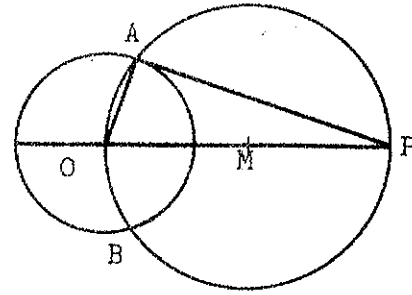
(1) $\sqrt{34/2}$
(2) 17
(3) 15
(4) $\sqrt{15/2}$
(5) 5/3

6. Which one of the following attributes of a logical system is essential?

(1) the set of axioms must be consistent
(2) the set of axioms must be based on physical reality
(3) the set of axioms must first be proven
(4) the axioms must depend on one another
(5) the principles of the system do not rely wholly on the set of axioms

7. Which of the following regions, if any, is not convex?

(1)   (2)   (3)   (4)   (5) none of these

8. The theorem which states that if two chords intersect within a circle, the product of the segments of one chord is equal to the product of the segment of the other is not true if the point of intersection of the two chords is

(1) the center of the circle
(2) the midpoint of one of the chords
(3) on a diameter of the circle
(4) the midpoint of both the chords
(5) none of these

9. Each interior angle of a regular pentagon contains

(1) 135°
(2) 90°
(3) 150°
(4) 108°
(5) 180°

10. In the given figure $\angle TOP$ and $\angle ROS$ are

(1) supplementary angles
(2) perpendicular
(3) complementary angles
(4) vertical angles
(5) none of these
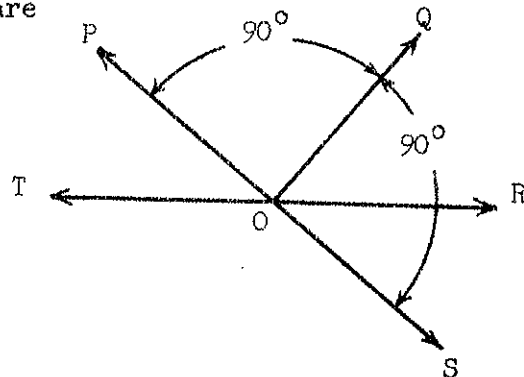
11. Which one of the following statements does not have a true converse?

    (1) If the angles of a triangle are equal, the triangle is isosceles.
    (2) A line parallel to one side of a triangle and cutting the other
         two sides divides these sides into proportional segments.
    (3) An angle inscribed in a semicircle is a right angle.
    (4) A square is a quadrilateral in which each angle is a right angle.
    (5) In the same circle, chords equidistant from the center are equal.

12. The radius of a circle is X and the radius of a second circle is 4X.
    What is the ratio of the area of the first circle to the second?

    (1) 1:4
    (2) 1:X
    (3) X:16
    (4) X:4
    (5) 1:16

13. Which one of the following statements is **not** correct?

    (1) right triangles are not all similar.
    (2) all equilateral triangles are similar.
    (3) an equilateral triangle is isosceles.
    (4) given a correspondence between two triangles such that two sides
         and an angle of the first are congruent to the corresponding parts
         of the second, then the correspondence is a congruence.
    (5) none of the above

14. In the given figure, AB is the diameter of circle O and $\angle DPB = 40°$.
    If arc AFC $= 20°$, then arc AED contains

    (1) $100°$
    (2) $80°$
    (3) $60°$
    (4) $160°$
    (5) $120°$
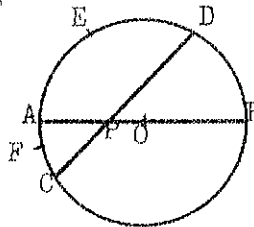
15. In a group of 120 students, 40 read French, 35 read German, 60 read
    Spanish, 10 read French and German, 15 read German and Spanish, 15 read
    French and Spanish, and 5 read all three. How many students cannot read
    any of these languages.

    (1) 0
   (2) 5
    (3) 10
    (4) 15
    (5) 20

16. Given that A, B, C are three non-collinear points. Which one of the following is not a line determined by subsets of A, B, C?

    (1) AC
    (2) AB
    (3) BC
    (4) ABC
    (5) none of these

17. If line segments are drawn connecting the midpoints of the sides of a triangle, how many similar triangles does the figure contain?

    (1) 3
    (2) 6
    (3) 4
    (4) 5
    (5) 7

18. Use the figure at the right. In circle O, AB is a chord perpendicular to the diameter CD at E. If AB is 16" long and CD is 34", what is the area of triangle OEB?
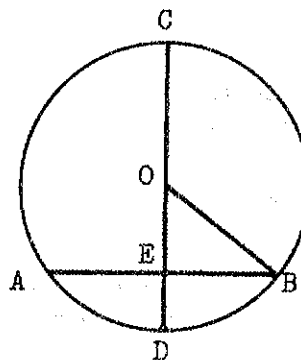
    (1) 120 square inches
    (2) 15   "   "
    (3) $\sqrt{33}$ "   "
    (4) 60   "   "
    (5) 64   "   "

19. Which one of the following statements is not correct in Euclidean Geometry?

    (1) Only one line can be drawn through the vertex of a triangle and parallel to the opposite side.
    (2) an exterior angle of a triangle can be bisected by only one line.
    (3) a triangle can have not more than one obtuse angle.
    (4) the exterior angles of a triangle are obtuse.
    (5) the sum of the lengths of two sides of a triangle is always greater than the length of the third side.

20. The locus of points from which a given line segment subtends a right angle is
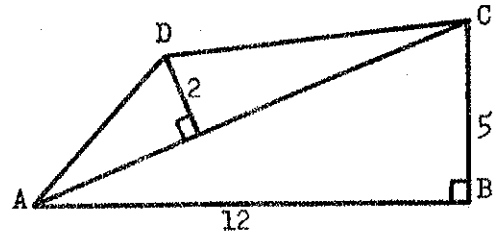
    (1) the circle having the given segment as radius
    (2) the circle having the given segment as diameter
    (3) the circle having one half the given segment as diameter
    (4) all of these
    (5) none of these

21. Find the base of a triangle whose area is $2X^2 + 5X-3$ and whose altitude's length is $X + 3$.

    (1) 4X - 1
    (2) X - 1/2
    (3) 4X - 2
    (4) 2X - 1
    (5) 4X

22. The area of quadrilateral ABCD is

    (1) 86
    (2) 43
    (3) 30 + $\sqrt{119}$
    (4) 56
    (5) none of the above

23. How far from a circle whose radius is 6 must a point be selected so that the length of the whole secant from that point through the center of the circle shall be twice the length of the tangent from that point to the circle?

    (1) 16
    (2) 8
    (3) 48/5
    (4) 18
    (5) 4

24. Let two sets be designated by A and B. Let their complements be designated by $\overline{A}$ and $\overline{B}$ respectively. Then $\overline{A \cup B}$ =

    (1) $\overline{A} \cup \overline{B}$
    (2) $\overline{A} \cap \overline{B}$
    (3) $\overline{A \cap B}$
    (4) the empty set
    (5) the universe

25. Given two similar triangles in which the ratio of the lengths of a pair of corresponding sides is 2/3. Which of the following is the ratio of the areas of the triangles?

    (1) 2/3
    (2) 4/9
    (3) $\sqrt{2} / \sqrt{3}$
    (4) $\sqrt{6}/3$
    (5) none of these

26. Three equal circles of radius 1 are drawn so that each is tangent to the other two. The area of the region bounded by the three arcs between the points of tangency is

    (1) $\sqrt{3} - \pi/6$

    (2) $\sqrt{3} - \pi/2$

    (3) $\pi/2$

    (4) $\sqrt{3}$

    (5) $(\pi/3) \sqrt{3}$

27. The ratio of the area of an equilateral triangle with side of length X
to its altitude is:

    (1) $2/\sqrt{3}$
    (2) $3/2$
    (3) $X/2$
    (4) $2/X$
    (5) $\dfrac{X\sqrt{3}}{2}$

28. Which one of the following statements is <u>not</u> correct?

    (1) if a diameter of a circle bisects a chord of that circle which is not
a diameter, then the diameter is perpendicular to the chord.
    (2) if a line bisects both the major and minor arcs of a given chord,
then it also bisects that chord.
    (3) an angle inscribed in a semi-circle is a right angle.
    (4) any two concentric circles have at least one point in common.
    (5) none of these.

29. The area of a circle is equal to:

    (1) the product of the length of its radius by its circumference.
    (2) one-half the product of π and the square of the length of diameter.
    (3) one-half the product of the length of its radius by its circumference.
    (4) two-thirds of the area of a square the length of whose side is equal to
the length of the radius of the circle.
    (5) none of these.

30. Given triangles whose areas are represented by A, B, C, and D. If we have
the following relationships among their areas: A > B, A + C < D, B < C,
which one of these relationships exists between B and D?

    (1) B < D
    (2) B > D
    (3) B = D
    (4) B $\geq$ D
    (5) the relationship cannot be found from the given conditions.

31. Given any triangle ABC with a, b and c as the sides opposite A, B, and C
respectively. Which of the following statements is not always true?

    (1) The bisector of angle A divides side "a" in the ratio b/c.
    (2) the medians to sides b and c intersect at the center of gravity
of the interior of the triangle.
    (3) the bisectors of angles A and C intersect at a point equidistant
from lines AB and BC.
    (4) the altitudes of the triangle meet at a point inside the triangle.
    (5) the perpendicular bisectors of sides a and c intersect at the
center of the circle through points A, B and C.

32. The line segment joining the midpoints of the non-parallel sides of a trapezoid is equal in length to

   (1) one-half the sum of the lengths of the bases.
   (2) three-fourths the sum of the lengths of the bases.
   (3) one-half the difference of the lengths of the bases.
   (4) one-half the product of the lengths of the bases.
   (5) none of these.

33. Use the figure at the right. Two secants AC and AB cut the circle with center at O, at D, and E respectively. $\angle CAB = 22^o$ $\angle CFB = 96^o$. What is the number of degrees in arc BC?
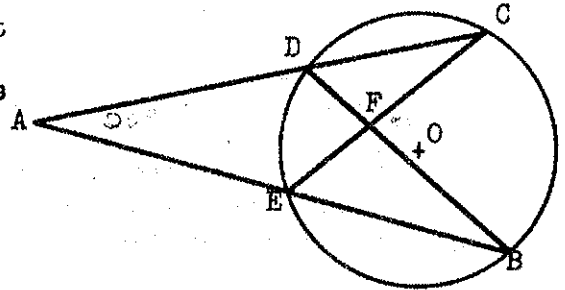
   (1) 96
   (2) 44
   (3) 84
   (4) 74
   (5) 118

34. The number of degrees in each of two angles of a triangle is 45. The ratio of the length of the longest side to the length of either of the other sides is

   (1) 1 to 2
   (2) 2 to 1
   (3) $\sqrt{2}$ to 1
   (4) 2 to $\sqrt{3}$
   (5) none of these

35. What is the locus of the vertex of a triangle that has a given area and a given line segment as its base?

   (1) A circle with the base as diameter.
   (2) The circumcircle of the triangle.
   (3) A circle with the vertex as center and radius length equal to the height of the triangle.
   (4) A line through the vertex and parallel to the base.
   (5) none of these.

36. In a triangle ABC, $\angle A = 60^o$, $\angle C = 90^o$, and the median to AB is 5 inches. What is the length of BC?

   (1) $5\sqrt{5}$ inches
   (2) $5\sqrt{3}$ "
   (3) 5 "
   (4) 10 "
   (5) $3\sqrt{5}$ "

37. The area of the figure in terms of the indicated lengths is given by

    (1) $ab + a(a - c)$
    (2) $ac + a(a - b)$
    (3) $ab - ac + a^2$
    (4) $ab + ac - a^2$
    (5) none of these
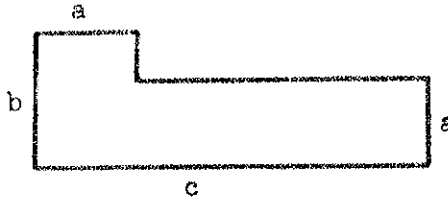
38. Which of the following is not a true statement?

    (1) Tangents to a circle from an external point are equal in length.
    (2) The areas of two circles are to each other as the squares of the lengths of their radii.
    (3) In a right triangle in which one angle contains 30°, the hypotenuse is double the shorter side.
    (4) The perpendicular is the shortest line segment that can be drawn from a point to a line.
    (5) The opposite sides of a parallelogram are equal, and the opposite angles are unequal.

39. The length and width of a rectangle are in the ratio of $4:1$. If the perimeter is 50 inches, the area is

    (1) 100 square inches
    (2) 400   "   "
    (3)   25   "   "
    (4)   50   "   "
    (5) 200   "   "

40. Which one of the following would <u>not</u> be sufficient information to prove a quadrilateral a parallelogram?

    (1) each two opposite sides are parallel.
    (2) each two opposite sides are equal in length.
    (3) three of its angles are right angles.
    (4) the diagonals bisect each other.
    (5) its diagonals are equal in length.