TWENTIETH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

1976

Scoring Formula: 4R - W

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This test was prepared from a list of Comprehensive questions submitted by Chattanooga State Technical Community College.

DIRECTIONS:

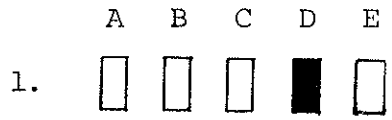
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a) 2/3. (b) 3. (c) 6.

   (d) 3/2 (e) none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. How many ordered pairs of values of x and y satisfy the following system of equations? $x^2 + y^2 = 25$

$x + 5 = y^2$

(a) 0

(b) 1

(c) 2

(d) 3

(e) 4

2. The value of x that satisfies the equation $\log x + \log(x + 3) = 1$ is:

(a) -1

(b) -5

(c) 2

(d) $\frac{3 \pm \sqrt{13}}{2}$

(e) none of these

3. Which of the following statements is true?

(a) $\sin 45^\circ = \sin 315^\circ$

(b) $\cos 45^\circ = \cos 315^\circ$

(c) $\tan 45^\circ = \tan 315^\circ$

(d) $\cot 45^\circ = \cot 315^\circ$

(e) $\csc 45^\circ = \csc 315^\circ$

4. The complex number $i^{13} - i^{48}$ $(i = \sqrt{-1})$ may be simplified and expressed as

(a) $(1 - i)$

(b) i^{-35}

(c) $(-1 + i)$

(d) i^{642}

(e) none of these

5. The solution to the inequality $x^2 < 14 - 5x$ is

   (a) $x < -7$

   (b) $x > 2$

   (c) $x < -7$ and $x > 2$

   (d) $-7 < x < 2$

   (e) none of these

6. The expression $(w^0x^{-1} - 2x^{-2})$, $w \neq 0$, may be simplified to

   (a) $\frac{x - 2}{x^2}$

   (b) $\frac{1}{x - 2x^2}$

   (c) $\frac{2x - 1}{2x^2}$

   (d) $\frac{x - 2w}{wx^2}$

   (e) none of these

7. An airplane travels non-stop from Town A to Town B, a distance of 1000 miles in 2 hours. The return trip requires 2 1/2 hours. If the difference in time is due to constant wind conditions, the wind may be described as

   (a) blowing 450 mph from Town A to Town B

   (b) blowing 450 mph from Town B to Town A

   (c) blowing 50 mph from Town A to Town B

   (d) blowing 50 mph from Town B to Town A

   (e) none of these

8. The number of subsets of the set $\{a, b, c, d, e\}$ is

(a) 5

(b) $5!$

(c) $2^5 - 1$

(d) $5! - 1$

(e) none of these

9. Given that $\log_{10} 62 = 1.7924$, then $\log_{10} .0620$ may not be written as

(a) $8.7924 - 10$

(b) $.7924 - 2$

(c) $18.7924 - 20$

(d) -1.2076

(e) $16.7924 - 10$

10. The line AB cuts three parallel planes in the points A,E and B; and the line CD cuts these same planes in the points C,F and D respectively. If AE = 4 in., EB = 6 in., and CD = 8 in., the length CF, in inches is:

(a) $4\frac{1}{4}$

(b) $3\frac{1}{5}$

(c) 5

(d) $2\frac{1}{2}$

(e) $3\frac{2}{3}$

11. If $\sin x = \sin y = 1/3$ and x and y are acute angles, then $\sin(x + y) =$

(a) $\frac{3}{4}$

(b) $\frac{2}{3}$

(c) 0

(d) $\frac{2}{9}$

(e) $\frac{4\sqrt{2}}{9}$

12. The value of $\sin 1110^{\circ} + \cos 1200^{\circ}$ is

(a) $\sqrt{3}$

(b) 1

(c) 0

(d) $\frac{1 + \sqrt{3}}{2}$

(e) $\frac{1 - \sqrt{3}}{2}$

13. If $2x + iy - 3y + 2i = 0$, x and y are real numbers, $i = \sqrt{-1}$, solve for x and y.

(a) $x = -3$
$y = -2$

(b) $x = 3$
$y = -2$

(c) $x = 3$
$y = 2$

(d) $x = 0$
$y = 0$

(e) $x = 2/3$
$y = -1$

14. The equation $\frac{3x - 6}{x - 1} + 6 = -\frac{3}{x - 1}$ has

(a) infinitely many roots

(b) one root

(c) two roots

(d) no roots

(e) four roots

15. $-\log (x + 2) + 2 \log (x - 1)$ is equal to

(a) $-\log \left[(x + 2)(x - 1)^2\right]$

(b) $-\frac{\log(x + 2)}{\log(x - 1)}$

(c) $\log \left[\frac{(x - 1)^2}{(x + 2)}\right]$

(d) $\log \left[\frac{(x + 2)}{(x - 1)^2}\right]$

(e) none of the above

16. If $\log_4 32 = x$ then x equals

(a) 8

(b) 5/2

(c) 1/4

(d) 1/8

(e) 2

17. A man is walking directly toward a vertical cliff. At one point the angle of elevation of the top of the cliff is 30°, four miles further down the road the angle of elevation of the top of the cliff is 45°. How high in miles is the cliff above the road?

(a) $\dfrac{4}{\sqrt{3} - 1}$

(b) $\dfrac{\sqrt{3} - 1}{4}$

(c) $\dfrac{3}{\sqrt{2} + 3}$

(d) $\dfrac{4}{1 - \sqrt{3}}$

(e) $\dfrac{4}{1 - \sqrt{2}}$

18. A box contains two white and three red balls. Two balls are drawn in succession without replacement. What is the probability that both are white?

(a) .1

(b) .4

(c) .15

(d) .6

(e) .65

19. $\sin^2 x =$

(a) $\dfrac{1 - \cos^2 x}{1 + \cos x}$

(b) $\dfrac{1 - \cos^2 x}{1 + \cos^4 x}$

(c) $\tan^2 x - 1$

(d) $\dfrac{\tan^2 x + 1}{\tan^2 x - 1}$

(e) $\dfrac{1 - \cos^4 x}{1 + \cos^2 x}$

20. The definition of $a \equiv b \pmod{k}$ states that there exists an integer n such that $a - b = kn$. Which of the following is a solution set to $x^2 + 2x \equiv 3 \pmod{7}$?

(a) {1,4,6}

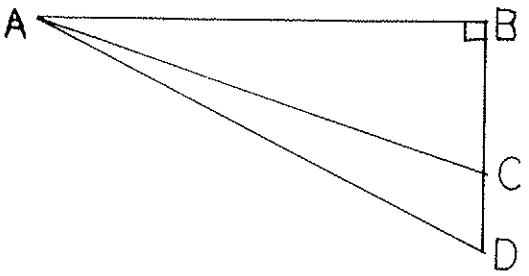
(b) {1,2}

(c) {2,4}

(d) {1,4}

(e) none of these

21.



Given:

$m \measuredangle CAD = 7^\circ$

$\overline{BC} = 10$

$\overline{CD} = 5$

$\measuredangle B$ is a right angle

Then $\overline{AB}$ is

(a) $\dfrac{+5 \pm \sqrt{25 - 600 \tan^2 7^\circ}}{2 \tan 7^\circ}$

(b) $\dfrac{-5 \pm \sqrt{25 - 600 \tan^2 7^\circ}}{2 \tan 7^\circ}$

(c) $\dfrac{-5 \pm \sqrt{25 + 600 \tan^2 7^0}}{2 \tan 7^\circ}$

(d) $\dfrac{+5 \pm \sqrt{25 + 600 \tan^2 7^\circ}}{2 \tan 7^\circ}$

(e) none of these

22. If a and b are any real numbers then $a^2 - ab + b^2$ is:

(a) sometimes a pure imaginary number

(b) a non-negative real number

(c) sometimes positive and sometimes negative

(d) always rational

(e) answer not given

23. The product of the elements of the solution set for the equation $\sqrt{4 - x} + \sqrt{x - 9} = \sqrt{x - 14}$ is

(a) 30

(b) -138

(c) 45

(d) 138

(e) does not exist

24. A unit vector $\vec{U}$ perpendicular to the vector $\vec{V} = 5i - 12j$ is:

(a) $\frac{12i}{13} - \frac{5j}{13}$

(b) $\frac{5i}{13} - \frac{12j}{13}$

(c) $12i - 5j$

(d) $\frac{12i}{13} + \frac{5j}{13}$

(e) answer not given

25. In the drawing, the triangle has sides of length 4. Find the area between the two circles (if the triangle is inscribed in one circle and circumscribed in the other circle).

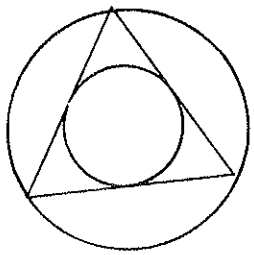
(a) 4π square units

(b) 5π square units

(c) $\frac{16\pi}{3}$ square units

(d) $\frac{17\pi}{3}$ square units

(e) answer not given



26. The sum of the elements of the solution set for the equation

$(2 \sin \theta - 1)(\sin^3 \theta + \sin \theta \cos^2 \theta - \sqrt{3} \cos \theta) = 0$ when $0 \leq \theta \leq 90^\circ$ is

(a) 60°

(b) 75°

(c) 53°

(d) 90°

(e) answer not given

27. If $0 < a < 1$ and $0 < b < 1$, a and b irrational, then $a + b$ is:

(a) always an irrational number

(b) always a rational number

(c) sometimes a rational number

(d) sometimes a pure imaginary number

(e) answer not given

28. The center of the circle passing through A(1,4), B(-5,-2), C(-5,4) is:

(a) (1,-2)

(b) (-2,1)

(c) (1,2)

(d) (-1,2)

(e) answer not given

29. There are two urns, A and B, urn A contains one blue and two red balls, urn B contains two blue and three red balls. A ball is chosen at random from urn A and put into urn B. A ball is then drawn at random from urn B. What is the probability that both balls drawn are the same color?

(a) 4/9

(b) 1/2

(c) 5/9

(d) 3/4

(e) answer not given

30. If the equation of a conic is $x^2 + 6x + 12y + 9 = 0$, the equation of its directrix is

(a) $x = 0$

(b) $y = 0$

(c) $x + 3 = 0$

(d) $y + 3 = 0$

(e) $y - 3 = 0$

31. The reciprocal of $3 + 5i$ is

(a) $3 - 5i$

(b) $1/3 + i/5$

(c) $3/34 - .5i/34$

(d) $-3/16 + 5i/16$

(e) $\sqrt{34}$

32. The area of the triangle whose vertices are $A(-2,3)$, $B(3,-2)$ and $C(3,6)$ is

(a) 40 square units

(b) 35 square units

(c) 25 square units

(d) 20 square units

(e) 15 square units

33. The roots of $x^2 - x + 1 = i$ are

(a) $\{1 + i, 1 - i\}$

(b) $\{1 + i, -i\}$

(c) $\{1 - i, i\}$

(d) $\{i, -i\}$

(e) $\emptyset$

34. If $f(x) = 2x^3 + 3x^2 - 2x$, at which of the following points does the tangent to the graph of f have a slope of +10?

(a) $(-2,0)$

(b) $(3,-1)$

(c) $(-1,3)$

(d) $(2,24)$

(e) $(0,0)$

35. The equation $4x^2 - 9y^2 - 32x - 36y + 28 = 0$ represents

(a) a circle

(b) an ellipse

(c) a parabola

(d) an hyperbola

(e) two straight lines

36. The equation of the ellipse with center at the origin passing through (2,4) and having one end-point of the minor axis at (4,0) is:

(a) $4x^2 + 3y^2 = 64$

(b) $3x^2 + 4y^2 = 64$

(c) $9x^2 + 8y^2 = 18$

(d) $x^2 + 2y^2 = 64$

(e) $2x^2 + y^2 = 32$

37. The value of the determinant $\begin{vmatrix} 1 & 2 & 1 & 2 \\ -1 & 2 & 1 & 2 \\ -1 & -2 & 1 & 2 \\ -1 & -2 & -1 & 2 \end{vmatrix}$ is:

(a) -16

(b) 32

(c) 64

(d) 0

(e) 24

38. The values of a,b, and c that require the graph of $y = ax^2 + bx + c$ to pass through the points $(0,3)$, $(-1,6)$, and $(2,9)$ are:

(a) $a = 2,\ b = 1,\ c = 3$

(b) $a = -2,\ b = 1,\ c = 3$

(c) $a = 2,\ b = 1,\ c = -3$

(d) $a = 2,\ b = -1,\ c = 3$

(e) none of these

39. If ten fair coins are thrown upon a table, what is the probability that at least 4, but no more than 6, will be heads?

(a) $\frac{210}{1024}$

(b) $\frac{420}{1024}$

(c) $\frac{252}{1024}$

(d) $\frac{672}{1024}$

(e) $\frac{3}{10}$

40. Semi-circles are constructed on AB, BC and AC. If AC = 10, AB = 6 and BC = 8, the area of the shaded portion of the figure in square units is

(a) $12 + 16\pi$

(b) $24 - 9\pi$

(c) 24

(d) $24 + 16\pi$

(e) $12 + 9\pi$