COMPREHENSIVE TEST

1977

Scoring Formula: 4R - W

EDITED BY:

Billy Edwards
    and
Clinton Smullen
The University of Tennessee
    at Chattanooga
Chattanooga, Tennessee

This test was prepared from a list of Comprehensive questions submitted by Union University.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

    (a) 2/3.   (b) 3.   (c) 6.

    (d) 3/2.   (e) none of these

	A	B	C	D	E
1.				■	

The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. The value of the determinant $\begin{vmatrix} 2 & -3 & -2 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix}$ is

   (a) 7

   (b) 8

   (c) 9

   (d) undefined

   (e) none of these

2. In a geometric progression the seventh term is 320 and the ninth term is 1280. The first four terms of the progression could be

   (a) 5, 10, 15, 20

   (b) 2, 4, 8, 16

   (c) 5, 10, 20, 40

   (d) 320, $320\sqrt[3]{4}$, $320\sqrt[3]{16}$, 1280

   (e) 320, 640, 960, 1280

3. A student either will succeed in college mathematics or will fail.
   Allie does not enjoy college mathematics.
   In order to succeed in college mathematics, it is necessary to work hard.
   A student can work hard only if the student enjoys a subject.

   Which of the following is a valid conclusion from the above premises?

   (a) If a student enjoys college mathematics then the student will succeed in it.

   (b) Allie will succeed in college mathematics.

   (c) Allie will fail in college mathematics.

   (d) Allie will work hard in college mathematics.

   (e) Both a and c.

4. On a day when the wind was a steady 20 miles per hour, an airplane traveled 250 miles against the wind in the same time it took to travel 350 miles with the wind. What is the speed of the airplane in still air?

    (a) 300 miles per hour

    (b) 270 mph

    (c) 120 mph

    (d) 180 mph

    (e) none of the above

5. Assuming that the numbers 2.123 and 12.1 are approximate numbers correct to the significant digits given, the product of these two numbers should be stated as

    (a) 25.6883

    (b) 25.69

    (c) 25.6

    (d) 25.7

    (e) 25.68830

6. The period of the trigonometric function $y = 4 + 2\cos\left[\frac{\pi}{3}(x-7)\right]$ is

    (a) 6 radians

    (b) 2π radians

    (c) $\frac{\pi}{3}$ radians

    (d) 4 radians

    (e) -7 radians

7. If $\log_{27} 9 = x$ then x is equal to

    (a) .9542   (d) 1/2

    (b) 2/3   (e) 1/3

    (c) 3

8. If a polynomial f(x) is divided by the expression (x-3) and the remainder is equal to f(4), are we justified in concluding that f(4) = f(3)?

   (a) yes

   (b) no

   (c) only if f(3) is a root of the polynomial

   (d) only if f(x) is a first degree polynomial

   (e) none of the above

9. On the final day of a sale a merchant hastily disposes of two lamps at the bargain price of $12.00 each. If he made a profit of 25% on one lamp but took a 25% loss on the other lamp, the total transaction resulted in:

   (a) a gain of $1.40

   (b) a loss of $1.40

   (c) a gain of $1.60

   (d) a loss of $1.60

   (e) the merchant breaking even

10. The diagonal of a cube whose volume is numerically equal to its total surface area is:

   (a) $6\sqrt{2}$

   (b) $5\sqrt{3}$

   (c) 6

   (d) $3\sqrt{5}$

   (e) $6\sqrt{3}$

11. At the closest point, what is the distance between the x-axis and the curve $4y = x^2 - 6x + 13$?

   (a) 0

   (b) 3

   (c) $\sqrt{3}$

   (d) 1

   (e) $\sqrt{10}$

12. Given   log(3.22) = .5079
             log(3.23) = .5092

Using linear interpolation in the table, which of the following is the best approximation to log(3.227)?

(a)   .5082

(b)   .5085

(c)   .5088

(d)   .5091

(e)   none of the above

13. In which of the following bases is the computation   (11)(101) = 1111 correct?

                        I.   Two
                       II.   Five
                      III.   Ten

(a)   I only

(b)   III only

(c)   I and III only

(d)   II and III only

(e)   I, II, and III

14. The square root of half the number of bees in a swarm has flown out upon a bush; one female bee flies about a male who was lured into a lotus flower by its sweet odor, but is now imprisoned in it; 8/9 of the swarm remained. The number of bees in the swarm is:

(a)   162

(b)   98

(c)   72

(d)   64

(e)   36

15. If the two solutions of $x^2 + 4x + c = 0$ are real and unequal, which of the following describes all possible values of the constant c?

   (a) $c \neq 0$

   (b) $c = 0$

   (c) $c > 1$

   (d) $c < 4$

   (e) $c > 4$

16. If $\tan x = \frac{3}{2}$, then $\tan(2x)$ is equal to:

   (a) -12/5

   (b) -12/13

   (c) 12/13

   (d) 12/5

   (e) 3

17. The following procedure defines a function $f(x)$.

   Given a value for x:

   STEP 1. If $x \geq 2$ then $f(x) = \sqrt{x}$ and stop.
   STEP 2. Otherwise, replace x by $-x$.
   STEP 3. Go back to STEP 1.

   What is the domain of $f(x)$?

   (a) all real values for x

   (b) all $x \geq 0$

   (c) all $x \geq 2$

   (d) all $x \leq -2$

   (e) all $x \geq 2$ or $x \leq -2$

18. Given the following functions   1. $x^2 + \cos x$

                                   2. $x^3 + \sec x$

                                   3. $x^3 + \tan x$

                                   4. $\sin x + \tan x$

                                   5. $x^2 + \sin x$

Which of these are neither even nor odd functions?

(a) 1.

(b) 3. and 4.

(c) 2. and 5.

(d) 4.

(e) 2. and 3.

19. For which values of A and B will $\cos(Ax) = \sin(Bx)$ for all values of x, $0^0 \le x \le 360^0$?

(a) $A = 0$, $B = 0$

(b) $A = 1/2$, $B = 1$

(c) $A = 1$, $B = 1/2$

(d) infinitely many values for A and B

(e) no values for A and B

20. The hypotenuse of a right triangle is 10 inches and the radius of the inscribed circle is 1 inch. The perimeter of the triangle in inches is:

(a) 15

(b) 22

(c) 24

(d) 26

(e) 30

21. A church in a small town in Vermont has only two bells; one sounds a "ding" and the other a "dong". By the laws of this town, a ding cannot be the second note after a ding, and a dong cannot be the third note after a dong. If the sexton starts a tune with ding, ding what is the <u>longest</u> tune he can legally play?

    (a)   five notes

    (b)   seven notes

    (c)   ten notes

    (d)   he can keep playing forever

    (e)   none of the above

22. Let a, b, c be in a number field where the commutative law for addition holds but the associative law for addition does <u>not</u> hold. In how many algebraically distinct ways can you add up a, b, and c ?

    (a)   1

    (b)   2

    (c)   3

    (d)   6

    (e)   none of the above

23. What is the probability that when 2 dice are rolled, the result will either be "a sum of 8" or both dice will show the same number?

    (a)   11/36

    (b)   5/216

    (c)   5/18

    (d)   2/9

    (e)   none of the above

24. A single die is rolled three times. What is the probability that at least one 2 results?

    (a)   5/6   (d)   91/216

    (b)   25/216   (e)   none of the above

    (c)   75/216

25. A <u>line of symmetry</u> for a plane figure is a line so that if the figure is folded along that line the two pieces lie on top of each other. How many lines of symmetry does a regular hexagon have?

    (a) none

    (b) 2

    (c) 3

    (d) 6

    (e) 9

26. Define an SF to be:
          (1) one of the letters p, q, r
          (2) a legal SF preceeded by W
          (3) two legal instances of SF preceeded by C
    Which of the following expressions is an example of an SF ?

    (a) p C Wq

    (b) C pt

    (c) C Wp Wq

    (d) Cp Cq

    (e) Wp Cq

27. If log 2 = a and log 5 = b then log $\sqrt{62.5}$ is equal to:

    (a) $3b/2 - a$

    (b) $(3b - a)/2$

   (c) $a - 3b$

    (d) $3b - a$

    (e) none of the above

28. If $y\sqrt{.04} = 1$, what is the value of y ?

    (a) .05   (d) 6.25

    (b) .5   (e) 50

    (c) 5

29. The graph of $x^2 - 2x + 4y^2 - 16y + 13 = 0$ is

    (a) a straight line

    (b) a circle

    (c) a hyperbola

    (d) an ellipse

    (e) a parabola

30. The hypothenuse of a right triangle is 100 ft. The area of the triangle is 2400 sq. ft. The <u>sum</u> of the two legs is

    (a) 140 ft.

    (b) 200 ft.

    (c) 20 ft.

    (d) 110 ft.

    (e) 97.98 ft.

31. If we insert 5 arithmetic means between 55 and -8 the difference between successive terms will be

    (a) 47

    (b) -63

    (c) -11

    (d) 11

    (e) -10.5

32. One coin is honest and another has 2 heads. One of these coins is selected at random and is tossed twice. The probability that both tosses are heads is:

    (a) 1/8   (d) 5/8

    (b) 3/4   (e) does not exist

    (c) 1/16

33. Find x if $\log_{10}(\log_{10} \sqrt{x}) = 0$.

   (a)   1

   (b)   10

   (c)   100

   (d)   0

   (e)   none of the above

34. The following base eight numeral written as a base two numeral is

$$4201.12$$

   (a)   100010000001.00101

   (b)   110000001.101

   (c)   100010000001.101

   (d)   110000001.101

   (e)   none of the above

35. The line $3x + 5y = 47$ is tangent to a circle with center at (1, 2). The point of tangency is

   (a)   (5.4, 6.6)

   (b)   (4, 7)

   (c)   (5.3, 6.2)

   (d)   (2, 8)

   (e)   (3, 7.6)

36. Given the triangle whose sides are 2, 3, and 4, the length of the median to the side whose length is 4 is

   (a) $\frac{1}{2} \sqrt{10}$

   (d) $2\sqrt{2}$

   (b)   3/2

   (e) $8 - 8 \cos 30^\circ$

   (c)   4/3

37. Three pipes each of diameter 7 inches, are bound with a string as shown below. Assuming the string to be tight and with no knots, find the length of the string.
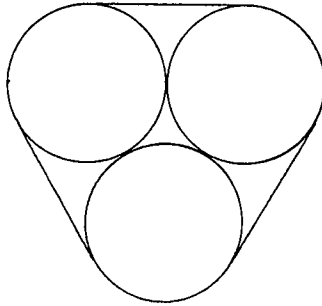
   (a) $21 + 14\pi$ inches

   (b) 42π inches

   (c) $21 + \pi$ inches

   (d) $21 + 7\pi$ inches

   (e) 21π inches

38. Determine the limit: $\lim\limits_{h \to 0} \left(\dfrac{\sqrt[3]{h + 1} - 1}{h} \right)$.

   (a)   0

   (b)   1/3

   (c)   1/2

   (d)   1

   (e)   none of the above

39. Let z be a complex number. The solution to the equation $|z| - z = 1 + 2i$ is

   (a) $3/2 - 2i$

   (b) $-1/2 - i$

   (c) $1 + 2\sqrt{5}/2 - 2i$

   (d) $1 - 2\sqrt{5}/2 - 2i$

   (e)   none of the above

40. A person drives to work at 20 kilometers per hour and drives home over the same route at 10 kilometers per hour. What was the average speed?

   (a)   15 kph

   (b)   13 $\frac{1}{3}$ kph

   (c)   20 kph

   (d)   17 kph

   (e)   none of the above