TENNESSEE MATHEMATICS TEACHER' ASSOCIATION

SIXTY-FOURTH ANNUAL MATHEMATICS CONTEST

2022

Geometry/Integrated II

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

Directions:

Do not open this booklet until you are told to do so

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.
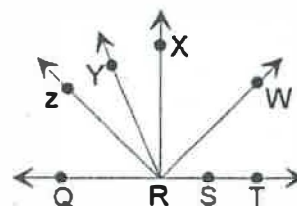
When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. What are the coordinates of R', the image of $R(2, 4)$ after a counterclockwise rotation of 90 degrees about the origin?

(a) (-2, 4)
(b) (-2, -4)
(c) (-4, 2)
(d) (-4, -2)
(e) (2, 4)

2. Given R is the midpoint of $\overline{QT}$, S is the midpoint of $\overline{RT}$, $\overrightarrow{RX}$ bisects $\angle WRZ$, and $\overrightarrow{RY}$ bisects $\angle XRZ$. If $m\angle YRZ = 26$, what is the measure of $\angle WRZ$?
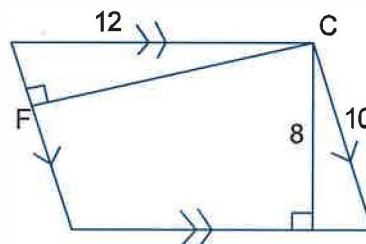
(a) 38 degrees
(b) 52 degrees
(c) 78 degrees
(d) 104 degrees
(e) 138 degrees

3. A supplement of an angle is 7 times the complement of the angle. What is the measure of the supplement of the angle?

(a) 22.5 degrees
(b) 25 degrees
(c) 75 degrees
(d) 105 degrees
(e) 157.5 degrees

4. What is the length of segment CF in the parallelogram shown below?

(a) 8
(b) 9
(c) 9.6
(d) 10
(e) 11.83

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5. A lighthouse operator at a point 25 meters above sea level sights a sailboat in the ocean. The angle of depression of the sighting is 10 degrees. Approximately how far is the boat from the base of the lighthouse?

(a)   4.4 meters
(b)   22.9 meters
(c)   24.6 meters
(d)   132.1 meters
(e)   141.8 meters

6. The coordinates of points F and E are 5 and 33 respectively. If C is the midpoint of $\overline{FE}$, and D is the midpoint of $\overline{CE}$, what is the length of segment FD?

(a)   19
(b)   24
(c)   26
(d)   31
(e)   38

7. Point Q (-2, -5) is reflected in the coordinate plane to Q'. If the coordinates of Q' are (-2, 5), which was the line of reflection?

(a) x-axis
(b) y-axis
(c) $x = 1$
(d) $y = x$
(e) $y = -x$

8. The measure of the largest angle of a triangle is twice the measure of the smallest angle. The third angle is 20 less than twice the smallest angle. What is the measure of the largest angle?

(a)   40 degrees
(b)   50 degrees
(c)   80 degrees
(d)   100 degrees
(e)   120 degrees

9. The perimeter of a rhombus is 32 and one diagonal is 8. Find the length of the other diagonal.

(a) $4\sqrt{3}$
(b) $4\sqrt{3}$
(c) $4\sqrt{5}$
(d) $8\sqrt{3}$
(e) $8\sqrt{5}$

10. What is the area of an isosceles trapezoid with bases of length 8 cm and 40 cm, and legs of length 20 cm?

(a)   12 cm²
(b) $\frac{20\sqrt{5}}{3}$ cm²
(c)   96 cm²
(d) $160\sqrt{5}$ cm²
(e)   1,920 cm²

11. What is the average rate of change of $g(x) = x^3 - 5x$ over the interval $1 \le x \le 7$?

(a) $\frac{152}{3}$
(b)   52
(c)   152
(d)   156
(e)   312

12. What is the number of sides in a regular polygon whose interior angles each measure 140°?

(a)   3
(b)   5
(c)   7
(d)   9
(e)   11

13. $\overline{AD}$ and $\overline{AE}$ are secants. If $m\angle A = 40$ and $m\widehat{DE} = 130$, then what is the $m\widehat{BC}$?

(a) 45
(b) 50
(c) 55
(d) 65
(e) 90
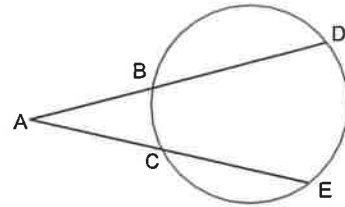
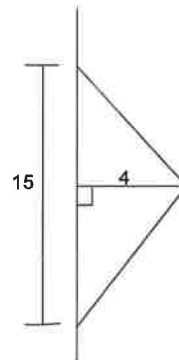14. What is the vertex of the following quadratic function, $f(x) = 3(x-2)^2 - 4$?

(a) (-2, -4)
(b) (-2, 3)
(c) (-2, 4)
(d) (2, -4)
(e) (2, 4)

15. What is the equation of a circle centered at (-2, -1) with a diameter of 24?

(a) $(x-2)^2 + (y-1)^2 = \sqrt{6}$
(b) $(x+2)^2 + (y+1)^2 = 144$
(c) $(x-2)^2 + (y-1)^2 = 2\sqrt{6}$
(d) $(x+2)^2 + (y+1)^2 = 2\sqrt{6}$
(e) $(x-2)^2 + (y-1)^2 = 576$

16. A triangle with base 15 and height 4 is rotated about that base to form a solid. What is the volume of the solid?

(a) 40π
(b) 75π
(c) 80π
(d) 225π
(e) 300π

4
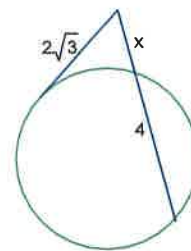
17. Given that $5x^2 = 2020$, what is the value of $\frac{7(x^4+6x^3+5x^2)}{x^2+6x+5}$.

(a) $28\sqrt{101}$

(b) $\frac{10,100}{7}$

(c)   2,020

(d)   2,828

(e)   14,140

18. Given $\overline{AB}$ is tangent to circle E, what is the length of AC?

(a) $2\sqrt{3} - 4\sqrt{3}$

(b) $\frac{\sqrt{3}}{2}$

(c) $\sqrt{3}$

(d) $2\sqrt{3}$

(e)   2

19. A company is going to make wooden frames out of reclaimed wood. To keep down on weight, the area of the frame will be 28 cm². The inside of the frame will be 5 cm by 7 cm. What should the width of the frame be?

(a)   1 cm

(b)   2 cm

(c)   4 cm

(d)   6 cm

(e)   7 cm

20. The cube shown below has side lengths of 6. What is the volume of the pyramid that has triangle BDE as its base and A as the vertex.

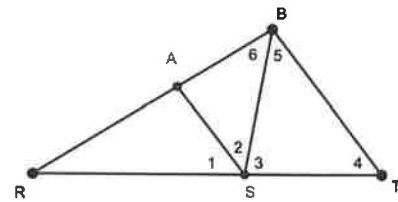(a)   6

(b)   36

(c)   72

(d)   108

(e)   216
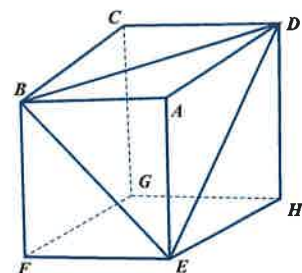
21. Two concentric circles with radii of lengths 16 and 20. What is the length of the chord of the larger circle that is tangent to the smaller circle?
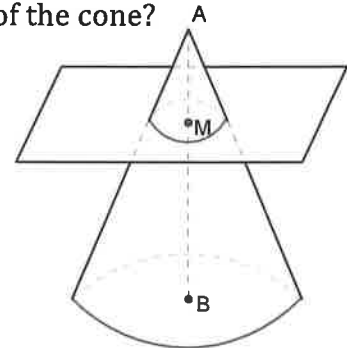
(a) 8
(b) 12
(c) 16
(d) 24
(e) 32

22. Given: $\overline{AS} \parallel \overline{BT}, m\angle 4 = m\angle 5$, and $\overrightarrow{SB}$ bisects $\angle AST$. What is the measure of $\angle 1$?

(a) 30
(b) 45
(c) 50
(d) 60
(e) 70

23. Given $AM = 4$ and $MB = 9$ and plane M is parallel to the base of the cone, what is the ratio of the volume of the top part to the volume of the bottom part of the cone?

(a) 4: 9
(b) 64: 729
(c) 64: 665
(d) 16: 81
(e) 9: 4

24. What is the area of the circle formed by the intersection of a plane and a sphere of radius 4 cm when the plane passes 3 cm from the center of the sphere?
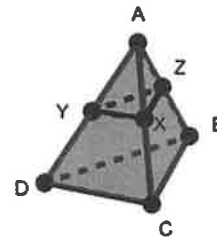
(a) $\sqrt{7}$
(b) π
(c) $\frac{7\pi}{2}$
(d) 7π
(e) 25π

25. If a, b, c are real numbers such that $a^2 + b^2 + 2c^2 - 6a + 2c - 2bc + 10 = 0$, then what is the value of $a + b - c$?

(a)   -1
(b)   0
(c)   3
(d)   4
(e)   5

26. If A, B, C, and D are noncoplanar, $\triangle ABC$, $\triangle ACD$, $\triangle ABD$ and are equilateral, X and Y are midpoints of $\overline{AC}$ and $\overline{AD}$, respectively, and Z is a point on $\overline{AB}$. What kind of triangle is $\triangle XYZ$?

(a)   Equilateral
(b)   Isosceles
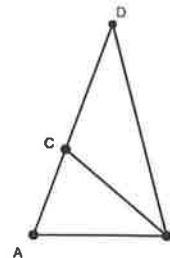(c)   Obtuse
(d)   Right
(e)   Scalene

27. A right prism has height h and bases that are regular hexagons with sides s. What is the volume of the prism?

(a) $\dfrac{3s^2 h\sqrt{3}}{2}$
(b) $3s^2 h\sqrt{3}$
(c) $6s^2 h\sqrt{3}$
(d) $3s^2 h$
(e) $6s^2 h$

28. In triangle ABD below, $AB = BC = CD$ and $AD = BD$. What is the measure of angle D?

(a)   30 degrees
(b)   36 degrees
(c)   52 degrees
(d)   60 degrees
(e)   72 degrees

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29. Two vertical poles have heights 6ft and 12 ft. A rope is stretched from the top of each pole to the bottom of the other. How far above the ground do the ropes cross?

(a) 2 ft
(b) 3 ft
(c) 4 ft
(d) 12 ft
(e) 18 ft

30. Points A, B, C, and D are consecutive vertices of a regular decagon with sides 20 cm long. $\overrightarrow{AB}$ and $\overrightarrow{DC}$ are drawn and intersect at X. What is the approximate length of BX?

(a) 10.51 cm
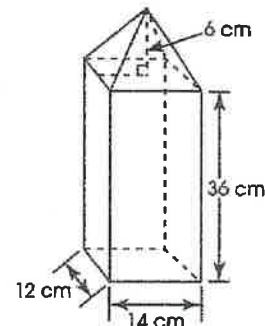(b) 12.36 cm
(c) 17.01 cm
(d) 32.36 cm
(e) 36 cm

31. Given the rectangle with dimensions 8 and 16 and with two semicircles of radius 8, what is the area of the shaded region?

(a) $16\pi\sqrt{3}$
(b) $\frac{128\pi}{3} - 16\sqrt{3}$
(c) $\frac{32\pi}{3} - 8\sqrt{3}$
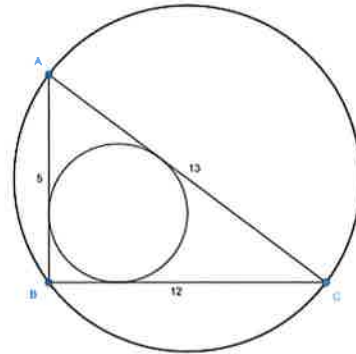(d) $\frac{32\pi}{3} - 32\sqrt{3}$
(e) $\frac{128\pi}{3} - 32\sqrt{3}$
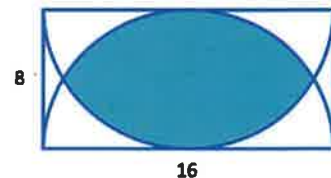
32. What is the surface area of the composite figure below?

(a) $2040 + 24\sqrt{85}$ cm²
(b) $2040 + 168\sqrt{2}$ cm²
(c) $2040 + 12\sqrt{85} + 84\sqrt{2}$ cm²
(d) $2208 + 12\sqrt{85} + 84\sqrt{2}$ cm²
(e) 6384 cm²

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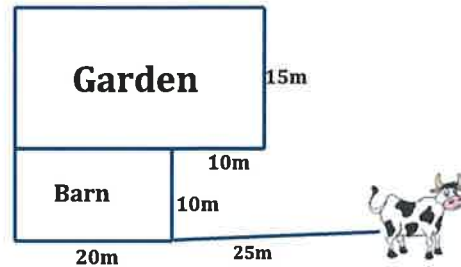33. A triangle has sides 5, 12, 13, What is the distance between the incenter and circumcenter of the triangle?

(a) $\dfrac{5\sqrt{2}}{3}$

(b) $\dfrac{3\sqrt{5}}{2}$

(c) $\dfrac{10}{3}$

(d) $\dfrac{25}{6}$

(e) $\dfrac{\sqrt{299}}{6}$

34. A cow is tied by a 25m rope to the corner of a barn as shown. A fence keeps the cow out of the garden. What is the approximate area of the grazing area?

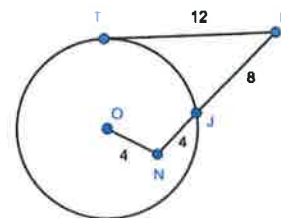(a) 1297 m²
(b) 1343 m²
(c) 1464 m²
(d) 1492 m²
(e) 1546 m²

35. What is the circumference of a circle inscribed in a rhombus with diagonals 12 cm and 16 cm?

(a) $\dfrac{24\pi}{5}$

(b) $2\pi\sqrt{11}$

(c) $\dfrac{48\pi}{5}$

(d) $2\pi\sqrt{39}$

(e) $\dfrac{64\pi}{5}$

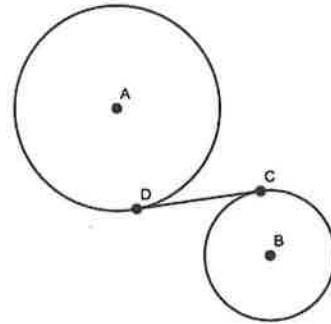36. In the diagram below, $\overline{PT}$ is tangent to circle O and $\overline{PN}$ intersects circle O at J. What is the length of the radius of circle O?

(a) 2

(b) $\dfrac{9}{2}$

(c) 5

(d) $\sqrt{34}$

(e) $2\sqrt{10}$


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37. BD is tangent to both circle A and circle B. The area of circle A is 196π and circle B is 36π. If $BD = 16$, what is the length of AB?

(a) $\dfrac{20\sqrt{41}}{5}$

(b) $\dfrac{74+6\sqrt{89}}{5}$

(c) $\dfrac{4\sqrt{1261}}{5} + \dfrac{14\sqrt{41}}{5}$

(d) $\dfrac{3\sqrt{141649}}{10} + \dfrac{6\sqrt{41}}{5}$

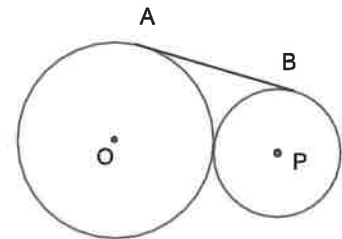(e) $\dfrac{3\sqrt{141649}}{10} + \dfrac{4\sqrt{1261}}{5}$

38. In the square below, circle H is tangent to square and the smaller square intersects the circle H at one point. What is the length of the radius of circle H?

(a) $\dfrac{6\sqrt{2}}{2+\sqrt{2}}$

(b) $\dfrac{6\sqrt{2}}{1+\sqrt{2}}$

(c) $3\sqrt{2}$

(d) $\dfrac{9\sqrt{2}}{1+\sqrt{2}}$

(e) $9\sqrt{2}$
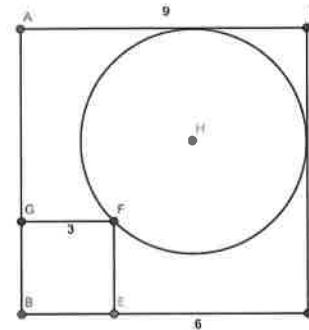
39. Given: Circles O and P with a common external tangent $\overline{AB}$, $OA = 8, PB = 2, OP = 10$. What is the length of AB?

(a) 8
(b) 10
(c) $4\sqrt{6}$
(d) $2\sqrt{34}$
(e) $4\sqrt{10}$

40. O is the center of square $ABCD$ and $\overline{VO}$ is perpendicular to the plane of the square. If $AB = 30$ and $VO = 20$, then what is the length of OE, the distance from O to the plane of triangle VBC?

(a) 9
(b) 12
(c) 16
(d) 25
(e) 36

10