SIXTEENTH ANNUAL MATHEMATICS CONTEST

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COMPREHENSIVE TEST

1972

Scoring Formula: 4R-W

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This test was prepared from a list of Comprehensive questions submitted by eleven colleges and universities across Tennessee.

DIRECTIONS:

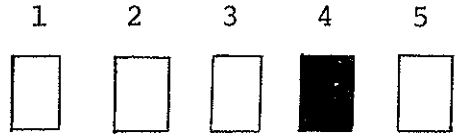
Do not open this booklet until you are told to do so.

This is a test of your competence in high school comprehensive. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 lead or softer). A sample problem follows:

1. If $2x = 3$. then x equals

(1). 2/3 (2). 3 (3). 6

(4). 3/2 (5). none of these



The correct answer for the sample problem is 3/2, which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. Given log 2 = 0.3010 and log 3 = 0.4771, the log 60 is equal to:

   (1). 0.1436

   (2). 1.7781

   (3). 0.7781

   (4). 1.1761

   (5). none of these

2. If A and B are sets and A' is the complement of A, then $(A \cup B)'$ is always the same as:

   (1). $A' \cup B'$

   (2). $B' \cup A'$

   (3). $(A \cap B)'$

   (4). $B' \cap A'$

   (5). $A \cap B$

3. For which of the following does f(-x) = f(x)?

   (1). log x

   (2). sin x

   (3). cos x

   (4). tan x

   (5). none of these

4. In what base(s) is the computation, (11)(101) = 1111, correct?

   (1). Base two only.

   (2). Base ten only.

   (3). Bases two and ten only.

   (4). Bases five and ten only.

   (5). Any natural number greater than one.

5. Foghorn C sounds every 12 seconds and foghorn D every 14 seconds. They sound together at noon. How many seconds later will they next sound together?

(1). 24

(2). 26

(3). 72

(4). 84

(5). none of these

6. If sin x = 3/5, then sec x is equal to:

(1). 4/5

(2). 4/5 or -4/5

(3). 5/4

(4). 5/3 or -5/3

(5). 5/4 or -5/4

7. Let X = {1,2,3,4,5} and Y = {-2,-1,0,1,2}. Which of the following is not a function of X into Y?

(1). {(1,0), (2,-1), (3,1), (4,-2), (5,2)}

(2). {(1,1), (2,1), (3,1), (4,1), (5,1)}

(3). {(4,0), (2,2), (3,0), (1,1), (5,0)}

(4), {(0,1), (1,2), (-1,3), (2,4), (-2,5)}

(5). {(5,-2), (4,-1), (1,0), (3,1), (2,-1)}

8. What must k + t be in order for AB = C where $A = \begin{pmatrix} 2 & 3 \\ 1 & 7 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 2 \\ 3 & 6 \end{pmatrix}$, $C = \begin{pmatrix} 19 & 2t \\ 2y & 4k \end{pmatrix}$.

(1). 11

(2). 22

(3). 44

(4). 2t

(5). none of these

9. The real value of x such that 64^{x-1} divided by 4^{x-1} equals 256^{24} is:

(1). -2/3

(2). -1/3

(3). 49

(4). 1/4

(5). 3/8

10. If A, B, C, and D are sets such that $A \subseteq (B \cup C)$ and $B \subseteq (C \cap D)$, then which of the following must be true?

(1). $A \subseteq D$

(2). $C \neq \phi$

(3). $C \subseteq D$

(4). $C \not\subseteq D$

(5). $(A \cap B) \subseteq D$

11. $\dfrac{2^{n+4} - 2(2^n)}{2(2^{n+3})}$ when simplified is:

(1). $2^{n+1} - 1/8$

(2). -2^{n+1}

(3). $1 - 2^n$

(4). 7/8

(5). 7/4

12. If $f(x) = \dfrac{1}{x^2-1}$ and $g(x) = 3x$, then the solution set for $f(g(x)) = g(f(x))$ is:

(1). {1}

(2). {2, 1}

(3). $\{\sqrt{2}/2\}$

(4). $\{\pm (\sqrt{13}) / 13\}$

(5). none of these

13. The weight of a rectangular block of metal varies jointly as the length, the width, and the thickness. If the weight of a 12 by 8 by 6 inch block of aluminum is 57.6 pounds, find the weight of a 16 by 10 by 4 inch block.

(1). 64 lbs.

(2). 61.3 lbs.

(3). 73.4 lbs.

(4). 68 lbs.

(5). 70 lbs.

14. Which of the following is not an identity?

(1). $\tan t + \cot t = (\sec t)\ (\csc t)$

(2). $1 - 2\sin^2 t = 2\cos^2 t - 1$

(3). $\cot t + \tan t = (\cot t)\ (\csc^2 t)$

(4). $\dfrac{1 - \cos x}{\sin x} = \dfrac{\sin x}{1 + \cos x}$

(5). $\sec y - \tan y = \dfrac{\cos y}{1 + \sin y}$

15. $\dfrac{\log_5 5}{\log_5 25}$ is equal to which of the following:

(1). log 5- log 25

(2). log (1/5)

(3). 1/2

(4). 1/5

(5). -1

16. If f is a function from the real numbers into the real numbers, then the domain of f, where f is defined by $f(x) = \sqrt{4 - x^2}$, is:

(1). $\{x \mid x \text{ is a real number}\}$

(2). $\{x \mid x \leq 2\}$

(3). $\{x \mid -2 < x < 2\}$

(4). $\{x \mid -2 \leq x \leq 2\}$

(5). none of these.

17. A circle is inscribed in an equilateral triangle. A second equilateral triangle is inscribed in the circle. The ratio of the areas of the two triangles is :

(1). 1 : $\sqrt{16}$

(2). 1 : $\sqrt{3}$

(3). 2 : 3

(4). 5 : 7

(5). $\sqrt{2}$: $\sqrt{8}$

18. The graph of J = sin x + cos(x + π /2) is:

(1). a straight line

(2). congruent to y = sin x + cos x

(3). the mirror image of y = tan x

(4). described by an amplitude of $\pm$ 1 and a period of π /2

(5). none of the above

19. The center of the graph $x^2 + 4y^2 + 6x - 8y - 10 = 0$ is at:

(1). (3,-1)

(2). (0,0)

(3). (-3,1)

(4). (-3,4)

(5). (1,4)

20. IF the angles of a triangle are A, B, and C and the lengths of the opposite sides are a, b, and c respectively, and if B = 2A, then cos A is equal to:

(1). b/a

(2). (a)(cos B)/b

(3). $\sqrt{3}/2$

(4). b/(2a)

(5). 1/2

21. The equation $x^{x^{x^{\cdots}}} = 2$ is satisfied when x is equal to:

(1). 2

(2). $\sqrt{2}$

(3). $\sqrt[4]{2}$

(4). infinity

(5). none of these

22. $3 \cos^4 x + 6 \sin^2 x$ is equal to:

(1). $3 - 3 \sin^4 x$

(2). $3 \sin^4 x + 12 \sin^2 x + 1$

(3). $(3 \sin^2 x + 3)(\sin^2 x + 1)$

(4). $3 + 3 \sin^4 x$

(5). $4 + 2 \sin^2 x$

23. Find the value of the following determinant: $\begin{vmatrix} 2 & 4 & 0 & 1 \\ 0 & 0 & 2 & 4 \\ 1 & 3 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix}$

(1). 0

(2). -4

(3). 3

(4). -5

(5). 1

24. The solution set for $2 \cos^2 x + \cos x = 0$, where $0 \leq x \leq \pi$, is:

(1). $\{\pi/3, 2\pi/3\}$

(2). $\{\pi/3, \pi\}$

(3). $\{\pi/3, 0\}$

(4). $\{\pi/2, \pi/3\}$

(5). $\{\pi/2, 2\pi/3\}$

25. $\{x \mid e^{-x \log_e 5} = 25\}$ is equal to:

(1). $\{e\}$

(2). $\{2\}$

(3). $\{-2\}$

(4). $\{5\}$

(5). none of these

26. Let $A = \{(x,y);\ 0 \leq x \leq 2,\ 0 \leq y \leq 2\}$ and $B = \{(x,y);\ 1 < x < 3,\ 1 < y < 3\}$. Then the union of these two sets, $A \cup B$, is the set:

(1). $\{(x,y);\ 1 \leq x \leq 2,\ 1 \leq y \leq 2\}$

(2). $\{(x,y);\ 0 \leq x \leq 3,\ 0 \leq y \leq 3\}$

(3). $\{(x,y);\ 0 \leq x < 3,\ 0 \leq y < 3\}$

(4). $\{(x,y);\ 1 < x < 3,\ 1 < y < 3\}$

(5). none of these

27. Solve for x: $1/x < [1/(x - 1)] - 1/2$

(1). $\{x;\ -1 < x < 4,\ x \neq 0,\ x \neq 1\}$

(2). $\{x;\ 0 < x < 2,\ x \neq 1\}$

(3). $\{x;\ -1 < x < 2,\ x \neq 0,\ x \neq 1\}$

(4). $\{x;\ -1 < x < 1,\ x \neq 0\}$

(5). none of these

28. During a gale a maypole was broken in such a manner that it struck the ground 20 ft. from the base of the pole. It was repaired, and broken by the wind a second time at a point 5 ft. lower down, and struck the ground 32 ft. from the base of the pole. What was the original height of the pole?

(1). 52.3 ft.

(2). 57.2 ft.

(3). 62.4 ft.

(4). 65 ft.

(5). none of these

29. The radius of the circle passing through A(2, 1), B(2,-7), and C(6, -3) is:

(1). $\sqrt{13}$

(2). four

(3). $\sqrt{15}$

(4). five

(5). one

30. A unit vector $\vec{u}$ in the direction of the vector $\vec{v} = 4i - 3j$ is:

(1). $i - j$

(2). $(4/7)i + (3/7)j$

(3). $(4/5)i + (3/5)j$

(4). $(4/7)i - (3/7)j$

(5). $(4/5)i - (3/5)j$

31. The product of the roots of $2x^3 - 3x^2 - 11x + 6 = 0$ is:

(1). -3

(2). 4

(3). 3/2

(4). -2

(5). none of these

32. If a and b are any real numbers such that $a > b$, then:

(1). $a^2 > b^2$

(2). $a^2 < b^2$

(3). $a^2 - b^2 > a$

(4). $a^3 - b^3 > 0$

(5). none of these

33. Let R^- be the set of negative real numbers. $R^- \times R^-$ is the cartesian product of R^- with itself. Let f be a function, with domain $R^- \times R^-$, defined as follows: for each (x, y) in $R^- \times R^-$, $f(x, y) = xy - y$. Which of the following statements is true?

(1). The range of f is the set of real numbers.

(2). f is not a binary operation.

(3). f is a one to one function.

(4). R^- and the range of f are disjoint.

(5). none of these

34. Let x_1 and x_2 be the roots of the equation $x^2 - ax + a - 1 = 0$, where a is a real number. Find the value of a for which the magnitude of the expression $x_1^2 + x_2^2$ will be the least.

(1). $a = 0$

(2). $a = 1/2$

(3). $a = 1$

(4). $a = 2$

(5). none of these

35. $\sin^2 230° - \sin^2 40°$ is equal to:

(1). $\cos^2 80°$

(2). $\cos 80°$

(3). $\sin^2 190°$

(4). $\sin 190°$

(5). none of these

36. A red card is removed from a bridge deck of 52 cards; 13 cards are then drawn and found to be the same color. Find the probability that all will be black.

(1). 2/3

(2). 13/51

(3). 1 - (26/51)

(4). (13! 38!) / 51!

(5). none of these

37. A man sets out at 12:00 to walk from town A to town B, and a friend of his started out at 2:00 to walk from town B to town A. They meet on the road at 4:05, and each man reaches his destination at the same time. What time did they arrive?

(1). 8:00

(2). 6:00

(3). 8:30

(4). 6:30

(5). 7:00

38. Two gamblers A and B are playing a game of chance where each player has staked $32 and has an equal chance to win. They are playing for three points; but when A has gained two points and B one point, they decide to stop playing. In what ratio should they divide the $64 in order to be fair to A in the light of his chances to win all of it?

(1). 3:1

(2). 2:1

(3). 1:1

(4). 4:1

(5). none of these

39. Suppose $f(x)$ is a function from the real numbers to the real numbers such that $|x - f(x)|^2 > |x|^2 + |f(x)|^2$ for all real numbers x. Then $f(x)$ is:

(1). always positive

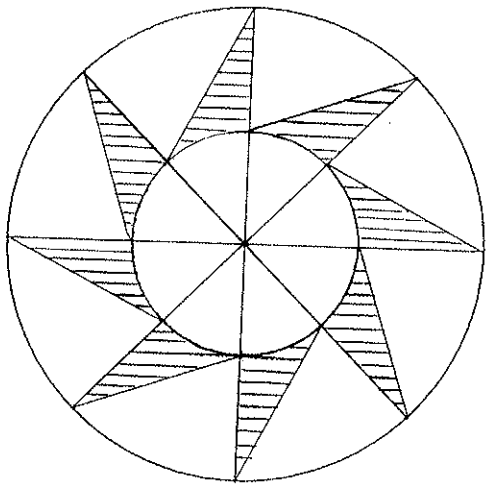
(2). always negative

(3). identically 0

(4). increasing

(5). none of these

40. In the drawing, the large circle has radius 5 and the small circle has radius 2. Find the area of the shaded portion.



(1). 8

(2). $\frac{20 - 4\pi}{3}$

(3). $8\sqrt{2} - 4\pi$

(4). $20\sqrt{2} - 4\pi$

(5). $\frac{2\pi}{3}$