FOURTEENTH ANNUAL MATHEMATICS CONTEST

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COMPREHENSIVE TEST

1970

Scoring Formula: 4R - W

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DIRECTIONS:

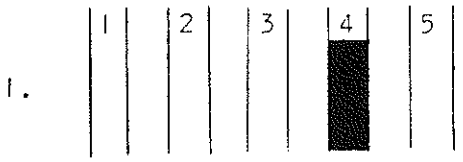
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these.



The correct answer for sample (1) is "$\frac{3}{2}$", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 30 minutes.

1. Which of the following is a counterexample to the statement: "The set $\{0,1\}$ is closed under addition?"

   (1) $0 + 1 = 1$

   (2) $1 + 0 = 1$

   (3) $1 + 1 = 2$

   (4) $0 + 0 = 0$

   (5) none of these.

2. Which of the following is <u>false</u>?

   (1) If $3 = 4 + 1$ then $5 = 9$

   (2) If $7 = 5 + 2$ then $6 = 8$

   (3) If $7 = 5 + 1$ then $3 = 2 + 9$

   (4) If $4 + 3 = 5 + 2$ then $8 + 1 = 9$

   (5) none of these.

3. The number of units in the perimeter of a triangle with vertices at the points $(-3,2)$, $(1,1)$, and $(13,-4)$ is:

   (1) $20 + \sqrt{65}$

   (2) $20\sqrt{65}$

   (3) 65

   (4) $18 + 2\sqrt{65}$

   (5) none of these.

4. An expression which is <u>not</u> equivalent to the expression $(a^4 - 16)$ is

   (1) $(a^2 - 4)(a^2 + 4)$

   (2) $(a + 2)(a - 2)(a^2 + 4)$

   (3) $(a + 2)(a - 2)(a + 2i)(a - 2i)$

   (4) $(a - 2)(a - 2)(a - 2)(a - 2)$

   (5) none of these.

5. The graph of the equation $4x^2 + y^2 + 8x - 4y - 8 = 0$ is

   (1) a circle (2) an ellipse (3) a hyperbola (4) a parabola

   (5) none of these.

6. The perimeter of a rectangle is 80 and its area is A. Its diagonal is

   (1) $2\sqrt{400 - A}$

   (2) $20\sqrt{2}$

   (3) $\sqrt{1600 - 2A}$

   (4) $10\sqrt{10}$

   (5) none of these.

7. The ratio of the area of a square inscribed in a circle to the area remaining when the square is removed is

   (1) 2 to 1

   (2) 2 to $(\pi - 2)$

   (3) π to 2

   (4) $(\pi - 1)$ to 2

   (5) depends on the radius of the circle.

8. An expression which is equivalent to $\log_b \left(x^3y/z^2\right)$ is

   (1) $\log_b x + \log_b y - \log_b z$

   (2) $3 \log_b x + \log_b y - 2 \log_b z$

   (3) $\frac{3}{2}\left(\log_b x + \log_b y - \log_b z\right)$

   (4) $3\left(\log_b x + \log_b y\right) - 2 \log_b z$

   (5) none of these.

9. With respect to the usual binary operations of addition and multiplication, which of the following is a field?

   (1) the set of all integers

   (2) the set of all positive integers

   (3) the set of all even integers

   (4) the set of all complex numbers

   (5) none of these.

10. $\sum_{j=1}^{5} 4^j - 1$ is equal to

    (1) 20 (2) 1024 (3) 256 (4) 341 (5) none of these.

11. The equation of the line with slope - 2/5 which passes through the point (1,3) is

(1) $2x + 5y = 11$

(2) $2x + 5y = 14$

(3) $5x - 2y = 11$

(4) $2x + 5y = 17$

(5) none of these.

12. Given a function $f(x) = 3x + 1$, its inverse function $g(x)$ is given by

(1) $g(x) = x + 3$

(2) $g(x) = 1 - 3x$

(3) $g(x) = \frac{x - 1}{3}$

(4) $g(x) = \frac{x - 3}{3}$

(5) none of these.

13. If $f(x) = x^2 + 2x - 3$, then $\frac{f(x + h) - f(x)}{h}$ is

(1) $2x + 2 + h$

(2) $h + 2$

(3) $2x + 2$

(4) $h + 2 - \frac{3}{h}$

(5) none of these

14. The vertical asymptotes of the graph of the function $\left\{(x,y) \mid y = \frac{x^2}{x^2 - 5x - 6}\right\}$ are

(1) $x = -6$, $x = 1$

(2) $x = 2$, $x = 3$

(3) $x = -2$, $x = -3$

(4) $x = 6$, $x = -1$

(5) none of these

15. The coefficient of x^4 in the expansion of $(1 + x)^n$ is 495. n equals

(1) 10 (2) 11 (3) 12 (4) 13 (5) none of these.

16- The binary operation * is defined on the set of all real numbers by $x * y = y$ for each of the real numbers x and y. This operation is

(1) commutative and associative.

(2) commutative and not associative.

(3) not commutative and not associative.

(4) associative and not commutative.

(5) none of these.

17. Three coins of diameter 2a are placed flat on a table top. What is the diameter of the smallest circle that can enclose them?

(1) $4a$

(2) $\frac{4a\sqrt{3} + 6a}{3}$

(3) $\frac{8a\sqrt{3}}{3}$

(4) $4a\sqrt{3} - 3a$

(5) $3a\sqrt{2}$

18. If m and n are positive integers greater than 1, which of the following is the largest?

(1) $m + n$

(2) $\sqrt{2mn}$

(3) $\frac{m^2 + n^2}{m + n}$

(4) $\frac{m^3 + n^3}{m^2 + n^2}$

(5) more information needed.

19. If m and k represent real numbers, $m \neq 1$, the graphs of the linear equations $mx - y + k = 0$ and $x - my + k = 0$

(1) coincide.

(2) intersect but are not perpendicular.

(3) are parallel.

(4) are perpendicular.

(5) none of these.

20. Given: $2 \cosh x = e^{x} + e^{-x}$ and $2 \sinh x = e^{x} - e^{-x}$. The expression $\cosh^2 x - \sinh^2 x$ is equal to

(1) one

(2) zero

(3) $\cos^2 x - \sin^2 x$

(4) $\frac{1}{2} e^{-2x}$

(5) none of these.

21. If $a = c \cos y$, $b = c \sin y$, then $a \sin x + b \cos x$ is equivalent to

(1) $c \cos (x + y)$

(2) $c \sin (x + y)$

(3) $c \cos (x - y)$

(4) $c \sin (x - y)$

(5) none of these.

22. The shaded region of the graph represents the solution set of a system of inequalities. Two of the inequalities are $y \leq 2$ and $2y \geq 3x - 6$. The third inequality of the system is

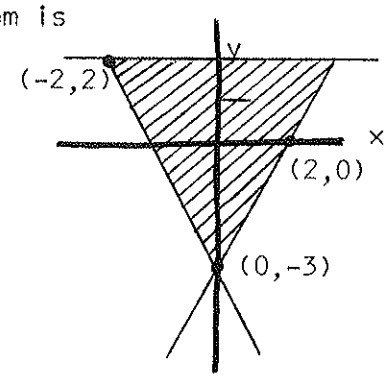
(1) $2y \geq 5x + 6$

(2) $5x + 2y + 6 \geq 0$

(3) $2x + 5y + 15 \leq 0$

(4) $2x - 5y + 15 \geq 0$

(5) none of these.



23. An open top box is made from a rectangular piece of metal of length x inches and width y inches by cutting squares of area z^2 square inches from each corner and turning up the edges. The volume of the box is

(1) xyz cubic inches.

(2) $(x - z)(y - z)z$ cubic inches

(3) xyz^2 cubic inches

(4) $(x - z)(y - z)z^2$ cubic inches

(5) none of these.

24. Let $A = \{3, \sqrt{3}, 1/3, -3, \sqrt{-3}, 3i, 3 + i\}$ and $B = \{x \mid x \text{ is a rational number}\}$. $A \cap B$ is the set

(1) $\{\sqrt{3}, \sqrt{-3}, 3i, 3 + i\}$

(2) $\{3, 1/3, -3\}$

(3) $\{3, \sqrt{3}, 1/3, -3\}$

(4) $\{1/3\}$

(5) none of these.

25. The solution set for the inequality $x^2 + 2x \leq 3$ is

(1) $\{x \mid -3 \leq x \leq 1\}$

(2) $\{x \mid x \geq 3\} \cup \{x \mid x \leq -1\}$

(3) $\{x \mid x \leq -3\} \cup \{x \mid x \geq 1\}$

(4) $\{x \mid x \leq -3\} \cap \{x \mid x \geq 1\}$

(5) none of these.

26. The roots of the equation $3x^2 + 7x + k + 4 = 0$ are real if and only if

(1) $k \geq 1/12$

(2) $k \geq -1/12$

(3) $k \leq 1/12$

(4) $k \leq 12$

(5) none of these.

27. The solution set of the equation $|x^2 - 2x - 1| = 2$ is

(1) $\{-1, 3\}$

(2) $\{-1, 1, 3\}$

(3) $\{-1, 1, -3, 3\}$

(4) $\{1\}$

(5) none of these.

28. The solution set of $\sqrt{x + 13} - \sqrt{7 - x} = 2$ is

(1) $\{3, -9\}$ (2) $\{3\}$ (3) $\{-9\}$ (4) $\{-1\}$ (5) none of these

29. If we consider the domain of the trigonometric functions to be real numbers, an expression equivalent to $2 \sin (x + \pi/6)$ is

(1) $\sqrt{3} \cos x + \sin x$

(2) $\sqrt{3} \sin x + \frac{1}{2} \cos x$

(3) $\frac{1}{2} \sin x + \sqrt{3} \cos x$

(4) $\sqrt{3} \sin x + \cos x$

(5) none of these.

30. The solution set for $\frac{x + 1}{x - 1} < 0$ is

(1) $\{x \mid -1 < x < 1\}$

(2) $\{x \mid -1 \leq x \leq 1\}$

(3) $\{x \mid 1 < x < 3\}$

(4) $\{x \mid x > 1 \text{ or } x < -1\}$

(5) $\{x \mid x < 1 \text{ or } x > 3\}$

31. If $\log_{10}(2x - 1) + 2 = 0$ then $x =$

(1) $\frac{101}{2}$ (2) $\frac{101}{200}$ (3) $\frac{99}{100}$ (4) $-\frac{1}{2}$ (5) none of these.

32. If r_1 and r_2 are the roots of the equation $ax^2 + bx + c = 0$, select, from the following, the equation whose roots are r_1^2 and r_2^2.

(1) $a^2x^2 + b^2x + c^2 = 0$

(2) $(ax^2 + bx + c)^2 = 0$

(3) $a^2x^4 + b^2x^2 + c^2 = 0$

(4) $a^2x^2 - (b^2 - 2ac)x + c^2 = 0$

(5) $a^2x^2 - (b^2 - 4ac)x + c^2 = 0$

33. The solution set of the equation $\begin{vmatrix} x^2 & x & 1 \\ 4 & 2 & 3 \\ 3 & 1 & 2 \end{vmatrix} = 0$ is

(1) $\{-2, 1\}$

(2) $\{-1, 2\}$

(3) $\left\{\frac{1}{2}(-1 + i\sqrt{7}), \frac{1}{2}(-1 - i\sqrt{7})\right\}$

(4) $\left\{\frac{1}{2}(-1 + \sqrt{7}), \frac{1}{2}(-1 - \sqrt{7})\right\}$

(5) none of these.

34. What is the probability of getting 80% or more of the questions correct on a ten question true-false exam by guessing?

(1) $\frac{1}{16}$

(2) $\frac{7}{128}$

(3) $\frac{3}{16}$

(4) $\frac{5}{32}$

(5) none of these.

35. If $a = x + x^3 + x^5 = \ldots$ and $b = 1 + x^2 + x^4 + \ldots$, then $a^2 - b^2$ equals

(1) $1 + x - x^2 - x^3 + x^4 + x^5 - \ldots$

(2) $1 - x^2 + x^4 - x^6 + \ldots$

(3) $\frac{1}{x^2 - 1}$

(4) $x^2 + 1$

(5) $\frac{x^2 - 1}{x^2 + 1}$

36. Two polynomials are identical if and only if the coefficients of like powers are the same. The ordered number triple (A,B,C) such that $A(x - 3) + (Bx + C)(2x - 1) = (x + 1)(4x + 3)$ is

(1) $\left(-\frac{11}{5}, -2, -\frac{18}{5}\right)$

(2) (3,2,3)

(3) $\left(-\frac{9}{5}, 2, \frac{12}{5}\right)$

(4) (-3,2,6)

(5) none of these.

37. The equation $4x^4 - 8x^3 + 7x^2 - 8x + 3 = 0$ has at least one root x such that

(1) $1 < x < 2$

(2) $-2 < x < -1$

(3) $x > 2$

(4) $x < -2$

(5) none of these.

38. If $i^2 = -1$, which of the following is not a cube root of i ?

(1) $-i$

(2) $1/2\ (\sqrt{3} - i)$

(3) $1/2\ (\sqrt{3} + i)$

(4) $1/2\ (-\sqrt{3} + i)$

(5) none of these.

39. Solve for x: $2^x + 2^{-x} = 3$

(1) $\log_2 3$

(2) $-1 + \log_2\ (3 \pm \sqrt{5})$

(3) $3 \pm \log_2 3$

(4) 2^3

(5) none of these.

40. The scalar product of two n-dimensional vectors $A = (a_1, a_2, \ldots, a_n)$ and $B = (b_1, b_2, \ldots, b_n)$ is defined by $A \cdot B = \sum_{i=1}^{n} a_i b_i$. If $x = (2,1,-3)$ and $y = (3,-2,1)$, then $x \cdot y =$

(1) -12

(2) 2

(3) 1

(4) 10

(5) none of these.