TWENTY-FIFTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST, 1981

Scoring Formula: 4R - W + 40

Edited by: Otto Bassler
Delmer DeBoer
Horace E. Williams

This test was prepared from a list of Comprehensive questions submitted by Vanderbilt University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

Acme Boot Company, Clarksville, Tennessee
Anderson Ford, Inc., Kingsport, Tennessee
Berkline Company, Morristown, Tennessee
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee
Coca-Cola Bottling Company of Memphis, Memphis, Tennessee
Commercial and Industrial Bank, Memphis, Tennessee
Department of Mathematics, Shelby State Community College, Memphis, TN
Exxon Company, U.S.A., Memphis, Tennessee
First National Bank of Sullivan County, Kingsport, Tennessee
First National Bank, Jefferson City, Tennessee
First People's Bank, Jefferson City, Tennessee
Fulton Sylphon Division, Robertshaw Controls, Knoxville, Tennessee
Great Lakes Research Corporation, Elizabethton, Tennessee
Harris Foundation, Johnson City, Tennessee
Holiday Inns, Inc. Memphis, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
Home Federal Savings and Loan Association, Knoxville, Tennessee
IBM Corporation, Chattanooga, Tennessee
IBM Corporation, Kingsport, Tennessee
IBM Corporation, Nashville, Tennessee
Jefferson County Bank, Dandridge, Tennessee
Johnson City Spring and Bedding Company, Johnson City, Tennessee
Klopman Mills, Incorporated, Johnson City, Tennessee
McDonald's Restaurants, Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-O-TOM), Memphis, Tennessee
Morristown Rotary Club, Morristown, Tennessee
New Jersey Zinc Company, Jefferson City, Tennessee
Provident Life and Accident Insurance Company, Chattanooga, Tennessee
Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, TN
Sears, Madison, Tennessee
Shoney's Inc., Nashville, Tennessee
Tennessee Eastman Company, Kingsport, Tennessee
Tennessee Handbag Company, Dandridge, Tennessee
Tri-State Container Corporation, Elizabethton, Tennessee
TRW, Ross Gear Division, Lebanon, Tennessee
Mr. Meeks B. Vaughan, Kingsport, Tennessee

1. In right ΔABC, the hypotenuse is 20 units and one of the angles measures 30°. The area of the triangle in square units is

   (a) $200\sqrt{3}$

   (b) 200

   (c) $100\sqrt{3}$

   (d) 100

   (e) $50\sqrt{3}$

2. In a group of 200 college students, 138 are enrolled in a course in psychology, 115 are enrolled in sociology, and 91 are enrolled in both courses. How many of these students are not enrolled in either course?

   (a) 0

   (b) 38

   (c) 53

   (d) 62

   (e) 85

3. Find the value of x which satisfies the equation, $\log_{27} 9 = x$.

   (a) 2/3

   (b) -1

   (c) $\sqrt[27]{9}$

   (d) 3

   (e) 3/2

4. The algebraic expression $3(x^2 - 1)^{-\frac{1}{2}}(x + 1) + (x^2 - 1)^{-\frac{1}{2}}$ is equivalent to

   (a) $\dfrac{3(x + 1) + (x^2 - 1)^{\frac{1}{2}}}{(x^2 - 1)^{\frac{1}{2}}}$

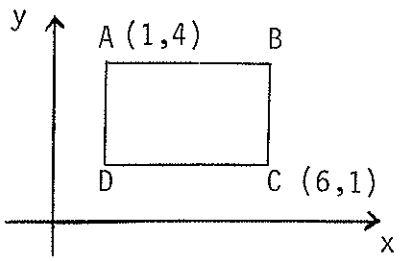
   (b) $\dfrac{(x + 1) + (x^2 - 1)^{\frac{1}{2}}}{3(x^2 - 1)^{\frac{1}{2}}}$

   (c) $\dfrac{x + 1}{3(x^2 - 1)^{\frac{1}{2}}} + (x^2 - 1)^{\frac{1}{2}}$

   (d) $\dfrac{(x + 1)(x + 2)}{(x^2 - 1)^{\frac{1}{2}}}$

   (e) $\dfrac{3x + 4}{(x^2 - 1)^{\frac{1}{2}}}$

5. For the rectangle shown, $\overline{AB}$ is parallel to the x-axis and $\overline{BC}$ is parallel to the y-axis. The area of the rectangle is



(a) 18 sq. units

(b) 15 sq. units

(c) 12 sq. units

(d) 10 sq. units

(e) none of the above

6. When simplified, the expression $\frac{(1/a) + (1/b)}{(1/a^2) - (1/b^2)}$ is equal to

(a) $a + b$

(b) $a - b$

(c) $\frac{ab}{a + b}$

(d) $\frac{ab}{a - b}$

(e) $\frac{ab}{b - a}$

7. A set of three positive integers, x, y, z, is called a Primitive Pythagorean Triple provided $x^2 + y^2 = z^2$ and provided x, y, z have no common integral factors greater than 1. Which of the following is <u>not</u> a Primitive Pythagorean Triple?

(a) 3, 4, 5

(b) 5, 12, 13

(c) 20, 21, 29

(d) 35, 84, 91

(e) 45, 28, 53

8. Insert two geometric means between 6 and 48.

(a) $12\sqrt{2}$, 48

(b) 20, 34

(c) 14, 28

(d) 27, 27

(e) 12, 24

9. Simplify the expression $\left(\frac{27x^{-9}y^{6}}{8x^{-3}y^{3}}\right)^{2/3}$. Leave the result with no negative and no zero exponents.

(a) $\frac{3\sqrt{3}\, y^{8/3}}{2\sqrt{2}\, x^{1/3}}$

(b) $\frac{9y^2}{4x^4}$

(c) $\frac{9y^{4/3}}{4x^2}$

(d) $\frac{3y^{4/3}}{2x^2}$

(e) $\frac{9x^{-4}}{4y^2}$

10. Give the largest solution set of real numbers for the following inequality: $3x^2 + 4x + 2 > 0$.

(a) all real x-values

(b) no real x-values

(c) $\{x \mid x \geq 0\}$

(d) $\{x \mid \frac{-2 - \sqrt{2}}{3} < x < \frac{-2 + \sqrt{2}}{3}\}$

(e) $\{x \mid x < \frac{-2 - \sqrt{2}}{3} \text{ or } x > \frac{-2 + \sqrt{2}}{3}\}$

11. Evaluate: $\lim_{x \to 2} \frac{x^2 + x - 6}{x^2 - 4}$. The value is

(a) 0

(b) 5/4

(c) does not exist

(d) undefined because the numerator is zero

(e) 3/2

12. Find the values of x and y so that each determinant will be equal to 3. $A = \begin{vmatrix} 4 & 5 \\ y & x \end{vmatrix}$, $B = \begin{vmatrix} -1 & -x \\ 2 & y \end{vmatrix}$. $(x + y)$ is then equal to

(a) 4

(b) 1

(c) -1

(d) 3

(e) 2

13. Let C be the circle with center at the origin and radius 5. The line which is tangent to C at (3, 4) intersects the x-axis and the y-axis, respectively, at the points

(a) (10, 0) and (0, 12)

(b) (25/3, 0) and (0, 25/4)

(c) (16, 0) and (0, 9)

(d) (9, 0) and (0, 16)

(e) (25/4, 0) and (0, 25/3)

14. A central angle of a circle of radius 13 inches intercepts an arc of 41 inches. The measure of the central angle in degrees to the nearest tenth of a degree is

(a) 18.2

(b) 3.2

(c) 180.7

(d) 41.0

(e) $\frac{180}{\pi}$

15. The equation of the perpendicular bisector of the line segment $\overline{PQ}$ from point P(-3, 5) to point Q(1, -4) is

(a) $4x - 9y - 40 = 0$

(b) $9x + 4y + 7 = 0$

(c) $8x - 18y + 17 = 0$

(d) $y = \frac{4}{9}x$

(e) $x = -1$

16. The domain of the function $f(x) = \sqrt{\frac{x - 2}{x + 1}}$ in interval notation is

(a) $x \in (-\infty, -1] \cup [2, \infty)$

(b) $x \in [-1, 2]$

(c) $x \in [2, \infty)$

(d) $x \in (-\infty, -1) \cup [2, \infty)$

(e) $x \in (-\infty, -1) \cup (-1, \infty)$

17. The measure in <u>radians</u> of all angles between -360^0 and 360^0 for which $\sec \theta = 2$, is

(a) $\frac{-11\pi}{6}, \frac{-\pi}{6}, \frac{\pi}{6}, \frac{11\pi}{6}$

(b) $\frac{-5\pi}{3}, \frac{-4\pi}{3}, \frac{\pi}{3}, \frac{2\pi}{3}$

(c) $\frac{-11\pi}{6}, \frac{-7\pi}{6}, \frac{\pi}{6}, \frac{5\pi}{6}$

(d) $\frac{-5\pi}{3}, \frac{-\pi}{3}, \frac{\pi}{3}, \frac{5\pi}{3}$

(e) $-300^0, -60^0, 60^0, 300^0$

18. When simplified, $\csc 2t + \cot 2t$ (where defined) equals

(a) $\tan t$

(b) $\sec t$

(c) $\csc^2 t$

(d) $\cot t$

(e) $2 \csc t + 2 \cot t$

19. Determine all x such that $\frac{x - 2}{x - 3} < 1$.

(a) $x > 3$

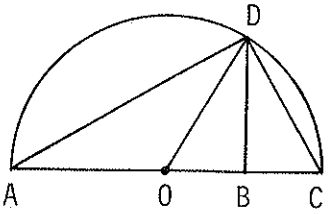
(b) $2 < x < 3$

(c) $2 \leq x \leq 3$

(d) $x \leq 3$

(e) $x < 3$

20. In the semi-circle with center at O, the length of $\overline{AB}$ is a and the length of $\overline{BC}$ is b. $\overline{BD}$ is perpendicular to $\overline{AC}$. The arithmetic mean and the geometric mean of a and b are, respectively,



(a) OC and OB

(b) OD and BD

(c) AD and DC

(d) AD and BD

(e) OD and BC

21. If f and g are functions such that $g(x) = e^x$ and $f(g(x)) = x$, then $f(x)$ equals

(a) e^x

(b) $\log_e x$

(c) $\log_x e$

(d) xe^x

(e) x/e^x

22. The solution set for the inequality $x^2 - 3x - 4 > 0$ is

(a) $\{x|-1 < x < 4\}$

(b) $\{x|x < -1 \text{ or } x > 4\}$

(c) $\{x|x < 4\}$

(d) $\{x|x < -1 \text{ or } x < 4\}$

(e) $\{x\ x < -1\}$

23. Give the complete solution set for θ if $\tan \theta = 1$.

(a) $\{\frac{\pi}{4} + k\pi$, where k is any integer$\}$

(b) $\{\frac{\pi}{4} + 2k$, where k is any integer$\}$

(c) $\{\frac{\pi}{4} + k\pi$ or $\frac{3\pi}{4} + k\pi$, where k is any integer$\}$

(d) $\{\frac{\pi}{4}, \frac{5\pi}{4}, \frac{9\pi}{4}, \frac{13\pi}{4}, \ldots\}$

(e) $\{\pm \frac{\pi}{4}, \pm \frac{5\pi}{4}, \pm \frac{9\pi}{4}, \pm \frac{13\pi}{4}, \ldots\}$

24. If $\sec \alpha = 5/4$, $\csc \beta = 2/\sqrt{3}$, $0 \leq \alpha \leq \pi/2$, $0 \leq \beta \leq \pi/2$, then $\cos(\alpha - \beta)$ is equal to

(a) $\frac{3 + 4\sqrt{3}}{10}$

(b) $\frac{3\sqrt{3}}{25}$

(c) $\frac{4 + 3\sqrt{3}}{10}$

(d) $\frac{3 - 4\sqrt{3}}{10}$

(e) $\frac{4 - 3\sqrt{3}}{10}$

25. If $\log_a b^2 = c^2$, $b > 0$, $c \neq 0$, then $\log_b a$ is equal to

(a) $2/c^2$

(b) $2c^2$

(c) $c/2$

(d) c^2

(e) $\sqrt{c}$

26. Find all values for x that satisfy the equation $\sec^2 x + \tan^2 x = 1$ in the interval $0 \leq x \leq 2\pi$.

(a) $\frac{\pi}{2}, \frac{3\pi}{2}$

(b) $0, \pi, 2\pi$

(c) $0, 2\pi$

(d) all values of x in the interval

(e) π

27. $f\left(\frac{x}{x + 1}\right) = x^2$, then $f(x)$ is

(a) $\frac{x}{1 + x}$

(b) $\frac{x}{1 - x}$

(c) $\frac{x^2}{(1 + x)^2}$

(d) $\frac{x^2}{(1 - x)^2}$

(e) x^2

28. The solution set for the following system of equations

$$\begin{cases} x \sin \alpha + y \cos \alpha = 3 \\ x \cos \alpha - y \sin \alpha = 4 \end{cases}$$ is

(a) $x^2 + y^2 = 25$

(b) $x^2 - y^2 = 25$

(c) $(x + y)^2 = 0$

(d) $(x - y)^2 = 0$

(e) $(x + y)^2 = 25$

29. The limit of the sum, $\frac{1}{n^2} + \frac{2}{n^2} + \frac{3}{n^2} + \cdots + \frac{n}{n^2}$, as $n \to \infty$ is

(a) 2

(b) 1/2

(c) 1

(d) 0

(e) ∞

30. When simplified, the expression $e^{-\log_e x^2}$ is equal to

(a) x^2

(b) 2.718

(c) $\frac{1}{x^2}$

(d) e^{x^2}

(e) -1

31. The complete solution set of $\left|\frac{3 - 2x}{2 + x}\right| \leq 4$ is

(a) $[\frac{-5}{6}, \infty)$

(b) $[\frac{-11}{2}, \frac{-5}{6}]$

(c) $(-\infty, \frac{-11}{2}]$

(d) $(-\infty, \frac{5}{6}] \cup [\frac{11}{2}, \infty)$

(e) none of the above

32. The point on the circle $4x^2 + 4y^2 - 8x = 1$ which is nearest to the point (2, 2) is

(a) (3/2, 1)

(b) (3/2, -1)

(c) (1/2, -1)

(d) (1/2, 1)

(e) (1, 0)

33. The sum of the coefficients in the expansion of $(x - 2y)^7$ is

(a) -1

(b) +1

(c) +7

(d) -2^7

(e) -3^7

34. A man is in a boat 6 miles out in the water from the nearest point on a straight shore, point A, and he wishes to reach a point B that is 12 miles up the shore from point A. He can row at 4 mph and walk at 5 mph. If he rows straight to a point C on the shore that is x miles from point A and then walks the rest of the way, it will take t hours. Then t, expressed in terms of x is

(a) $4\sqrt{x^2 + 36} + 5(12 - x)$

(b) $\frac{\sqrt{x^2 + 36}}{4} + \frac{12 - x}{5}$

(c) $\frac{x^2 + 36}{4} + \frac{12 - x}{5}$

(d) $\frac{x}{4} + \frac{12 - x}{5}$

(e) $\frac{27}{10}$

35. What relation exists between the area A of a triangle with sides of length 13, 13, and 10 units and the area B of a triangle with sides of lengths 13, 13, and 24 units?

(a) $A < B$

(b) $A > B$

(c) $A = B$

(d) The relation cannot be determined from the given information.

(e) It is impossible to construct such triangles.

36. When simplified to lowest terms, the value of the expression $(2\sqrt{3} + 5\sqrt{7})^8 + (2\sqrt{3} - 5\sqrt{7})^8$ is

(a) an even integer

(b) an odd integer

(c) a rational number but not an integer

(d) a negative irrational number

(e) a positive irrational number

37. The product $(\cos a)(\cos 2a)(\cos 4a)(\cos 8a) \ldots (\cos 2^{n-1}a)$ equals

(a) $\cos 2^n a$

(b) $\dfrac{\cos 2^n a}{\sin a}$

(c) $\dfrac{\sin 2^n a}{2^n \sin a}$

(d) $\dfrac{\cos 2^n a}{2^n \cos a}$

(e) none of the above

38. The smallest positive integer with exactly 28 different positive integer divisors is

(a) 28!

(b) 28

(c) 2^{27}

(d) $2^6 \cdot 3 \cdot 5$

(e) $2^6 \cdot 3^3$

39. The numerical value of $\sqrt{2\sqrt{2\sqrt{2\sqrt{2}\ldots}}}$ is

(a) $2\sqrt{2}$

(b) $\sqrt{2}$

(c) 1

(d) 2

(e) 1/2

40. The equation $x^3 + ax^2 + (10 + 20i)x + b = 0$ has roots $3 + i$ and $3 - i$. The third root is

(a) 10/3

(b) 5

(c) $5i$

(d) $10i$

(e) $(10/3)i$