THIRTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE 1987

Scoring formula: 4R - W + 40

Prepared by:
Mathematics Department
East Tennessee State University
Johnson City, TN
Marjorie Midyett, Co-ordinator

Edited by: Larry Bouldin, Roane State Community College

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
Sears, Madison, Tennessee
TRW, Ross Gear Division, Lebanon, Tennessee

1. Find x if $2^x \cdot 4^x \cdot 8^x = 1/16$.

   (a) 1

   (b) 0

   (c) 3/2

   (d) 2/3

   (e) -2/3

2. Each of two angles of a triangle is 60^o and the included side is 4 inches. The area of the triangle in square inches is

   (a) $8\sqrt{3}$

   (b) 8

   (c) $4\sqrt{3}$

   (d) 4

   (e) $2\sqrt{3}$

3. If matrix $A = \begin{bmatrix} 2 & 1 \\ -3 & -1 \end{bmatrix}$ and matrix $B = \begin{bmatrix} 1 & 4 \\ -1 & 0 \end{bmatrix}$

   what is $A^{-1} \times B$?

   (a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

   (b) $\begin{bmatrix} 1 & 8 \\ -2 & -12 \end{bmatrix}$

   (c) $\begin{bmatrix} 0 & -4 \\ 1 & 12 \end{bmatrix}$

   (d) $\begin{bmatrix} -1/2 & 1 \\ -1/3 & -1 \end{bmatrix}$

   (e) Not defined

4. The vertex of the parabola $y = 2x^2 + 4x + 3$ is located at the point

(a) (-1, 9)

(b) (-1, 1)

(c) (1, 1)

(d) (-1, 3)

(e) (-1, -1)

5. $\begin{vmatrix} 1 & -\sin x \\ \sin x & -\cos^2 x \end{vmatrix} =$

(a) 1

(b) -1

(c) sin2x

(d) cos 2x

(e) -cos2x

6. How many different sums of money can be formed from a cent, a nickel, a dime, a quarter, and a half-dollar, using at least one coin at a time?

(a) 8

(b) 16

(c) 32

(d) 33

(e) 31

7. If $\log_6 x + \log_6 (x - 1) = 1$, then x is

(a) 6

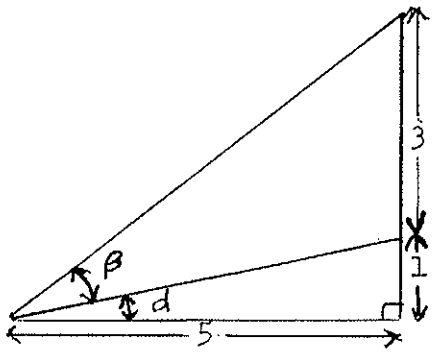
(b) -3

(c) -2

(d) 3

(e) 1

8. Recalling $\tan(x - y) = \dfrac{\tan(x) - \tan(y)}{1 + \tan(x)\tan(y)}$ and using the sketch below, find $\tan(\beta)$.



(a) 25/29

(b) 15/29

(c) 3/5

(d) 4/5

(e) 5/8

9. If $f(x) = 4x - 2$, then $f^{-1}(x)$ is

(a) $\dfrac{1}{4x - 2}$

(b) $\dfrac{4}{x} - 2$

(c) $\dfrac{4}{x} + 2$

(d) $1/4(x + 2)$

(e) Not defined

10. The value of $x - y^{x-y}$ when $x = 2$ and $y = -2$ is

(a) -18

(b) -14

(c) 18

(d) 14

(e) 256

11. If $x = 2 \cos(\theta)$ and $y = 3 \sin(\theta)$, then

(a) $x^2/4 + y^2/2 = 1$

(b) $x^2 + y^2 = 1$

(c) $x^2/4 - y^2/9 = 1$

(d) $x^2/4 + y^2/9 = 1$

(e) $y^2/9 - x^2/4 = 1$

12. A raffle offers a first prize of \$100 and two second prizes of \$40 each. One ticket costs \$1 and 500 tickets are sold. Find the expected winnings for a person who buys one ticket.

(a) -\$0.634

(b) -\$0.640

(c) -\$0.633

(d) -\$0.635

(e) -\$0.636

13. If $x + y = 1$, then the largest value of xy is

(a) 1

(b) 0.5

(c) 0.4

(d) 0.25

(e) 0

14. Find $\log_{2}(10000)$ where 10000 is in binary notation.

(a) 4

(b) $\log_{10}(16)$

(c) $\log_{2}(16)$

(d) 5

(e) 2.1

15. It is given that x varies directly as y and inversely as the square of z and that x = 10 when y = 4 and z = 14. Then, when y = 16 and z = 7, x equals

(a) 180

(b) 190

(c) 100

(d) 160

(e) 120

16. In how many different ways can a committee of three men and two women be chosen from a group of fifteen men and twelve women?

(a) 385

(b) 30030

(c) 5005

(d) 858

(e) 35

17. The operations of $\cup$ (union) and $\cap$ (intersection) in the algebra of sets are analagous to the operations of + (addition) and x (multiplication) respectively in the algebra of numbers. Which of the following true propositions in the algebra of sets is false for the corresponding property in the algebra of numbers?

(a) $A \cap B = B \cap A$

(b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(c) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

(d) $A \cup (B \cup C) = (A \cup B) \cup C$

(e) $A \cup B = B \cup A$

18. Find the 7th term of the binomial expansion of $(2x - y)^{10}$.

(a) $-5760x^3y^7$

(b) $10080x^4y^6$

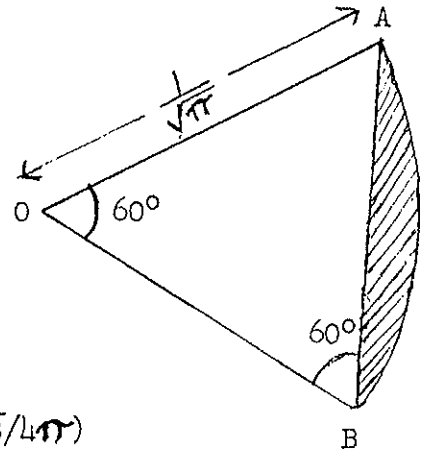
(c) $3360x^4y^6$

(d) $1440x^3y^7$

(e) $-1440x^3y^7$

Comprehensive

19. Find the area of the shaded region which is part of a circle with radius OA and chord AB.



(a) 1/12

(b) $(1/6 - \sqrt{3}/4\pi)$

(c) $(\sqrt{\pi}/6 - \sqrt{3}/4)$

(d) $\sqrt{3}/4\pi$

(e) 1

20. What is the probability that, on a single throw of a pair of dice, the absolute value of the difference will be 3?

(a) 3/36

(b) 5/36

(c) $(1/3)^2$

(d) 1/12

(e) 1/6

21. The product $\log_a b \cdot \log_b a$ is equal to

(a) a

(b) b

(c) ab

(d) 1

(e) 0

22. For what value or values of a is the matrix $\begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & 2 \\ 2 & 3 & a \end{bmatrix}$ singular?

(a) 0

(b) 1 and 3

(c) 1

(d) 3

(e) -1

23. Three people work independently at deciphering a message in code. The respective probabilities that they will decipher it are 1/7, 1/6, and 1/5. What is the probability that the message will be deciphered?

(a) 90/210

(b) 1/210

(c) 107/210

(d) 3/210

(e) 120/210

24. When simplified and expressed with negative exponents, the expression $(x + y)^{-1}(x^{-1} + y^{-1})$ is equal to

(a) $x^{-2} + 2x^{-1}y^{-1} + y^{-2}$

(b) $x^{-2} + 2^{-1}x^{-1}y^{-1}$

(c) $x^{-1}y^{-1}$

(d) $x^{-2} + y^{-2}$

(e) $1/x^{-1}y^{-1}$

25. Find the complete solution set for $\sin(x) = 1/2$.

(a) $\{\pi/6 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\} \cup \{5\pi/6 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\}$

(b) $\{\pi/3 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\} \cup \{\pi/4 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\}$

(c) $\{\pi/4 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\} \cup \{0\}$

(d) $\{\pi/2 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\} \cup \{\pi/3 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\}$

(e) $\{\pi/6 \pm 2k\pi,\ k = 0, 1, 2, 3, \ldots\}$

26. For $0 \leq \theta < 2\pi$, the solution set for $2\sin^2\theta - \cos\theta - 1 = 0$ is

(a) $\emptyset$

(b) $\{\pi/3, \pi\}$

(c) $\{\pi/3, 5\pi/3, \pi\}$

(d) $\{\pi/4, 3\pi/4\}$

(e) $\{\pi/6, 5\pi/6, 11\pi/6\}$

27. If $9^{2-x} = 27^{x+2}$, then x is

(a) $-2/5$

(b) $2/3$

(c) $5/2$

(d) $-5/2$

(e) $-1/3$

28. The diagonal of Square I is $a + b$. The perimeter of Square II with twice the area of Square I is

(a) $(a + b)^2$

(b) $\sqrt{2}(a + b)^2$

(c) $2(a + b)$

(d) $\sqrt{8}(a + b)$

(e) $4(a + b)$

29. If $\tan(x) = 1/2$ and $\tan(y) = 1/3$, find $\tan(x + y)$.

(a) $5/6$

(b) 1

(c) $1/7$

(d) $1/6$

(e) 7

30. Find all x which satisfy $\log_7 (x^2) = 3 \log_7 (2x)$.

(a) 0, 6

(b) 1

(c) 1/8

(d) 1/8, 0

(e) 0

31. The digits 2, 3, 5, 6, 7, and 9 are written on separate cards and placed in a container. Three cards are drawn. If repetitions are not permitted, what is the probability that these three digits will name a number which is less than 400?

(a) 1/4

(b) 1/3

(c) 1/2

(d) 38/162

(e) 1/5

32. Obtain the equation of a line parallel to $8x + 15y = -51$ so that the origin will be between the lines and equidistant from them.

(a) $8x - 15y = 51$

(b) $8x + 15y = 51$

(c) $15x - 8y = -51$

(d) $15x + 8y = -51$

(e) $-8x + 15y = 51$

33. The value of $|x \sin (x)|$ is always

(a) ≤ 1

(b) > 1

(c) ≥ 1

(d) < 1

(e) ≥ 0

34. By adding the same constant to each of 20, 50, 100 a geometric progression results. The common ratio is

(a) 5/3

(b) 4/3

(c) 3/2

(d) 1/2

(e) 1/3

35. The distance between the point (-3, 4) and the line $y = \frac{3}{4}x$ is

(a) 4

(b) 3

(c) 5

(d) 7

(e) 6

36. The exponential function $y = 2^{-x}$ takes values

(a) $y > 0$

(b) $y \geq 0$

(c) $y < 0$

(d) $y \leq 0$

(e) $y = -1/2$

37. Find the center of the circle represented by $2x^2 + 2y^2 + 2x - 6y - 3 = 0$.

(a) (1/2, -3/2)

(b) (1/2, 3/2)

(c) No circle

(d) (-1/2, 3/2)

(e) (-1/2, -3/2)

38. $\frac{\cos(x)}{\sec(x) + \tan(x)}$ is equal to

(a) $1 - \sin(x)$

(b) $1 + \sin(x)$

(c) $\frac{\cos(x)}{1 + \sin(x)}$

(d) $\frac{\cos(x)}{1 - \sin(x)}$

(e) $\frac{1 + \sin^2(x)}{1 + \sin(x)}$

39. Jim and Jan each toss three coins. What is the probability that they obtain the same number of heads?

(a) 5/16

(b) 1/2

(c) 19/20

(d) 1/4

(e) 1/8

40. The number of diagonals that can be drawn in a polygon of 100 sides is

(a) 4850

(b) 4950

(c) 9900

(d) 98

(e) 8800