COMPREHENSIVE TEST 1982   Edited by: The University of
                                                                Tennessee at Martin

Scoring Formula: 4R - W + 40

     This test was prepared from a list of Comprehensive questions submitted by
East Tennessee State University.

DIRECTIONS:

     Do not open this booklet until you are told to do so.

     This is a test of your competence in high school mathematics. For each problem
there are listed 5 possible answers; one and only one is correct. You are to work
each problem, determine the correct answer, and indicate your choice by making a
heavy black mark in the correct place on the separate answer sheet provided. You
must use a pencil with a soft lead (No. 2 lead or softer).

     This test has been constructed so that most of you are not expected to answer
all questions. Do your very best on the questions you feel you know how to work.
You will be penalized for incorrect answers, so it is advisable not to do much wild
guessing.

     If you should change your mind about an answer, be sure to erase completely.
Do not mark more than one answer for any problem. Make no stray marks of any kind
on your answer sheet. The answer sheets will not be returned to you. If you wish a
record of your performance, mark your answers in this booklet also. You will be able
to keep this booklet after the test is completed.

     When told to do so, open your test booklet to page 2 and begin. When you have
finished one page, go on to the next. The working time for the entire test is 80
minutes.

Contributors to TMTA for Annual Mathematics Contest:

1. In the circle below with center O, suppose that point B is chosen so that AO/BO = BO/AB and point C is chosen so that AC = BO. What is angle AOC?

    a) 18°

    b) 30°

    c) 36°

    d) 45°

    e) 72°
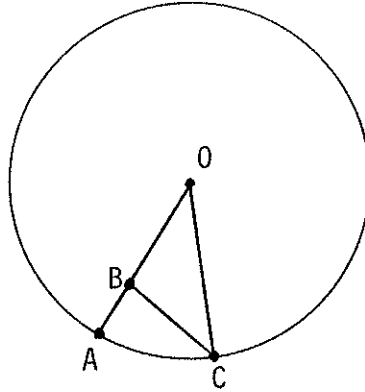
2. A circle is inscribed in an isosceles triangle ABC, where angle A = angle B = 30°, and side AC has length 1. What is the radius of the circle?

    a) $1 - \sqrt{3}/2$

    b) $\sqrt{3} - 3/2$

    c) $2\sqrt{3} - 3$

    d) $2 - \sqrt{3}$

    e) $\sqrt{3}/4$

3. Let m and n be integers greater than 0. Define the three quantities

    $$A = (m + n)/2 \qquad G = \sqrt{mn} \qquad H = 2mn/(m + n)$$

    Which of the following relationships must hold?

    a) $A \leq G \leq H$

    b) $G \leq H \leq A$

    c) $H \leq G \leq A$

    d) $H \leq A \leq G$

    e) more information needed

4. Two dice are each loaded so that the probability of a 1 is twice the probability of each other number. When both dice are thrown, what is the probability of obtaining a total of 6?

   a) 1/7

   b) 2/7

   c) 5/36

   d) 1/6

   e) 5/49

5. If t is an angle such that sin t ≠ 0, then $\dfrac{\sin t \cdot \tan t}{\tan t - \sin t} =$

   a) $\dfrac{\sin t}{1 + \cos t}$

   b) $\dfrac{\sin t}{\cos t - 1}$

   c) $\dfrac{\tan t + \sin t}{\sin t \cdot \tan t}$

   d) $\dfrac{\tan t - \sin t}{\cos t \cdot \cot t}$

   e) $\dfrac{\tan^2 t}{1 - \sec t}$

6. The base of a pyramid is an equilateral triangle with sides of length $2\sqrt{3}/3$. The other three edges of the pyramid have lengths $2\sqrt{3}/3$, $2\sqrt{3}/3$, and 1. What is the height of the pyramid?

   a) $\sqrt{3}/2$

   b) 1

   c) $2\sqrt{3}/3$

   d) $\sqrt{3}$

   e) $\sqrt{6}$

7. In triangle ABC below AB has length 5, BC length 12, and AC length 13. Moreover, AC is perpendicular to BD. What is the length of AD?
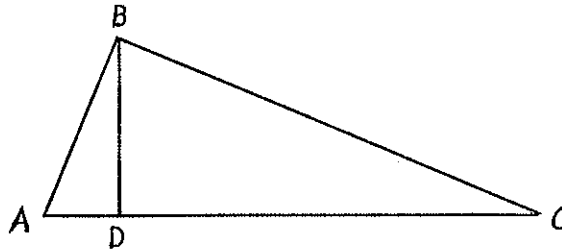
   a) 65/17

   b) 12 - $\sqrt{5}$

   c) 13/5

   d) 25/13

   e) 12/5

8. Consider the convex quadrilateral (that is, each interior angle is < 180°) ABCD with diagonals AC and BD. Which of the following must be true?

   a) AB < AC

   b) AB + CD < AC + BD

   c) AD + CD < AB + BC

   d) AB + BC + CD + DA < AC + BD

   e) None of the above

9. What point on the conic $x^2 + 8x + y^2 - 6x = 11$ is closest to the origin?

   a) (-4, 3)

   b) $(4\sqrt{5}/5, -3\sqrt{5}/5)$

   c) (4/5, -3/5)

   d) $(-\sqrt{2}/2, \sqrt{2}/2)$

   e) $(\sqrt{2}/2, -\sqrt{2}/2)$

10. If the line $y = ax$ is tangent to the conic $4x^2 + (y - 4)^2 = 4$, one possible value of a is

   a) 1

   b) $\sqrt{2}$

   c) $\sqrt{3}$

   d) $2\sqrt{3}$

   e) $2\sqrt{7}$

11. A party of six people is seated randomly at a circular table. Included in the party are two mutual enemies. What is the probability that no two of the enemies are seated next to each other?

   a) 3/20

   b) 1/3

   c) 2/5

   d) 3/5

   e) 2/3

12. A triangle has vertices (0, 0), (2, 1), and (4, 4). What is its area?

   a) 2

   b) 4

   c) 8

   d) $\sqrt{65}/4$

   e) $\sqrt{130}/3$

13. When the polynomial $p(x)$ is divided by $x^2 - 1$ the remainder is $x + 1$. What is the remainder when $p(x)$ is divided by $x - 1$?

   a) -2

   b) 0

   c) 1

   d) 2

   e) Need more information

14. When 2^{1025} is divided by 5, the remainder is

   a) 0

   b) 1

   c) 2

   d) 3

   e) 4

5

15. The equation $x^2 + 4xy + 8y^2 = 0$ can best be described as the equation of a(n)

    a) circle

    b) ellipse

    c) hyperbola

    d) parabola

    e) point

16. In the figure below 0 is the center of the circle and ABCD is a square. If the radius of the circle is 1, what is the area of the square?

    a) 1/4

    b) 1/2

    c) 4/5

    d) 1

    e) 4/3
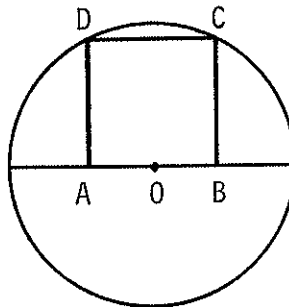
17. If x, y, and z satisfy the equations
$$\begin{array}{l} x + 2y + 3z = 7 \\ x + 3y + 5z = 10 \\ 2x + 3y + 8z = 15 \end{array},$$
    what is the value of x?

    a) 0

    b) 1

    c) 2

    d) 3

    e) 4

18. The real solutions of $2^x - 2^{-x} = 2^2$ are

    a) $\log_2 (2 \pm \sqrt{5})$

    b) $\log_2 (2 + \sqrt{5})$

    c) $2 \pm \sqrt{5}$

    d) $2 + \sqrt{5}$

    e) 1

19. If $i^2 = -1$, which of the following represents $(1/2 + \sqrt{3}/2\ i)^{3/2}$?

    a) 1

    b) i

    c) $\sqrt{2}/2 + \sqrt{2}/2\ i$

    d) $\sqrt{2}/2 - \sqrt{2}/2\ i$

    e) -1

20. The equation $\sin(t) + \cos(t) = 1$ has how many roots in the interval $0 < t < 2\pi$?

    a) 0

    b) 1

    c) 2

    d) 3

    e) 4

21. How many distinct permutations can be made from the letters of the word algebra?

    a) 28

    b) 720

    c) 2520

    d) 5040

    e) 10080

22. The sum of the roots of the polynomial $x^4 + 2x^3 + 3x^2 + 4x + 5$ is

    a) 2

    b) -2

    c) 3

    d) 5

    e) -5

23. If the radius of a circle is 4, what is the area of the inscribed equilateral triangle?

    a) $6\sqrt{3}$

    b) $12\sqrt{3}$

    c) $24\sqrt{3}$

    d) $\pi\sqrt{3}$

    e) 16π

24. What is the solution set of $|x + 1| < |x - 1|$?

    a) $0 \leq x$

    b) $0 < x$

    c) $1 < x$

    d) $x < -1$

    e) $x < 0$

25. How many real roots are there for the equation $x^3 - 6x^2 + 11x - 6 = 0$?

    a) 0

    b) 1

    c) 2

    d) 3

    e) 4

26. If $x^4 + 64 = (x^2 - ax + b)(x^2 + ax + c)$ with $a \geq 0$, then $a =$

    a) -2

    b) 0

    c) 2

    d) 4

    e) Not possible

8

27. Given that $f(x) = x^2 + 2x$ on the interval $-1 \leq x < \infty$, the inverse function $f^{-1}(x)$ is

    a) $-1 + \sqrt{1 + x}$

    b) $1/(x^2 + 2x)$

    c) $-1 \pm \sqrt{1 + x}$

    d) $x + \sqrt{2x}$

    e) $-1 - \sqrt{1 + x}$

28. For a set A, let $|A|$ denote the number of members of A. If $|A| = 8$, $|B| = 6$, $|C| = 5$, $|A \cap B| = 2$, $|A \cap C| = 1$, $|B \cap C| = 1$ and $|A \cup B \cup C| = 16$, what is $|A \cap B \cap C|$?

    a) 3

    b) 2

    c) 1

    d) 0

    e) Not possible

29. If $10^{-x \log 5} = 25$, where log is to the base 10, then $x =$

    a) -4

    b) -2

    c) 0

    d) 2

    e) 4

30. If the greatest common divisor of the numbers 26 and 35 is 8, what is the base?

    a) 7

    b) 8

    c) 9

    d) 10

    e) 16

9

31. A man drives up a hill at 30 MPH. How fast must he drive back down to average 60 MPH for the entire trip?

    a) 30 MPH

    b) 60 MPH

    c) 90 MPH

    d) 120 MPH

    e) Can't be done

32. Let AB and CD be two chords of a circle, intersecting in the point E. If AE = 4, BE = 3, and CE = 2, what is DE?

    a) 5

    b) 6

    c) $\sqrt{5}$

    d) $\sqrt{6}$

    e) 7

33. The length of a rectangular field is 3 times the width. The formula for the area in terms of the perimeter is $A = xP^2$.

    a) 3/64

    b) 3/16

    c) 1/4

    d) 3/4

    e) 1

34. A bucket can be filled by two hoses in 2 and 3 minutes, respectively. In how many minutes can it be filled by both hoses acting together?

    a) 1/5

    b) 5/6

    c) 1

    d) 6/5

    e) 5

35. If the lines $y = 4mx + 1981$ and $y = -mx - 1982$ are perpendicular and $m \geq 0$, what is m?

    a) 0

    b) 1/2

    c) 1

    d) 3

    e) There is no such m.

36. If $f(x) = 1 + x^2$ and $g(x) = 2x$, then $f(g(x)) =$

    a) $1 + 4x^2$

    b) $1 + (4x)^2$

    c) $2(1 + x^2)$

    d) $2x + 2x^3$

    e) $1 + 2x + 2x^3$

37. In the expansion of $(2x + xy)^4$, what is the coefficient of x^4y^2?

    a) 6

    b) 12

    c) 24

    d) 48

    e) There is no such term.

38. Let ABC be a triangle. If AB = 9 inches, BC = 8 inches and CA = 7 inches, which angle is largest?

    a) $< A$

    b) $< B$

    c) $< C$

    d) Not enough information

    e) There is no such triangle.

39. A circle has circumference 4π. What is its area?

    a) 2π

    b) 4π

    c) 8π

    d) $4\pi^2$

    e) 64π

40. The algebraic expression $\dfrac{\dfrac{1}{x+1} + \dfrac{2}{x-1}}{3x+1}$ is equal to

    a) $\dfrac{3}{(x^2-1)(3x+1)}$

    b) $\dfrac{3}{x(3x+1)}$

    c) $\dfrac{3x-1}{3x+1}$

    d) $\dfrac{1}{x^2-1}$

    e) 1