TWENTY-FOURTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST 1980

Scoring Formula: 4R - W + 40

Edited by Bruce Myers and Leon McQueen,
Austin Peay State University, Clarksville,
Tennessee

This test was prepared from a list of Comprehensive questions submitted by University of Tennessee at Martin.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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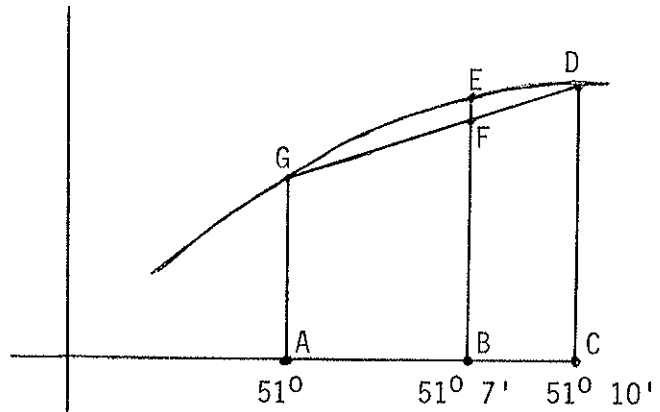Mr. Meeks B. Vaughan, Kingsport, Tennessee

1. For the triangle shown, the numerical ratio that corresponds to a : b : c is

   (a) $1 : 2 : \sqrt{3}$

   (b) $1 : \sqrt{2} : 2$

   (c) $\sqrt{2} : 2 : 1$

   (d) $1 : \sqrt{3} : 2$

   (e) $\sqrt{3} : 1 : 2$
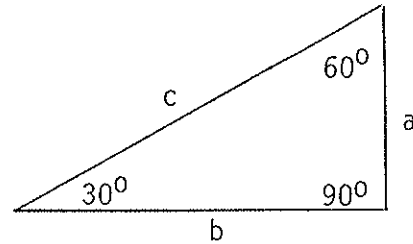
2. For the diagram shown, if one is using interpolation to find the value of $\sin 51^0\ 7'$ when $\sin 51^0$ and $\sin 51^0\ 10'$ are known, then the resulting number will be the length of which one of the following line segments?

   (a) $\overline{BF}$

   (b) $\overline{BE}$

   (c) $\overline{GD}$

   (d) $\overline{AB}$

   (e) $\overline{AC}$

3. The set of all solutions for
   $$2 \sin 2x - 2 \sin x - 2\sqrt{3} \cos x + \sqrt{3} = 0$$
   which are contained in the interval $[0, \frac{\pi}{2}]$ is

   (a) $\left\{ \frac{\pi}{6}, \frac{\pi}{3} \right\}$

   (b) $\left\{ \frac{\pi}{3} \right\}$

   (c) $\left\{ \frac{\pi}{6} \right\}$

   (d) $\emptyset$

   (e) none of the above

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4. What is the value for $\dfrac{\sin^2\theta + \cos^2\theta}{\cos^2\theta - \sin^2\theta}$?

(a) $\dfrac{1}{1 + 2\cos^2\theta}$

(b) 1

(c) $\dfrac{1}{\cos 2\theta}$

(d) $\dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta}$

(e) none of the above

5. If the matrix is $\begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$, then the determinant of the matrix is

(a) 0

(b) 1

(c) 2

(d) -2

(e) none of the above

6. If the $\sin\theta = \dfrac{8}{17}$ and θ is in quadrant II, then $\cos\theta$ is

(a) $-\dfrac{17}{8}$

(b) $\dfrac{15}{17}$

(c) $\dfrac{17}{8}$

(d) $-\dfrac{15}{17}$

(e) none of the above

7. A single fair die is rolled three times. What is the probability that at least one two results?

   (a) $\frac{5}{6}$

   (b) $\frac{25}{36}$

   (c) $\frac{91}{216}$

   (d) $\frac{75}{216}$

   (e) $\frac{30}{216}$

8. In a geometric progression the seventh term is 320 and the ninth term is 1280. The first four terms of the progression could be

   (a) 2, 4, 8, 16

   (b) 5, 10, 15, 20

   (c) 5, 10, 20, 40

   (d) 24, 36, 54, 81

   (e) none of the above

9. Consider the equation $x^2 + ax + 1 = 0$. A single fair die is rolled to determine the value of the coefficient a. The value chosen for a is the number of dots on the face of the die which turns up. The probability that the equation will have real, unequal roots is

   (a) $\frac{1}{12}$

   (b) $\frac{5}{6}$

   (c) $\frac{1}{2}$

   (d) $\frac{1}{3}$

   (e) $\frac{2}{3}$

10. If the graph of the equation $cy = dx^2 - 4$ passed through the points $(2, 0)$ and $(-4, 3)$, then the value of c is

    (a) -20

    (b) 0

    (c) $-\frac{9}{2}$

    (d) $-\frac{20}{3}$

    (e) 4

11. What is the intersection of the cylinder $x^2 + y^2 = 4$ with the plane $y = 2$?

    (a) a point

    (b) a circle

    (c) a plane

    (d) a line

    (e) They do not intersect.

12. The graph of the equation $(x - 1)(y + 1) = 7$ is

    (a) a hyperbola

    (b) a parabola

    (c) an ellipse

    (d) a circle

    (e) a straight line

13. Transform $r = 2 \cos \theta$ from polar to rectangular coordinates.

    (a) $x^2 + y^2 + 2y = 0$

    (b) $x^2 + y^2 + 2x = 0$

    (c) $x^2 + y^2 - 2y = 0$

    (d) $x^2 + y^2 - 2x = 0$

    (e) $x^2 + y^2 - 4 = 0$

14. The graph of $y = 4 - \cos 3x$ crosses the y-axis at the point where y is

    (a) 4

    (b) -1

    (c) 1

    (d) 3

    (e) 5

15. If x is a positive number less than 180^0, what are all values of x for which $|\cos x| > |\sin x|$?

    (a) $45^0 < x < 135^0$

    (b) $0^0 < x < 45^0$ or $135^0 < x < 180^0$

    (c) $90^0 < x < 180^0$

    (d) $45^0 < x < 90^0$

    (e) none of the above

16. For all real x, $f(x) = \frac{1}{3}x^3 - x - 1$. Find the values of x for which the function is increasing.

    (a) $|x| > 1$

    (b) $x \geq 1$ or $x \leq -1$

    (c) $x > 1$

    (d) $-1 < x < 1$

    (e) none of the above

17. For all real values of a for which $\begin{vmatrix} 1 & 0 & 2 \\ -1 & a & -2 \\ a & 1 & 2 \end{vmatrix} = 0$.

    (a) $a = 0, -1$

    (b) no real values

    (c) $a = 0, 1$

    (d) $a = 0, 1, -1$

    (e) none of the above

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18. Find the solution set of the inequality $\dfrac{x - 2}{x^2(1 - x)} \leq 0$.

    (a) $x \leq 2$

    (b) $x < 1$ or $x > 2$

    (c) $1 < x \leq 2$

    (d) $0 < x < 2$

    (e) none of the above

19. The possible set of all possible rational roots of $3x^4 - x^3 + x^2 + x = 4$ is

    (a) $\left\{1, -1, \frac{1}{3}, -\frac{1}{3}\right\}$

    (b) $\left\{-\frac{1}{3}, -\frac{2}{3}, -\frac{4}{3}, -1, -2, -4\right\}$

    (c) $\left\{1, -1, \frac{1}{3}, -\frac{1}{3}, 2, -2, \frac{2}{3}, -\frac{2}{3}, 4, -4, \frac{4}{3}, -\frac{4}{3}\right\}$

    (d) $\left\{0, 1, -1, \frac{1}{3}, -\frac{1}{3}\right\}$

    (e) none of the above

20. The value of y where $y = 10 \cdot 2^{-1/3 \log_2 x}$ when $x = 8$ is

    (a) $\dfrac{5}{256}$

    (b) -20

    (c) $-\dfrac{10}{3}$

    (d) 5

    (e) none of the above

21. If $f'(x) = \lim\limits_{h \to 0} \dfrac{1}{h}\left[\dfrac{3}{2 - x - h} - \dfrac{3}{2 - x}\right]$, then $f'(x) =$

    (a) 3

    (b) $-\dfrac{3}{(2 - x)^2}$

    (c) $\dfrac{3}{(2 - x)^2}$

    (d) Indeterminate

    (e) none of the above

22. If $\log_e x = -1$, then the value of x is

   (a) 0.1

   (b) $\dfrac{1}{2.7183}$

   (c) $\dfrac{1}{e}$

   (d) 10

   (e) none of the above

23. The solution set for the inequality $2^{x^2-x-2} < 16$ is

   (a) $\{x \mid x < -2 \ \text{ or } \ x > 3\}$

   (b) $\{x \mid x > 3\}$

   (c) $\{x \mid x < -2\}$

   (d) $\{x \mid x > -2 \ \text{ and } \ x < 3\}$

   (e) none of the above

24. The graph of $y = \frac{1}{3}x^3 - x^2 + 8x + 2$ has a point of inflection at

   (a) 1

   (b) 4

   (c) -2

   (d) 2

   (e) This graph has no point of inflection.

25. Let $p = \dfrac{x + 1}{x}$, $\dfrac{1}{100} \leq x \leq 100$. For what value of x does p have its smallest value?

   (a) $\dfrac{1}{100}$

   (b) 1

   (c) 2

   (d) 100

   (e) $\dfrac{1}{2}$

26. The set of all real numbers in the interval $[0, 2\pi]$ which satisfy the equation

$$\cos x \sin 2x + \sin x \sin 3x + \cos 2x \sin x = 0$$

is

(a) $\{0, \frac{\pi}{3}, \frac{3\pi}{2}\}$

(b) $\{\frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}\}$

(c) $\{0, \frac{3\pi}{2}\}$

(d) $\{0, \frac{\pi}{3}, \frac{2\pi}{3}, \frac{3\pi}{2}, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}, 2\pi\}$

(e) none of the above

27. If $\log_8 M + \log_8 \frac{1}{6} = \frac{2}{3}$, what is M?

(a) 48

(b) $\frac{2}{3}$

(c) 4

(d) 24

(e) $\frac{23}{6}$

28. One root of the equation $x^2 + ax + a = 0$ is 7. What is the other root?

(a) $-\frac{7}{8}$

(b) $\frac{49}{8}$

(c) $\frac{7}{8}$

(d) -7

(e) $-\frac{49}{8}$

29. P and Q each toss three fair coins. What is the probability that P and Q obtain the same number of heads?

(a) $\frac{1}{9}$

(b) $\frac{5}{16}$

(c) $\frac{3}{8}$

(d) $\frac{1}{4}$

(e) $\frac{3}{16}$

30. The simplified form of $\dfrac{\dfrac{0 - \dfrac{1}{x^2}}{1 + \dfrac{1}{x}}}{\log x}$ is

   (a) $-x(x + 1)\log x$

   (b) $\dfrac{-1}{x(x + 1)\log x}$

   (c) $\dfrac{-x(x + 1)}{\log x}$

   (d) $\dfrac{-\log x}{x(x + 1)}$

   (e) none of the above.

31. A tank in the form of an inverted cone has base radius 5 m. and height 30 m. The tank is filled with water and then water is allowed to drain from the tank for a period of time. A metal sphere of diameter 4 m. is placed in the tank. The height of the water in the tank is then 18 m. The volume of the water left in the tank is

   (a) 54

   (b) $\dfrac{130\pi}{3}$

   (c) $\dfrac{32\pi}{3}$

   (d) $\dfrac{64\pi}{3}$

   (e) none of the above

32. The vertical asymptote(s) for the graph of $y = \dfrac{x^2 + 3x + 2}{x^2 - 4x - 12}$ is(are)

   (a) $x = 6$

   (b) $x = -2$

   (c) $x = 6$ and $x = -2$

   (d) $x = -2$ and $x = -1$

   (e) $x = -2$, $x = -1$, and $x = 6$

33. If $\begin{vmatrix} 4x & 5 \\ x & x \end{vmatrix} = 9$, then the sum of the values for x is

   (a) -5

   (b) $\dfrac{5}{4}$

   (c) $\dfrac{4}{5}$

   (d) 5

   (e) none of the above

34. The vertices of the polygon formed by

$$x + 2y \leq 6$$

$$3x + 2y \leq 10$$

$$x \geq 0$$

$$y \leq 0$$

are

(a) $(0, 0)$, $(0, 3)$, $(\frac{10}{3}, 0)$, $(2, 2)$

(b) $(0, 0)$, $(0, 5)$, $(6, 0)$, $(2, 2)$

(c) $(0, 0)$, $(3, 0)$, $(6, 0)$, $(2, 2)$

(d) $(0, 0)$, $(\frac{10}{3}, 0)$, $(6, 0)$, $(2, 2)$

(e) $(0, 0)$, $(\frac{10}{3}, 0)$, $(0, 5)$, $(2, 2)$

35. Suppose r is doubled and h is tripled. If $V = \pi r^2 h$, V is multiplied by a factor of

(a) 18   (d) 36

(b) 6   (e) none of the above

(c) 12

36. Given the repeating decimals $x = 0.\overline{52}$ and $y = 2.\overline{4}$, then $x + y =$

(a) $\dfrac{294}{99}$ (d) $\dfrac{296}{100}$

(b) $\dfrac{292}{100}$ (e) none of the above

(c) $\dfrac{290}{99}$

37. If $f(x) = \dfrac{1}{x^2 + 1}$ and $g(x) = \sin x + \cos x$, then

(a) $f[g(x)] = \dfrac{1}{\sin(x^2 + 1) + \cos(x^2 + 1) + 1}$

(b) $f[g(x)] = \dfrac{1}{\sin^2 x + \cos^2 x + 1}$

(c) $f[g(x)] = \dfrac{1}{\sin x^2 + \cos x^2 + 1}$

(d) $f[g(x)] = \dfrac{1}{\sin 2x + 2}$

(e) none of the above

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38. The possible number of negative real roots for the equation

$$x^5 + 3x^4 - x^2 + 6x + 4 = 0$$

is

(a) 0

(b) 5 or 3 or 1

(c) 3 or 1

(d) 5

(e) none of the above

39. If the population of a particular species doubles each 1/4 hour and two are present in a culture at noon and 131,072 at 4 p.m., at what time will the population of this culture be 65,536?

(a) 2 p.m.

(b) 3 p.m.

(c) 3:45 p.m.

(d) 4:00 p.m.

(e) none of the above

40. A United States Senator has polled 100 of his constituents to determine which course he should pursue. Partial results from his poll are as follows:

1) 12 people favor the increased construction of nuclear plants only.

2) 20 people think that both solar energy research and increased nuclear plant construction are desirable.

3) 22 people favor both tax credits for oil companies and increased nuclear plant construction.

4) 14 people would like to see all three areas pursued.

From this information, determine the total number of people who favored increased nuclear plant construction.

(a) 40

(b) 12

(c) 14

(d) 28

(e) 100