ELEVENTH ANNUAL MATHEMATICS CONTEST

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COMPREHENSIVE TEST

1967

Scoring Formula: 4R - W.

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DIRECTIONS:

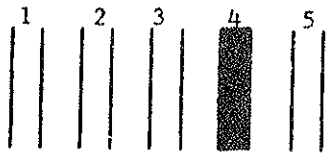
Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) $\frac{2}{3}$ (2) 3 (3) 6

(4) $\frac{3}{2}$ (5) none of these.

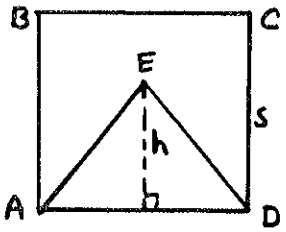
1. 

The correct answer for sample (1) is "$\frac{3}{2}$", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test if 80 minutes.

1. Find the ratio of s to h such that the area of triangle ADE, having altitude h, is equal to $\frac{1}{2}$ the area of the square ABCD having side s.



(1) $\frac{1}{2}$ (2) 1 (3) 2

(4) $\frac{3}{2}$ (5) none of the above.

2. Given the following algebraic operations: $a - 0$, $0 - a$, $a \times 0$, $0 \div a$, $a \div 0$. If a is any real number other than 0, how many of these have no answer?

(1) 1 (2) 2 (3) 3 (4) 4 (5) 5.

3. $\sqrt{4} = ?$

(1) 2

(2) -2

(3) ± 2

(4) all of the above.

(5) none of the above.

4. An angle that is four times its complement is:

(1) 144° (2) 80° (3) 72° (4) 60° (5) none of these.

5. The altitude of an equilateral triangle with side S is:

(1) $\frac{3}{4} S^2$

(2) $\frac{\sqrt{3}}{2} S$

(3) $\frac{1}{2} \sqrt{S}$

(4) $\frac{1}{2} (S + \sqrt{3})$

(5) $\sqrt{2} S$.

6. Two vertical poles, one 15 feet high and the other 27 feet high, are 16 feet apart. What is the distance between their tops?

(1) 20 ft. (2) 16 ft. (3) 12 ft. (4) 15 ft. (5) 36 ft.

7. If $ax^2 + bx - c = 0$, then x is:

   (1) 0

   (2) $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

   (3) $\dfrac{b \pm \sqrt{b^2 - 4ac}}{2a}$

   (4) $\dfrac{b}{2}$

   (5) $\dfrac{-b \pm \sqrt{b^2 + 4ac}}{2a}$

8. Find the solution set to the following inequality: $\frac{x}{3} - 2 < \frac{5x + 9}{2}$.

   (1) $\{x \mid x > 5\}$

   (2) $\{x \mid x = \frac{3}{2}\}$

   (3) $\{x \mid x = 2\}$

   (4) $\{x \mid x > -3\}$

   (5) $\{x \mid x < -5\}$

9. The base ten numberal 100 would be expressed in base nine as:

   (1) 90 (2) 900 (3) 127 (4) 121 (5) none of these.

10. Given a set with at least two distinct elements with operation @ such that $a@b = a$ and $b@a = b$, where a is any element of the set, and b is any element of the set, then:

    (1) @ is both associative and commutative

    (2) @ is commutative but not associative

    (3) @ is neither associative nor commutative

    (4) @ is associative but not commutative

    (5) @ is not defined.

11. The angles of elevation from a point level with the base of a tower, to the top of the tower and the top of a flag which is placed on the top of the tower are 30° and 60°, respectively. If the height of the flag with its pole is 50 feet, then the height of the tower is:

(1) 50 feet

(2) 100 feet

(3) 25 feet

(4) 75 feet

(5) none of these.

12. The coordinates of a focus of the ellipse $\frac{(x-1)^2}{25} + \frac{(y-2)^2}{9} = 1$ is:

(1) (6,2) (2) (5,2) (3) (-3,1) (4) (-4,1) (5) (-4,2).

13. If the radius of a sphere is doubled, the volume is multiplied by:

(1) 2 (2) 4 (3) 6 (4) 8 (5) none of these.

14. In triangle ABC, side AB = 9, BC = 8, and CA = 7. If CD bisects $\angle$ ACB, where D is a point on AB, then the length of DB is:

(1) 5 (2) $4\frac{1}{2}$ (3) $\frac{24}{5}$ (4) 4.85 (5) $\frac{21}{5}$.

15. The base of a triangle is 12 inches and its altitude is 9 inches. What is the altitude of a similar triangle whose base is 9 inches?

(1) 6 inches

(2) $7\frac{1}{2}$ inches

(3) $6\frac{3}{4}$ inches

(4) 8 inches

(5) none of the above.

16. If $\cos A = \frac{2}{5}$, then $\cos 2A$ is equal to:

(1) $+\frac{4}{5}$ (2) $+\frac{17}{25}$ (3) $-\frac{17}{25}$ (4) $-\frac{1}{5}$ (5) $+\frac{1}{5}$

17. A fish pond can be filled in 4 hours and drained in 7 hours. If both pipes are left open, how long will it take to fill the pond?

(1) 4 hours

(2) $9\frac{1}{3}$ hours

(3) $2\frac{6}{11}$ hours

(4) $12\frac{1}{5}$ hours

(5) $7\frac{1}{4}$ hours.

18. $\left\{(x,y) \mid y = 3 \tan \frac{1}{2} x\right\}$ is a periodic function with period:

(1) 3 (2) π (3) $\frac{\pi}{2}$ (4) 2π (5) $\frac{\pi}{3}$

19. If $(\bar{x},\bar{y})$ is a point mid-way between (x_1,y_1) and (x_2,y_2) then:

(1) $\bar{x} = \frac{x_2 - x_1}{2}$ and $\bar{y} = \frac{y_2 - y_1}{2}$

(2) $\bar{x} = 0$ and $\bar{y} = 0$

(3) $\bar{x} = \frac{x_1 + x_2}{2}$ and $\bar{y} = \frac{y_1 + y_2}{2}$

(4) $x_1 = y_2$ and $y_1 = y_2$

(5) $x_1 = y_2$ and $y_1 = x_2$

20. A radiator holds 16 gallons of solution containing 40% anti-freeze. How much solution should be removed and replaced by water to give a 25% solution of anti-freeze?

(1) 3 gallons

(2) 5 gallons

(3) 6 gallons

(4) 6.5 gallons

(5) none of the above.

21. The solution set of $\begin{vmatrix} x & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & x \end{vmatrix} < 1$ is:

(1) $\{x \mid 0 < x < 2\}$

(2) $\{x \mid x > 2 \text{ or } x < 0\}$

(3) $\{x \mid x = 1 \text{ or } x = -1\}$

(4) $\{x \mid x \geq 1 \text{ and } \leq 2\}$

(5) none of the above.

22. If $f(x) = 7x + 8$, then $f^{-1}(-x) =$

(1) $-7x - 8$

(2) $-\frac{7x + 8}{7}$

(3) $\frac{1}{7x + 8}$

(4) $-\frac{1}{-7x + 8}$

(5) none of the above.

23. If m and n are positive integers, $\frac{x^m \cdot x^n}{\frac{x^m}{x^n}}$, $x \neq 0$, simplifies to:

(1) x^{m+n}

(2) x^{2n}

(3) x^{2m}

(4) x^{m-n}

(5) none of the above.

24. Simplify: $\dfrac{a - \frac{ab}{a+b}}{a + \frac{ab}{a+b}}$, a and b are positive.

(1) 0

(2) 1

(3) $\frac{a}{a+2b}$

(4) $\frac{a^2+b^2}{a+b}$

(5) none of the above.

25. $\left|2 - \sqrt{5}\right| = ?$

(1) $2 + \sqrt{5}$

(2) $2 - \sqrt{5}$

(3) $-2 - \sqrt{5}$

(4) $-2 + \sqrt{5}$

(5) none of the above.

26. If a and b are real numbers such that $3a < b < a$, then $|b-a| + |5a-b|$ is equal to:

(1) 4a

(2) -4a

(3) 6a - 2b

(4) 2b - 6a

(5) there is no real number such that $3a < a$.

27. The solution set of the equation log x + log 2x = log 50 is:

(1) $\left\{\frac{50}{3}\right\}$

(2) $\{5\}$

(3) $\{5, -5\}$

(4) $\{10\}$

(5) none of the above.

28. There are 100 people in a room. Each person shakes hands with every other person exactly once. How many hand shakes took place?

(1) 10,000

(2) 1,000

(3) 4,950

(4) 5050

(5) none of the above.

29. Define the operation $\textcircled{x}$ for two real numbers a and b by $a \textcircled{x} b = a + b - ab$. Then the set of all real numbers satisfying the equation $a \textcircled{x} 1 = 1$ is:

(1) $\{1\}$

(2) $\{0\}$

(3) all positive numbers

(4) all real numbers

(5) none of the above.

30. A store owner pays $198 for a sofa and he adds 40% to the price. He then sells the sofa at a 10% discount. What is the selling price?

(1) $257.40

(2) $249.48

(3) $304.92

(4) $178.20

(5) none of the above.

31. $x^m y^n = ?$

(1) $(xy)^{m+n}$

(2) $(xy)^{mn}$

(3) $(x)^{m+n}(y)^{m+n}$

(4) $x^n y^m$

(5) none of the above.

32. If set A = $\{1,2,3,4,5\}$ and set B = $\{4,5,6,7\}$, then $A \cap B$ equals:

(1) $\{4,5\}$

(2) $\{1,2,3,4,5,6,7\}$

(3) $\{1,2,3,6,7\}$

(4) $\emptyset$

(5) $(A \cup B)'$.

33. If $\alpha = \beta$, $\sin(\alpha + \beta)$ is equal to:

(1) $\cos 2\alpha$

(2) $1-2\sin^2\alpha$

(3) $2\cos^2\alpha - 1$

(4) $2\sin\alpha\cos\alpha$

(5) $\sin^2\alpha$.

34. Two angles of a triangle have measure α and 2α. The side opposite the angle with measure α has length a. The side opposite the angle with measure 2α has length b. The value of $\cos\alpha$ is:

(1) $\frac{1}{2}$

(2) $\frac{b}{\sqrt{a^2+b^2}}$

(3) $\frac{2}{\sqrt{5}}$

(4) $\frac{\sqrt{3}}{2}$

(5) $\frac{b}{2a}$

35. Given: $\sin A = \frac{3}{5}$. The terminal side of A is in quadrant II. Find $\cos A$.

(1) $\frac{4}{5}$

(2) $\frac{2}{5}$

(3) $\frac{4}{5} - 1$

(4) $\frac{1}{5}$

(5) $\frac{-4}{5}$

36. For what value or values of X is tan X = sec X?

(1) $0 \leq X \leq \pi$

(2) $X = 0$

(3) $X = \pi$

(4) $0 \leq X \leq \frac{\pi}{2}$

(5) no values of X.

37. A set A is said to be closed with respect to an operation $*$ if $x \in A$ and $y \in A \Rightarrow x * y \in A$. Let $A = \{-1, 0, 1\}$. Then A is closed with respect to:

(1) addition

(2) subtraction

(3) division

(4) multiplication

(5) none of the above.

38. How many combinations of 4 distinct letters can be formed from (c,d,e,f,g,h,k), each including d?

(1) 20 (2) 6 (3) 720 (4) 120 (5) none of these.

39. A ball is drawn from a bag containing 5 red balls and 7 black balls. If (and only if) it is red, a second ball is drawn without replacing the first ball. What is the probability of drawing two red balls?

(1) $\frac{5}{33}$ (2) $\frac{5}{36}$ (3) $\frac{25}{144}$ (4) $\frac{5}{12}$ (5) none of these.

40. If $\frac{1}{1+x} - \frac{1}{x-1} = 0$ then $\frac{1}{1+x} = \frac{1}{x-1}$. This implication is:

(1) never valid

(2) always valid

(3) valid only if $x \neq 1$ and $x \neq -1$

(4) valid only if $x \neq 1$ or $x \neq -1$.

(5) none of the above.