NINETEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

1975

Scoring Formula: 4R - W

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This test was prepared from a list of Comprehensive questions submitted by Vanderbilt University.

DIRECTIONS:

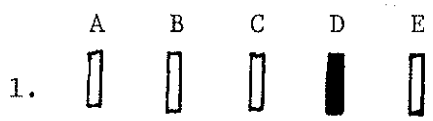
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$, then x equals

   (a). 2/3. (b). 3. (c). 6.

   (d). 3/2. (e). none of these



The correct answer for the sample problem is 3/2, which is answer (d); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 1 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If $f(x) = 2x + 1$, then $f[f(a+1)]$ is equal to

   (a) $2a + 3$

   (b) $4a + 7$

   (c) $4a + 6$

   (d) $2a + 2$

   (e) $2a + 1$

2. The sum of the infinite series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \ldots$ is

   (a) 1

   (b) $\frac{1}{2}$

   (c) $\frac{3}{2}$

   (d) 2

   (e) $\frac{4}{3}$

3. The value of the determinant $\begin{vmatrix} 2 & -3 & -2 \\ -1 & 1 & -1 \\ 2 & 0 & 1 \end{vmatrix}$ is

   (a) 9

   (b) -1

   (c) 7

   (d) 0

   (e) -2

4. In number base 7 the product (5) (4) is equal to

   (a) 54 (base 7)

   (b) 26 (base 7)

   (c) 5 (base 7)

   (d) 20 (base 7)

   (e) cannot be found

5. The function $y = -3 \sin (2x - \frac{\pi}{4})$ has a period of:

   (a) -3

   (b) $\frac{\pi}{4}$

   (c) 2

   (d) 2π

   (e) π

6. The complete set of values of k for which the quadratic equation $2x^2 + 3x + k = 0$ has no real roots is

   (a) $k < \frac{8}{9}$

   (b) $k < 1$

   (c) $k > 1$

   (d) $k > \frac{9}{8}$

   (e) $k \geq 2$

7. If 15 typists can type 240 pages in 8 hours, how many hours will it take 16 typists to type 384 pages if they type at the same rate?

   (a) 8

   (b) 10

   (c) 12

   (d) 15

   (e) 12.8

8. $\text{Log}_2 6$ is equivalent to which one of the following?

   (a) 3

   (b) $\log_{10} 3$

   (c) $\frac{1}{2} \log_{10} 6$

   (d) $\frac{\log_{10} 6}{\log_{10} 2}$

   (e) 2^3

9. The solution set for x in the equation $\frac{\log_{10}(7x-12)}{\log_{10}x} = 2$ is

(a) $\{3\}$

(b) $\{3,4\}$

(c) $\{\frac{12}{5}\}$

(d) $\{\frac{7}{3}\}$

(e) the empty set

10. Given $f(x) = x^2$ and $g(x) = \sqrt{x-2}$, then the composite function $f(g(x))$ is

(a) $\sqrt{x^2 - 2}$

(b) $x^2\sqrt{x - 2}$

(c) $x - 2$

(d) $(x - 2)^2$

(e) $x - \sqrt{2}$

11. The equation $4x^2 + 9y^2 + 16x - 18y - 11 = 0$ can best be described as:

(a) a circle with center $(-2,1)$

(b) a circle with center $(2,-1)$

(c) an ellipse with center $(2,-1)$

(d) a hyperbola with center $(2,-1)$

(e) an ellipse with center $(-2,1)$

12. In the following triangle $b = 30$ ft., angle $A = 75^\circ$, angle $B = 60^\circ$. The value of c is:

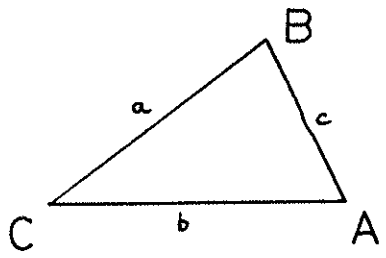
(a) $10\sqrt{6}$ ft.

(b) 30 ft.

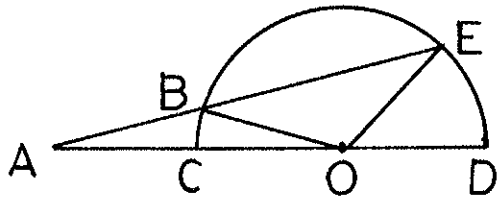
(c) $15\sqrt{2}$ ft.

(d) $30\sqrt{2}$ ft.

(e) $10\sqrt{3}$ ft.



13. In the figure $\overline{CD}$ is a diameter of semicircle CBED with center 0; $\overline{AB} = \overline{OD}$; $\measuredangle EOD = 24X$. Find $\measuredangle A$.



(a) 3X

(b) 4X

(c) 6X

(d) 8X

(e) 12X

14. The middle term of the binomial expansion $\left(\frac{2y^{\frac{1}{2}}}{x^3} - \frac{3x^{\frac{1}{2}}}{y^2}\right)^6$ is

(a) $\frac{2160}{x^{11}y^2}$

(b) $\frac{-4320}{x^{15/2}y^{9/2}}$

(c) $\frac{4860}{x^4y^7}$

(d) $\frac{-2430}{x^{1/2}y^{19/2}}$

(e) none of these

15. The common ratio in the geometric progression $3, \frac{2}{3}, \frac{4}{27}, \cdots$ is the common difference in an arithmetic progression in which the first term is 6. The nineteenth term of the arithmetic progression is:

(a) $\frac{9}{2}$

(b) 10

(c) $\frac{2}{9}$

(d) 7

(e) $\frac{88}{9}$

16. The term $\frac{-2 - 2i}{4i}$ may be simplified to

(a) $-\frac{1}{2}i - \frac{1}{2}$

(b) $-\frac{1}{2} + \frac{1}{2}i$

(c) $\frac{1}{2}i$

(d) $\frac{1}{2} + \frac{1}{2}i$

(e) $\frac{\sqrt{2}}{2}$

17. An equivalent expression for $\frac{1 + \cos 2\theta}{\sin 2\theta}$ is:

(a) $\cot \theta$

(b) $\tan \theta$

(c) $1 + \cot 2\theta$

(d) $2 \csc \theta(1 + \cos 2\theta)$

(e) $\frac{1 + 2 \cos \theta}{2 \sin \theta}$

18. Given $\sin(2 \text{ Cos}^{-1}(-\frac{3}{5})) = \sin(2 \text{ Arc cos } (-\frac{3}{5}))$. The common value is

(a) $-\frac{6}{5}$

(b) $\frac{4}{5}$

(c) $-\frac{12}{25}$

(d) $-\frac{24}{25}$

(e) $\frac{8}{5}$

19. The solution set for the inequality $\cos x \geq \sin x$ in the interval $\pi \leq x \leq 2\pi$ is

(a) $0 \leq x \leq \frac{\pi}{4}$

(b) all x

(c) $\pi \leq x \leq \frac{3\pi}{2}$

(d) $\frac{5\pi}{4} \leq x \leq 2\pi$

(e) no values of x

20. The solution set for the system of equations

$$\begin{cases} \frac{3}{x} + \frac{2}{y} = 7 \\ \frac{1}{x} - \frac{7}{y} = 10 \end{cases}$$ is:

(a) $\{3, -1\}$

(b) $\{-\frac{1}{3}, 1\}$

(c) $\{+\frac{1}{3}, -1\}$

(d) $\{-3, 1\}$

(e) $\{-3, -1\}$

21. The factorization of $x^4 + 64$ is

(a) $(x^2 + 8)^2$

(b) $(x^2 + 8)(x^2 - 8)$

(c) $(x^2 - 4x + 8)(x^2 - 4x + 8)$

(d) $(x^2 - 4x + 8)(x^2 + 4x + 8)$

(e) expression is prime

22. The solution set for the inequality $x^2 + 2x + 8 > 0$ is

(a) all real x

(b) $x > 8$

(c) $x > 0$

(d) $-2 < x < 4$

(e) $x < -2$ or $x > 4$

23. If $x^5 - 3x^3 + x^2 - 7$ is divided by $x - 2$, the remainder is

(a) -7

(b) 53

(c) -11

(d) $2x - 7$

(e) 5

24. The solution set for the inequality $|3 - 2x| > 7$ is:

(a) $\{x | x < -2 \text{ or } x > 5\}$

(b) $\{x | -2 < x < 5\}$

(c) $\{x | x < -2 \text{ and } x > 5\}$

(d) $\{x | x < -\frac{3}{2} \text{ or } x > +\frac{3}{2}\}$

(e) $\{x | x > -2\}$

25. One of the following is <u>not</u> a root of $x^6 - 64 = 0$. Which one is it?

(a) $+2$

(b) $-1 + i\sqrt{3}$

(c) $-1 - i\sqrt{3}$

(d) $\sqrt{3} - i$

(e) -2

26. The value of the determinant $\begin{vmatrix} 1 & 2 & -1 \\ 6 & -2 & 1 \\ -3 & x & 2 \end{vmatrix}$ is -49.

Then x must be equal to

(a) -2

(b) -3

(c) 2

(d) 3

(e) 1

27. If $P = \frac{s}{(1+K)^n}$ and $K > 0$, then n equals

(a) $\frac{\log_{10}(s/P)}{\log_{10}(1+K)}$

(b) $\log_{10} \frac{s}{P(1+K)}$

(c) $\log_{10} \frac{s-P}{1+K}$

(d) $\log_{10}(\frac{s}{P}) + \log_{10}(1+K)$

(e) $(\frac{s}{P})^{1/n}$

28. Given $f(x) = x^2 + 2x$, $x \geq 0$. Then the inverse function $f^{-1}(x)$ is

(a) $-1 + \sqrt{1 + x}$

(b) $\frac{1}{x^2 + 2x}$

(c) $-x^2 - 2x$

(d) $x + \sqrt{2x}$

(e) does not exist

29. The base of a triangle has length 80 and one of the base angles is 60°. The sum of the lengths of the other two sides is 90. The shortest side has length

(a) 45

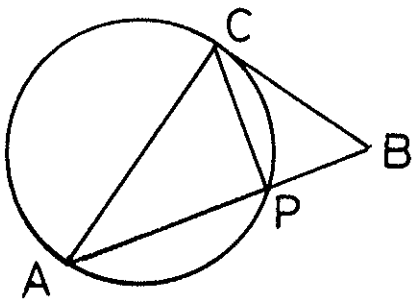
(b) 40

(c) 36

(d) 17

(e) 12

30. In the figure $\overline{AC}$ is a diameter. $\overline{BC}$ is a tangent at C. If $\overline{AB} = 20$ and $\overline{AC} = 16$, then $\overline{PB}$ is



(a) 8.4

(b) 7.4

(c) 6

(d) 7.2

(e) none of these

31. In how many ways can 9 books be arranged on a shelf so that 3 of the books are always together?

(a) 30,240

(b) 40,320

(c) 840

(d) 9!

(e) 7!

32. The square roots of the complex number $2 + 2\sqrt{3}i$ are

(a) $(\sqrt{3} \pm i)$

(b) $(i \pm \sqrt{3})$

(c) $(\sqrt{3} + i)$ and $(-\sqrt{3} - i)$

(d) $\pm \sqrt{2}$

(e) $\sqrt{2 + 2\sqrt{3}i}$

33. The definition of $a \equiv b \pmod{k}$ states that there exists an integer n such that $a - b = kn$. Which of the following is a solution to $44 \equiv x \pmod{7}$?

(a) $x = 6$

(b) $x = 0$

(c) $x = 4$

(d) $x = 11$

(e) $x = 23$

34. If $\sin(2\theta) - 4 \sin\theta \cos\theta + 1 = 0$, θ a real number, the solution set to the equation is

(a) $\{(\frac{K}{K+1})\pi \mid K \text{ is an integer}\}$

(b) $\{(\frac{4K+1}{4})\pi \mid K \text{ is an integer}\}$

(c) $\{\frac{K\pi}{4} \mid K \text{ is an integer}\}$

(d) $\{(\frac{2K-1}{2})\pi \mid K \text{ is an integer}\}$

(e) $\{(\frac{2K}{K+1})\pi \mid K \text{ is an odd integer}\}$

35. The solution set in the interval $0 \leq x < 2\pi$ for the trigonometric equation $\sin x + \cos x = 1$ is

(a) $x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{5\pi}{3}, \frac{5\pi}{6}$

(b) $x = 0, \frac{\pi}{2}$

(c) $x = \frac{\pi}{2}, \pi$

(d) $x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

(e) all values of x

36. The product of the elements of the solution set for the equation $\sqrt{3x + 4} - \sqrt{x - 3} = 3$ is

(a) -2

(b) 28

(c) -28

(d) $\frac{1}{8}$

(e) does not exist

37. What is the largest possible product xz if x and z are two real numbers such that $2x + z = 10$?

(a) $\frac{25}{2}$

(b) $\frac{125}{4}$

(c) 25

(d) 10

(e) 8

38. A man walks from A to B at x miles per hour and returns at y miles per hour. His average rate is

(a) $\sqrt{xy}$

(b) $\dfrac{2}{\frac{1}{x} + \frac{1}{y}}$

(c) $\dfrac{x + y}{2}$

(d) $\dfrac{1}{x} + \dfrac{1}{y}$

(e) $\dfrac{x + y}{2xy}$

39. The circle $x^2 + y^2 - 8x + 4y - 60 = 0$ has $y = 2x + b$ as a tangent line. One value of b can be

(a) $\frac{5}{2}$

(b) 30

(c) 6

(d) $-\frac{5}{2}$

(e) 10

40. Consider the equation $x^2 + ax + 1 = 0$. A single fair die is rolled to determine the value of the coefficient a. The value chosen for a is the number of dots on the face of the die which turns up. The probability that the equation will have real, unequal roots is

(a) $\frac{2}{3}$

(b) $\frac{5}{6}$

(c) $\frac{1}{2}$

(d) $\frac{1}{12}$

(e) $\frac{1}{3}$