COMPREHENSIVE 1985

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on the the next. The working time for the entire test is 80 minutes.

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1. $A^{1/2} \left(A^{1/2} + A^{-1/2} \right)$ simplifies to

   a) $A^{1/4} + \dfrac{1}{A^{1/4}}$

   d) $\sqrt[4]{A} + \dfrac{\sqrt[4]{A^3}}{A}$

   b) 0

   e) None of the above

   c) $A + 1$

2. $\dfrac{xy^{-2} - x^{-2}y}{y^{-2} - x^{-2}}$ simplifies to

   a) $\dfrac{xy}{x + y}$

   d) $\dfrac{x^2 - xy + y^2}{x + y}$

   b) $\dfrac{x^2 + xy + y^2}{x + y}$

   e) $x - y$

   c) $x + y$

3. If the graphs of $2y + x + 3 = 0$ and $3y + kx + 2 = 0$ are perpendicular lines, then k = _____.

   a) $\pm\, 2/3$

   d) 6

   b) $- 2/3$

   e) $- 6$

   c) $- 3/2$

4. Find all values of K such that the slope of the line through the points (K, 4) and (1, 3 − 2K) is less than 5.

   a) $K > 1$ or $K < 2$

   d) $1 < K < 2$

   b) $K < 1$ or $K > 2$

   e) None of the above

   c) $- 1 < K < 2$

5. If the location of the graph of $y = f(x)$ is known, where is the graph of $y = f(x + 2)$ in relation to the graph of $y = f(x)$?

   a) 2 units higher

   d) 2 units to the left

   b) 2 units lower

   e) None of the above

   c) 2 units to the right

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6. The largest subset of the real numbers which can serve as the domain of the function $f(x) = \dfrac{\sqrt{2x + 7}}{x^3 - 16x}$ is

   a)   All real numbers

   b)   All real numbers except 0, 4, or −4

   c) $[-7/2, \infty)$

   d) $[-7/2, \infty)$ except 4

   e)   None of these

7. If $h(x) = x^2 - 3x$, then $h(t + 2) = $ _____.

   a) $t^2 + t - 2$

   b) $t^2 - t - 2$

   c) $t^2 - 3t - 2$

   d) $4t^2 - 6t$

   e) $t^2 - 6$

8. If $\sin^{-1} t = w$, then _____.

   a) $\sin t = 1/w$

   b) $\sin t = w$

   c) $\sin w = -t$

   d) $\sin w = t$

   e) $\sin(-1) = tw$

9. Given the following functions: 1. $x + 3x^2$

                                         2. $\cos x + \sin x$

                                         3. $\sec x + 5x^2$

                                         4. $\tan x + x$

Which of these are <u>neither</u> even <u>nor</u> odd functions?

   a)   1 and 2

   b)   3 and 4

   c)   1 only

   d)   2 and 4

   e)   3 only

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10. How many distinct real number solutions does the equation $x^3 + 4x = 0$ have?

    a)   0   d)   3

    b)   1   e)   More than 3

    c)   2

11. The solution set of $2^{x^2 - 3x} = 16$ is _____.

    a) $\emptyset$ d) $\{1, -4\}$

    b) $\{2, 2\}$ e) $\{-1, 4\}$

    c) $\{1/2, 4\}$

12. The roots of $x^2 - 8x + 14 = 0$ are

    a)   both rational   d)   one rational and one complex

    b)   both irrational   e)   both complex

    c)   one rational and one
         irrational

13. The values of a, b, and c that require the graph of $y = ax^2 + bx + c$ to pass through the points (1, 4), (-2, -5), and (3, 0) are _____.

    a)   a = 1, b = -2, c = 3   d)   a = -1, b = 3, c = -2

    b)   a = -2, b = 1, c = 4   e)   None of the above

    c)   a = -1, b = 2, c = 3

14. If x is the highest power of 3 such that 3^x is a factor of 100!, then x is

    a)   27   d)   48

    b)   35   e)   49

    c)   47

4

15. A clock strikes once on each quarter hour, twice on each half hour, three times on each three-quarters hour, and (at the end of each hour) strikes the hour. How many times will it strike in 24 hours?

   a)   150
   b)   157
   c)   168
   d)   276
   e)   300

16. The equation of the circle with center $(3, -1)$ which is tangent to the perpendicular bisector of $\overline{AB}$ where A is the point $(-5, -2)$ and B is the point $(-1, -2)$ is

   a) $x^2 + y^2 - 6x + 2y + 1 = 0$
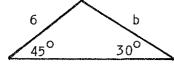   b) $x^2 + y^2 - 6x + 2y - 26 = 0$
   c) $x^2 + y^2 + 6x - 2y - 26 = 0$
   d) $x^2 - y^2 + 6x + 2y - 27 = 0$
   e) $x^2 + y^2 - 6x + 2y + 10 = 0$

17. The inside dimensions of a closed box (rectangular parallelepiped) are 3, 4, and 5 feet. What is the length of the longest straight rod that could be placed within the box?

   a)   5 feet
   b) $5\sqrt{2}$ feet
   c) $\sqrt{41}$ feet
   d) $\sqrt{34}$ feet
   e)   12 feet

18. Given the triangle   6 ⟋⟍ b   with angles 45° and 30° The length of side b is

   a) $2\sqrt{6}$
   b) $3\sqrt{6}$
   c) $3\sqrt{2}$
   d) $6\sqrt{2}$
   e) $\dfrac{3\sqrt{2}}{2}$

19. A square is inscribed in a circle whose radius is 4. Find the area of the square.

   a)   16   d)   64

   b) $16\sqrt{27}$ e)   None of the above

   c)   32

20. What is the intersection of the cylinder $x^2 + y^2 = 4$ with the plane $y = 2$?

   a)   A point   d)   A parabola

   b)   A circle   e)   None of the above

   c)   A line

21. Find the total surface area of a regular tetrahedron whose edge is 4 units in length.

   a) $4\sqrt{3}$ d)   32

   b)   16   e)   64

   c) $16\sqrt{3}$

22. Which of the following systems is (are) inconsistent?

   I. $x + y = 5$ II. $x - y = 0$ III. $2x + y = 1$ IV. $3x + 2y = 1$

   $2x + 2y = 6$ $7x - 2y = 0$ $3x - 4y = 2$ $6x + 4y = 0$

   a)   I only   d)   II and III

   b)   I and II   e)   I and IV

   c)   II and IV

23. Simplify $\left(\log_2 4^e\right)^{-1}$

   a) $-e$ d) $\dfrac{1}{2e}$

   b)   1

   c)   2e   e)   Cannot be simplified because 2 and e are different bases

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24. If $\log_2 \left[\log_{10}x\right] = 2$, then $x = ?$

    a) 1

    b) 10,000

    c) 16

    d) 4

    e) None of the above

25. If $\sin t = -4/5$ and the terminal side of an angle of t radians in standard position lies in the third quadrant, then $\cos t = ?$

    a) $-3/5$

    b) t is not in the domain of the cosine function

    c) $-\dfrac{\sqrt{41}}{5}$

    d) $3/5$

    e) $\cos t$ is defined, but there is not enough information to to calculate it.

26. What is the value of $\dfrac{\sin^2 \theta + \cos^2 \theta}{\cos^2 \theta - \sin^2 \theta}$?

    a) $\dfrac{1}{1 + 2\cos^2 \theta}$

    b) -1

    c) $\cos 2\theta$

    d) $\dfrac{\sin \theta + \cos \theta}{\sin \theta - \cos \theta}$

    e) $\sec 2\theta$

27. $\dfrac{\tan^2 x - \sin^2 x}{\sec^2 x}$ can be reduced to

    a) 0

    b) $\sin^4 x$

    c) $\cos^2 x$

    d) $\cos^2 x - \sin^2 x$

    e) None of the above

28. The number of distinct values for x on $[0, 2\pi)$ for which $4\sin^2 x - 3 = 0$ is

    a) 0

    b) 1

    c) 2

    d) 3

    e) 4

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29. The solution set for the equation $\sin 2\theta + \sin \theta = 0$ on the interval $[0, 2\pi)$ is

    a) $\{0, \ \pi/3, \ \pi, \ 5\pi/3\}$

    b) $\{0, \ \pi/2, \ \pi, \ 3\pi/2\}$

    c) $\{0, \ \pi/3, \ \pi/2, \ 2\pi/3, \ \pi\}$

    d) $\{0, \ 2\pi/3, \ \pi, \ 4\pi/3\}$

    e) $\{0, \ 2\pi/3, \ \pi, \ 4\pi/3, \ 2\pi\}$

30. As x varies from $\pi/4$ radians to $^{7}\pi/4$ radians, the graph of

    $y = 4 \sin (2x - \pi/8)$

    a)   does not cross the x-axis

    b)   crosses the x-axis once

    c)   crosses the x-axis twice

    d)   crosses the x-axis three times

    e)   crosses the x-axis four times

31. What is the solution set of the inequality $|x - 3| \leq |x + 5|$?

    a) $3 \leq x \leq 5$

    b) $-5 \leq x \leq 3$

    c) $x \leq 3$

    d) $x \geq -1$

    e) $x \geq 0$

32. Find the set of values for which $x^3 + 1 \geq x^2 + x$

    a) $x \leq 0$

    b) $x \geq 0$

    c) $0 < x \leq 2$

    d) $-1 \leq x \leq 1$

    e) $x \geq -1$

33. If $0 < r \leq s - t$, then which of the following statements is false?

    a) $s \geq r + t$

    b) $t - s \leq -r$

    c) $-r \geq s - t$

    d) $\dfrac{s - t}{r} \geq 0$

    e) $s - r \geq t$

34. What is the sum of the infinite geometric series $10 - 5 + 5/2 - \ldots$?

    a)   6

    b)   7

    c)   5

    d)   15/2

    e)   None of the above

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35. A child piles 91 cylindrical bars in layers so that the top layer contains one bar, and each lower layer has one more bar than the one above it. How many bars are there in the lowest layer?

    a) 9

    b) 11

    c) 12

    d) 13

    e) 14

36. The converse of $\sim p \rightarrow q$ is equivalent to which of the following?

    a) $p \rightarrow q$

    b) $q \rightarrow \sim p$

    c) $\sim p \rightarrow \sim q$

    d) $p \rightarrow \sim q$

    e) None of the above

37. All residents of this state who are registered voters are over 21 years of age. If John is a resident of this state, then it is correct to conclude that

    a) If John is over 21, he is a registered voter.

    b) If John is a registered voter, he is over 21.

    c) If John is not a registered voter, he is not over 21.

    d) If John is not over 21, he is a registered voter.

    e) None of the above.

38. A coin is flipped 4 times. The probability of exactly 2 heads and 2 tails is

    a) 1/2

    b) 1/16

    c) 3/8

    d) 1/4

    e) 1

39. How many distinguishable permutations can be made from the letters of the word SUCCESS if they are all used every time?

a) 6!

d) 210

b) 420

e) None of the above

c) 840

40. What is the value of the continued fraction

$$\cfrac{1}{2 + \cfrac{1}{2 + \ldots}}$$

assuming convergence?

a) $1 + \sqrt{2}$

d) $-1 + \sqrt{2}$

b) $1 - \sqrt{2}$

e) $-1 - \sqrt{2}$

c) 2/3