TWENTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

1978

Scoring Formula: 4R - W

EDITED BY:

Bruce Myers
and
Jim Ridenhour
Austin Peay State University
Clarksville, Tennessee

This test was prepared from a list of Comprehensive questions submitted by David Lipscomb College and Austin Peay State University.

DIRECTIONS:

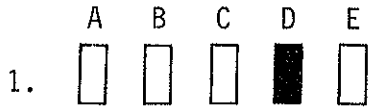
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a) 2/3. (b) 3. (c) 6.

   (d) 3/2 (e) none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. What is the solution for y in the following system of equations?

$$\begin{aligned} 2x - y + z &= 3 \\ x + 2y - z &= 3 \\ 3x - 4y + 2z &= -1 \end{aligned}$$

(a) -15

(b) -5

(c) -3

(d) 3

(e) 15

2. $3 \log_4 4 - 2 \log_4 8 =$

(a) $\log_4(-4)$

(b) $\log_4 0$

(c) $\log_4 1$

(d) $3/2 \log_4 5$

(e) none of the above

3. How many times does the graph of $f(x) = 3 \cos(\frac{x}{2})$ intersect the x-axis for $0 \leq x < 2\pi$?

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

4. Let $f(x) = 3x^2 - 3x - 6$. At what real number x does the function take its minimum value?

(a) -6

(b) 2

(c) 1

(d) $\frac{1}{2}$

(e) $-\frac{1}{2}$

5. The solution set for the equation $x^3 + 1 = 0$ is

   (a) $\left\{1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}\right\}$

   (b) $\left\{1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right\}$

   (c) $\left\{-1, \frac{1 + i\sqrt{3}}{2}, \frac{1 - i\sqrt{3}}{2}\right\}$

   (d) $\left\{-1, \frac{-1 + i\sqrt{3}}{2}, \frac{-1 - i\sqrt{3}}{2}\right\}$

   (e) none of the above

6. In what base is the equation $12 \times 5 = 104$ true?

   (a) 5

   (b) 6

   (c) 10

   (d) 56

   (e) none of the above

7. The product of the two complex numbers $[2(\cos 20^0 + i \sin 20^0)]$ and $[3(\cos 40^0 + i \sin 40^0)]$ is

   (a) $1 + i\sqrt{3}$

   (b) $2 + 2i\sqrt{3}$

   (c) $3 + 3i\sqrt{3}$

   (d) $6 + 6i\sqrt{3}$

   (e) none of the above

8. The inverse of $\sim p \to q$ is equivalent to

   (a) $\sim p \to \sim q$

   (b) $\sim q \to \sim p$

   (c) $q \to p$

   (d) $p \to q$

   (e) none of the above

9. What is the sum of all multiples of 10 between 99 and 999?

   (a) 48,505

   (b) 49,050

   (c) 49,410

   (d) 49,595

   (e) 49,959

10. How many distinguishable permutations can be made from the letters of the word WEEKEND if they are all used every time?

   (a) 35

   (b) 210

   (c) 840

   (d) 1680

   (e) 5040

11. If $\sin x = \frac{2}{3}$ and $0 < x < \frac{\pi}{2}$, then $(2 \sin 2x - 2 \cos x)$ is equal to

   (a) $\frac{8 - 2\sqrt{5}}{3}$

   (b) $\frac{8\sqrt{3}}{2}$

   (c) $\frac{2\sqrt{5}}{9}$

   (d) $\frac{10}{9}$

   (e) $\frac{2\sqrt{5}}{27}$

12. If $\log_2 [\log_{10} x] = 2$, then

   (a) $x = 1$

   (b) $x = 10{,}000$

   (c) $x = 16$

   (d) $x = 4$

   (e) none of the above

13. If a square of side 4 is revolved about a diagonal through a 360^{o} revolution, a vertex on the other diagonal will trace a distance of

(a) $4\sqrt{2}\,\pi$

(b) $2\sqrt{2}\,\pi$

(c) $8\sqrt{2}\,\pi$

(d) 4π

(e) none of the above

14. How many card hands of 5 cards can be selected from a 52-card deck?

(a) $52!$

(b) $\frac{52!}{47!5!}$

(c) $\frac{52!}{47!}$

(d) $\frac{52!}{5!}$

(e) $5!$

15. If x is a real number other than zero, we can infer that

(a) $x^2 > x$

(b) $x^n > x$ for sufficiently large n

(c) $x^2 > x$ if $x > 0$

(d) $x^2 < x$ if $x < 1$

(e) none of the above

16. Solve the equation $e^{-x \log_e 5} = 25$ for x.

(a) ± 5

(b) e^5

(c) 5^e

(d) $\log_e 5$

(e) -2

17. The solution set for $\log_{10}(x + 3) - \log_{10}x = 1$ is

(a) $\{\frac{1}{3}\}$

(b) $\{-5, 2\}$

(c) $\{2\}$

(d) $\emptyset$

(e) none of the above

18. If the arithmetic mean and geometric mean of two numbers, a and b, are equal, then it is necessary that the numbers be

(a) 1 and -1

(b) opposite in sign

(c) complex

(d) 1 and 1

e. none of the above are necessary

19. What is the largest functional value for $f(x) = -x^2 + 8x - 7$ where x is any real number?

(a) 4

(b) -4

(c) 9

(d) -7

(e) There is no largest value.

20. The solution set for the inequality $\frac{2x + 3}{x - 4} \geq 0$ is

(a) $\{x : x \geq -\frac{3}{2}\}$

(b) $\{x : x \leq -\frac{3}{2}\}$

(c) $\{x : x \leq -\frac{3}{2} \text{ or } x > 4\}$

(d) $\{x : x \leq -\frac{3}{2} \text{ or } x \geq 4\}$

(e) $\{x : x \neq 4\}$

21. The solution set of the equation $\log_9 x = \{\log_{16} 4\}\{\log_4 64\}$ is

(a) $\{18\}$

(b) $\{27\}$

(c) $\{81, 9\}$

(d) no solution

(e) $\{9\}$

22. If x and h are positive real numbers, then $\sqrt{x + h} - \sqrt{x}$ is equal to

(a) $\sqrt{x}$

(b) $\dfrac{h}{\sqrt{x + h} + \sqrt{x}}$

(c) 0

(d) h

(e) $\sqrt{h}$

23. What is the coefficient of x^{-3} in the binomial expansion of $(x - \frac{m}{x})^{11}$?

(a) $-330\ m^7$

(b) $-924\ m^7$

(c) $-495\ m^4$

(d) $-330\ m^8$

(e) $165\ m^8$

24. Let C(n,r) represent the binomial coefficient $\dfrac{n!}{r!(n - r)!}$.

What is the value of the sum $\sum_{r=0}^{25} C(n,r)(-1)^r\ 2^r\ 3^{25-r}$?

(a) 0

(b) 2835

(c) 625

(d) 1

(e) 5^{25}

25. A basketball player hits an average of 70% of his free throws. What is the probability that he will hit 2 of the next 4 shots?

(a) .0441

(b) .4900

(c) .2100

(d) .2646

(e) none of the above

26. The solution set of the equation $2^{2x^2-5x} = 8$ is

(a) $\emptyset$

(b) $\{3\}$

(c) $\{-1, 4\}$

(d) $\left\{-\frac{1}{3}, \frac{3}{8}\right\}$

(e) $\left\{-\frac{1}{2}, 3\right\}$

27. Factor $a^n - b^n$ (n a natural number).

(a) $(a - b)^n$

(b) $(a - b)(a^{n-1} + b^{n-1})$

(c) $(a + b)(a^{n-1} - a^{n-2}\, b + a^{n-3}\, b^2 - \cdots + ab^{n-2} - b^{n-1})$

(d) $(a - b)(a^{n-1} + a^{n-2}\, b + a^{n-3}\, b^2 + \cdots + ab^{n-2} + b^{n-1})$

(e) $(a + b)(a^{n-1} - b^{n-1})$

28. Write an equation whose roots are the roots of $x^3 - 6x^2 - x + 30 = 0$, each decreased by one.

(a) $x^3 - 3x^2 - 10x + 24$

(b) $x^2 + 9x - 22$

(c) $x^3 + 3x^2 - 16x + 12$

(d) $x^3 - 3x^2 - 18x + 40$

(e) $x^3 + 3x^2 + 16x - 12$

29. Which of the following functions is not a one-to-one function?

(a) $y = 3x - 2$

(b) $y = \sin x$

(c) $y = \log x$

(d) $y = e^x$

(e) $y = x^3$

30. Fourteen men carry rocks and place them in a truck. If the first man carries 2 rocks, the second man 4 rocks, and so on, where each man is required to carry twice as many as the previous man, what is the total number of rocks placed in the truck?

(a) 8674

(b) 16382

(c) 42728

(d) 32766

(e) 131070

31. The value of the continued fraction $1 + \cfrac{1}{1 + \cfrac{1}{1 + \cfrac{1}{1 + \cdots}}}$ is

(a) $\sqrt{5}$

(b) $\sqrt{3}$

(c) ∞

(d) $\frac{1 - \sqrt{5}}{2}$

(e) $\frac{1 + \sqrt{5}}{2}$

32. A bag contains 3 black balls and 5 red balls. One ball at a time is withdrawn and not replaced until four have been drawn. What is the probability that the third black ball is drawn on the fourth draw.

(a) 1

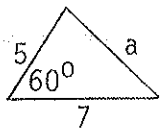
(b) 1/56

(c) 0

(d) 3/14

(e) $\frac{3}{56}$

33. Given a triangle as shown, then a equals



(a) $7\sqrt{\frac{3}{2}}$

(b) $5\sqrt{\frac{3}{2}}$

(c) $\sqrt{39}$

(d) $74 - 35\sqrt{3}$

(e) $\frac{7}{2}$

34. $\cos(2 \sin^{-1}(-\frac{4}{5})) =$

(a) $-\frac{7}{25}$

(b) $\frac{3}{5}$

(c) $\frac{6}{5}$

(d) $-\frac{24}{25}$

(e) $\frac{24}{25}$

35. A student has 4 blue and 6 black socks in a drawer and selects 2 socks at random from the drawer. What is the probability that both socks are of the same color?

(a) $\frac{2}{15}$

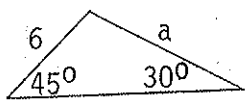
(b) $\frac{1}{3}$

(c) $\frac{7}{15}$

(d) $\frac{1}{2}$

(e) $\frac{2}{5}$

36. Given a triangle as shown, then a equals



(a) $2\sqrt{6}$

(b) $3\sqrt{6}$

(c) $\frac{3}{2}\sqrt{2}$

(d) $3\sqrt{2}$

(e) $6\sqrt{2}$

37. Let T be a set with subsets A, B and C satisfying the following where $\tilde{X}$ denotes the complement of a set X.

subset	$A \cap B$	$A \cap C$	$\tilde{B} \cap C$	$A \cap \tilde{B}$	$\tilde{A} \cap B \cap C$	$B \cup C$	$\tilde{A} \cup B$	$\tilde{A} \cap \tilde{B} \cap \tilde{C}$
number of elements in subset	12	14	11	15	3	30	22	1

The number of elements in $A \cap B \cap C$ is

(a) 26

(b) 0

(c) 25

(d) 5

(e) 7

38. Let f be a quadratic function with domain the closed interval $[a, b]$, $a < b$. If $f(x) = a_2x^2 + a_1x + a_0$, then under what condition is the tangent line at a point P on the graph of f parallel to the line segment drawn between $(a, f(a))$ and $(b, f(b))$.

(a) never

(b) always

(c) when P has x coordinate a or b

(d) when P has x coordinate $\frac{a + b}{2}$

(e) It depends on a_2, a_1, and a_0.

39. If 7^{820} is divided by 10, the remainder is

(a) 7

(b) 9

(c) 3

(d) 0

(e) 1

40. If $f(x) = 2 + \sin x$ and $f[g(x)] = \frac{1}{x}$, then

(a) $g(x) = 2 + \frac{1}{\sin x}$

(b) $g(x) = \frac{1}{2 + \sin x}$

(c) $g(x) = 2 + \sin \frac{1}{x}$

(d) $g(x) = \frac{1}{\sin^{-1}(2 + x)}$

(e) $g(x) = \sin^{-1}(\frac{1 - 2x}{x})$