EIGHTH ANNUAL MATHEMATICS CONTEST

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PLANE GEOMETRY TEST

1964

Scoring Formula: 4R - W

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in Plane Geometry. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. The sum of the angles of a triangle is:

(1) 360°
(2) 45°
(3) 90°
(4) 180°
(5) 270°

1. 

The correct answer for the sample question is "180°", which is answer (4); so you would answer this question by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

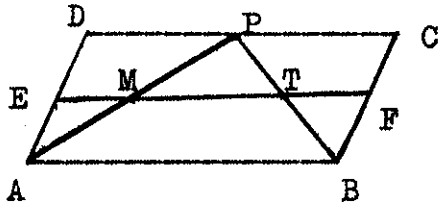
When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Which one of the following statements is <u>not</u> correct?

   (1) If two sides of a quadrilateral are parallel and equal, the figure is a parallelogram.
   (2) If the diagonals of a quadrilateral bisect each other, the figure is a parallelogram.
   (3) If two angles of a quadrilateral are supplementary, the figure is a parallelogram.
   (4) If the opposite sides of a quadrilateral are parallel, the figure is a parallelogram.
   (5) If the opposite sides of a quadrilateral are equal, the figure is a parallelogram.

2. In parallelogram ABCD, P is the midpoint of DC, M is the midpoint of AP, T is the midpoint of PB. MT extended meets AD at E and BC at F. If EF is 12 inches long, then the length of TF is

   (1) $2\sqrt{2}$ inches
   (2) 3 inches
   (3) $2\sqrt{3}$ inches
   (4) 4 inches
   (5) none of the above answers



3. The center of the circle circumscribed about a triangle can be found by

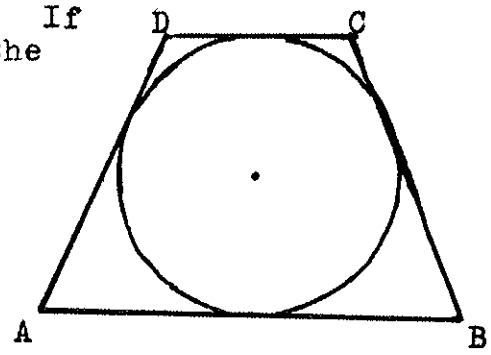
   (1) drawing the medians of the triangle.
   (2) drawing the altitudes to the sides.
   (3) drawing the angle bisectors.
   (4) drawing the bisectors of the exterior angles.
   (5) drawing the perpendicular bisectors of the sides.

4. The center of the circle inscribed in a triangle can be found by

   (1) drawing the medians of the triangle.
   (2) drawing the altitudes of the triangle.
   (3) drawing the angle bisectors.
   (4) drawing the bisectors of the exterior angles.
   (5) drawing the perpendicular bisectors of the sides.

5. The trapezoid ABCD is circumscribed about the circle, with BC = AD. If AB is 18 and DC is 6, what is the radius of the circle?

   (1) $3\sqrt{2}$
   (2) $4\sqrt{3}$
   (3) $5\sqrt{2}$
   (4) $3\sqrt{3}$
   (5) $6\sqrt{2}$



6. Which one of the following choices does <u>not</u> correctly complete the statement:

Two triangles are congruent if

(1) two sides and an angle are correspondingly equal.
(2) three sides of one triangle are equal to three sides of the other.
(3) two angles and a side are correspondingly equal.
(4) they are similar and have equal perimeters.
(5) they are similar and have equal areas.

7. The sum of the exterior angles of a polygon of six sides (angles formed by extending each side in succession) is

(1) 360°; (2) 720°; (3) 900°; (4) 480°; (5) 180°.

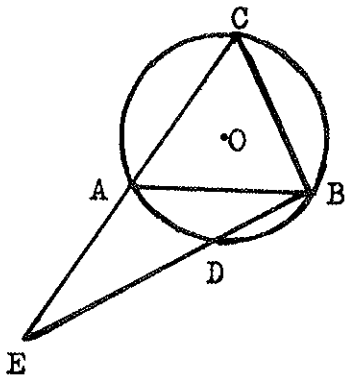
8. Which one of the following choices does <u>not</u> correctly complete the statement:

Two line segments are equal if

(1) they are chords of a circle equidistant from the center.
(2) they are corresponding elements of similar triangles.
(3) they subtend equal angles inscribed in a circle.
(4) they are any two sides of a rhombus.
(5) they are sides of a triangle opposite equal angles.

9. ABC is an equilateral triangle inscribed in circle O. If D is the midpoint of arc AB, and if the radius of the circle is 5, then CE is equal to

(1) 15
(2) $10\sqrt{3}$
(3) $15\sqrt{2}$
(4) 20
(5) $10\sqrt{2}$



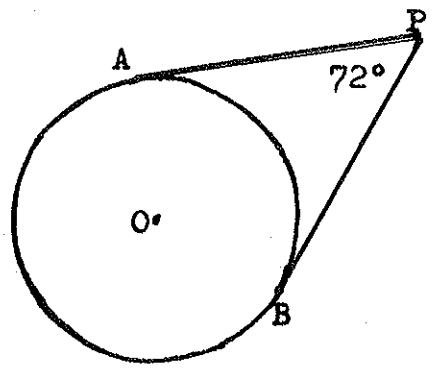
10. Given the following statements:

If I study hard, then I will make an "A" on the geometry test.
If I do not study hard, then I do not know geometry well.
If I am accepted in college, then I know geometry well.
I will not make an "A" on the geometry test.

Which of the following conclusions is valid?

(1) I know geometry well.
(2) I am accepted in college.
(3) I do not study hard, but I know geometry well.
(4) I study hard.
(5) I am not accepted in college.

23. Two tangents to a circle form an angle of 72°. If the radius of the circle is 10 inches, the length of the smaller arc is



(1) 2π inches
(2) 3π inches
(3) 4π inches
(4) 5π inches
(5) 6π inches

24. Given a straightedge one foot long and a compass, a line segment of length $\sqrt{3}$ feet long

(1) can be constructed by using no other instruments.
(2) can never be constructed.
(3) can be constructed only if permitted use of an additional instrument.
(4) can only be constructed approximately.
(5) none of the above choices complete the statement correctly.

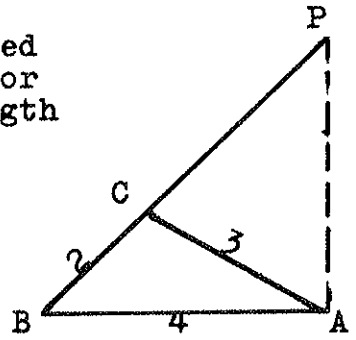
25. The radii of two concentric circles are 7 and 25. A chord of the larger circle is tangent to the smaller circle. The length of this chord is

(1) 32; (2) 40; (3) 48; (4) 49; (5) 50.

26. How long is the radius of a circle of which the number of inches in the circumference is equal to the number of square inches in the area?

(1) 1/2; (2) 2; (3) $1/\pi$; (4) $2/\pi$; (5) 1.

27. Given triangle ABC. BC extended and the bisector of the exterior angle at A meet at P. The length of CP is



(1) 5
(2) 5.5
(3) 6
(4) 6.5
(5) 7

28. Each interior angle of a regular polygon is equal to 144°. How many sides has the polygon?

(1) 6; (2) 8; (3) 10; (4) 12; (5) 16.

29. If the minute hand of a clock is 3 units long, how far does the tip of the hand travel in 36 minutes of time?

(1) 3π; (2) $16\pi/5$; (3) $7\pi/2$; (4) $18\pi/5$; (5) 4π.

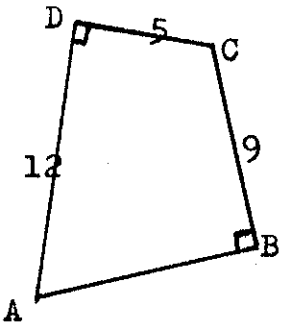
30. An equilateral triangle is inscribed in a circle of radius 2. The area inside the circle, but outside the triangle equals

(1) $4\pi - 3\sqrt{3}$; (2) $4\pi - 2\sqrt{3}$; (3) 4π; (4) $4\pi - 2\sqrt{2}$; (5) $4\pi - 3\sqrt{2}$.

31. The longer diagonal of a rhombus is 16 and the perimeter is 40. The area of the rhombus is

(1) 144; (2) 120; (3) 96; (4) 192; (5) 128.

32. What is the area of the given figure if angles B and D are right angles?



(1) $30 + 9\sqrt{22}$
(2) $30 + 12\sqrt{14}$
(3) $30 + 9\sqrt{17}$
(4) $30 + 5\sqrt{21}$
(5) $30 + 21\sqrt{5}$

33. Which of the following could <u>not</u> be the number of points that are both on a given circle and equidistant from two given lines in the plane of the circle?

(1) 1; (2) 2; (3) 3; (4) 4; (5) 5.

34. If the radius of a circle is doubled the area of the resulting circle equals the original area multiplied by

(1) 4; (2) π; (3) 3; (4) 2π; (5) 2.

35. An exterior angle of a triangle is 95°. The remote interior angles are in the ratio of 2:3. Find the smaller angle.

(1) 34°; (2) 38°; (3) 43°; (4) 51°; (5) 57°.

36. The circumference of a circle is 36. The length of an arc of 15° is

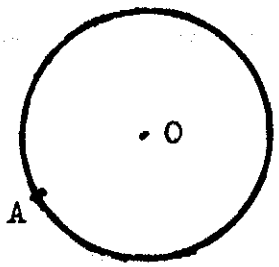
(1) 3/2; (2) 12/5; (3) $\pi/2$; (4) 3; (5) 5/2.

37. If the lengths of the sides of a right triangle are 3, 4, and 5, what is the area of the smaller of the two triangles into which it is divided by a perpendicular from the vertex of the right angle to the hypotenuse?

(1) 9/4 ; (2) 54/25 ; (3) 12/5 ; (4) 45/16 ;
(5) 7/3.

38. The locus of the midpoints of all chords passing through point A on the given circle with center O is

(1) an arc of a circle with center at A.
(2) a straight line segment.
(3) a circle with OA as diameter.
(4) two line segments with O as common endpoint.
(5) none of the above answers.



39. The length and width of a rectangle are in the ratio of 3:2. If the area is 96, the perimeter is

(1) 36; (2) 40; (3) 44; (4) 48; (5) 54.

40. The sides of a triangle are 12, 18, and 20. How long are the segments into which the bisector of the greatest angle divides the opposite side?

(1) 5 and 15
(2) 6 and 14
(3) 7.5 and 12.5
(4) 8 and 12
(5) 9 and 11