SEVENTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

COMPREHENSIVE TEST

1963

Scoring Formula: 4R - W.

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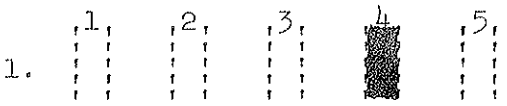
DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work the problems, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. A sample follows:

1. If $2x = 3$, then x equals:

(1) 2/3 ; (2) 3 ; (3) 6 ;
(4) 3/2 ; (5) none of these.



The correct answer for the sample problem is "3/2", which is answer (4); so you would answer this problem by making a heavy black mark under space 4 as indicated above.

If you should change your mind about an answer, be sure to erase completely. Avoid wild guessing, as wrong answers count against you. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. $(-2x + 1)(x - 2) - x(5 - 3x)$ can be expressed as:

   (1) $-5x^2 - 8x - 2$

   (2) $(x - 4 - 3\sqrt{2})(x - 4 + 3\sqrt{2})$

   (3) $(x - 2)(x - 1)$

   (4) $(x + \sqrt{2})(x - \sqrt{2})$

   (5) $-5(x^2 + \frac{2}{5})$

2. If $a^{-1} - 1$ is divided by $a - a^{-1}$ where a is neither 0 nor 1, the quotient is:

   (1) $\frac{-1}{a + 1}$

   (2) $\frac{1}{a + 1}$

   (3) $\frac{(1 - a)(a^2 - 1)}{a^2}$

   (4) $\frac{-(a + 1)}{2a}$

   (5) none of the above

3. If A and B are right circular cylinders, if the height of A is twice the height of B, and if the diameter of the base of A is half the diameter of the base of B, then the ratio of the volume of A to the volume of B is:

   (1) 8

   (2) 2

   (3) $\frac{1}{8}$

   (4) $\frac{1}{2}$

   (5) 1

4. If $x > 0$ and $\log_x 9 = -2$, then x equals:

   (1) $\frac{1}{3}$

   (2) 81

   (3) 3

   (4) $\frac{1}{81}$

   (5) $\frac{9}{2}$

5. Town A is 60 miles from town B. If a car travels from A to B at an average speed of 20 miles per hour and returns to A at an average speed of 30 miles per hour, then the average speed for the entire trip is:

   (1) 25 MPH

   (2) 24 MPH

   (3) 26 MPH

   (4) 50 MPH

   (5) not determined from the information given

6. The product $(\frac{1}{4})^{-\frac{1}{2}} \times \frac{4}{2^{-3}} \times (\frac{1}{8})^{\frac{2}{3}}$ is:

(1) $\frac{1}{4}$

(2) $2\sqrt{2}$

(3) 16

(4) 4

(5) $\frac{\sqrt{2}}{2}$

7. If the sides of a triangle are 3 in., 4 in., and 5 in. in length, then the altitude to the 5 in. side has length:

(1) 2.4

(2) 1

(3) $\frac{18}{5}\sqrt{6}$

(4) $2\frac{1}{2}$

(5) none of the above

8. If a is 6 and b is 2, then

$$\sqrt{a} \times \sqrt[3]{(a-b)^2} \times \frac{1}{\sqrt[4]{a^2}} \times \sqrt{a-b} \text{ is:}$$

(1) $\sqrt[3]{32}$

(2) $8\sqrt[3]{2}$

(3) 8

(4) $2\frac{2}{3}$

(5) none of the above

9. If the circumference of a circle is equal to the perimeter of a square, then the ratio of the area of the square to the area of the circle is:

(1) $\frac{\pi}{2}$

(2) 4

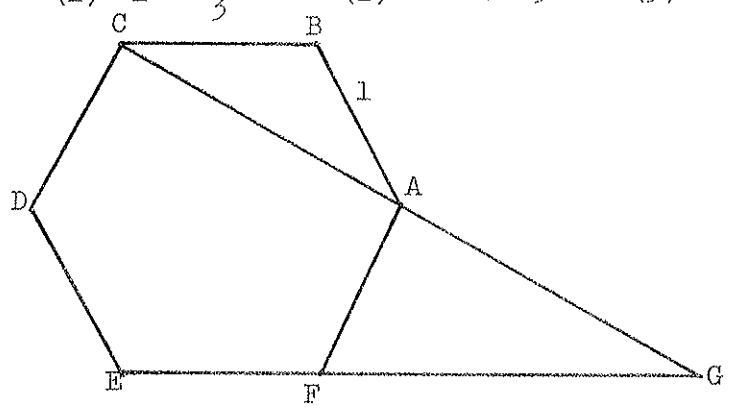
(3) $\frac{\pi}{4}$

(4) $\frac{1}{4}$

(5) $\frac{4}{\pi}$

10. In the figure below, if ABCDEF is a regular hexagon and $\overline{AB}$ has length 1, then $\overline{EG}$ has length:

(1) $1 + \frac{2\sqrt{3}}{3}$ (2) $1 + 2\sqrt{3}$ (3) $1 + \sqrt{3}$ (4) 2 (5) 3



(not a scaled figure)

11. The value of $8^{\log_2 5}$ is:

(1) 5 (2) 2 (3) 8 (4) 125 (5) none of these

12. The coefficient of x^2y^3 in the expansion of $(2x + y)^5$ is:

(1) 40 (2) 10 (3) 4 (4) 5 (5) 80

13. The number of positive roots of the equation

$$x^4 + x^3 - 3x^2 - 4x - 4 = 0 \text{ is:}$$

(1) 0 (2) 1 (3) 2 (4) 3 (5) 4

14. If $2^x = \frac{1}{4}$, then x^x is:

(1) 2 (2) 4 (3) -4 (4) $\frac{1}{4}$ (5) $-\frac{1}{4}$

15. The perimeter of a regular hexagon inscribed in a circle of radius 1 is:

(1) 6 (2) $6\sqrt{2}$ (3) $\frac{8\sqrt{3}}{2}$ (4) 16 (5) $4\sqrt{2}$

16. $\tan 50^\circ$ is equal to:

(1) $\tan 130^\circ$ (2) $2 \tan 25^\circ$ (3) $\frac{1}{2} \tan 100^\circ$ (4) $\tan 30^\circ + \tan 20^\circ$

(5) $\tan 230^\circ$

17. An automobile travels one mile at the speed of 30 MPH; how fast must the driver go during the second mile in order to have an average speed of 60 MPH?

(1) 60 (2) 150 (3) 90 (4) 180 (5) impossible

18. If a,b,c,d are all real numbers, then $(a + b)(c + d)$ and $ac + bd$ are equal:

(1) never (2) always (3) when $a = 0$ (4) when $ac + bd = 0$

(5) when $ad + bc = 0$

19. If $0 < a < b$, then $(\frac{a}{b})^{\frac{b}{a}}$ is:

(1) 1 (2) 2 (3) less than 1 (4) more than 1 (5) impossible to say

20. Let A, B, and C be the angles of a triangle and a, b, and c be the lengths of the sides opposite these angles, respectively. If $B = 120^\circ$, $C = 15^\circ$, and $b = \sqrt{6}$, then a is

(1) 1 (2) 2 (3) 3 (4) $\sqrt{2}$ (5) $\sqrt{3}$

21. The solutions of $|x|^2 - \sqrt{x^2} - 6 = 0$ are

(1) 3,2 (2) 3,-3 (3) 3,-2 (4) 2,-3 (5) 2,-3,3,-2

22. If $f(x) = \log_2 x$ and $g(x) = \sin x$, then $f(g(\frac{\pi}{6}))$ is

(1) -1 (2) 4 (3) 1 (4) -4 (5) not defined

23. Assume: (a) All unripe fruit is unwholesome.
(b) These apples are wholesome.
(c) No fruit grown in the shade is ripe.

Then:

(1) Wholesome fruit is grown in the shade.
(2) These apples are unripe.
(3) Ripe fruit is grown in the shade.
(4) These apples were not grown in the shade.
(5) Unwholesome fruit is never grown in the shade.

24. If an atheletic department charges \$.85 per ticket, 5,000 spectators will attend a basketball game; and for each \$.05 added to the price for admission the attendance will be cut by 200. How much should the department charge to get maximum gate receipts if the arena seats 8,000 people? (Assume that the price per ticket will be at least \$.85.)

(1) \$1.00 (2) \$.75 (3) \$1.05 (4) \$.85 (5) \$.50

25. In a certain state the first \$2,500 of a person's income is not taxed; on the next \$5,000 of income, the tax is 2 per cent; on all income over \$7,500 the tax is 5 per cent. If a person pays a tax of \$365, his income is:

(1) \$3,750 (2) \$15,000 (3) \$9,645 (4) \$12,000 (5) \$12,800

26. In a pile of logs, each layer contains one more log than the layer above and the top layer contains exactly one log. If there are 820 logs in the pile, then the number of layers is

(1) 41 (2) 15 (3) 40 (4) 20 (5) 21

27. If a coin is flipped 50 times, the probability that "heads" will turn up at least once is:

(1) $1 - (\frac{1}{2})^{50}$ (2) $(\frac{1}{2})^{50}$ (3) $\frac{1}{2}$ (4) $\frac{1}{50} \cdot \frac{1}{2}$ (5) $\frac{1}{50!} \cdot \frac{1}{2}$

28. A merchant has some coffee worth \$.80 per pound and some worth \$.90 per pound. How many pounds of each should he use to make 100 pounds of coffee worth \$.88 per pound?

(1) 20 lbs. of \$.80 coffee
80 lbs. of \$.90 coffee

(2) 80 lbs. of \$.80 coffee
20 lbs. of \$.90 coffee

(3) 50 lbs. of \$.80 coffee
50 lbs. of \$.90 coffee

(4) 30 lbs. of \$.80 coffee
70 lbs. of \$.90 coffee

(5) 70 lbs. of \$.80 coffee
30 lbs. of \$.90 coffee

29. The average of a set of forty numbers is 75 . Ten numbers are discarded from the set so that the remaining thirty numbers average 85 . The average of the discarded ten numbers is:

(1) 80 (2) 65 (3) 45 (4) 50 (5) 60

30. Let the operation $*$ be defined for positive integers by:

$$a * b = a^b$$

Consider the following statements:

(a) $a * b = b * a$ for all positive integers a and b .

(b) $(a * b) * c = a * (b * c)$ for all positive integers a and b .

(c) The positive integers are closed with respect to $*$.

Choose the correct answer.

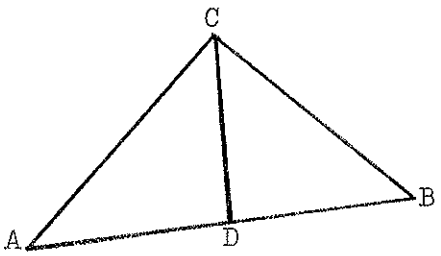
(1) (a) and (b) are false, and (c) is true.

(2) all are true.

(3) all are false.

(4) (b) and (c) are true and (a) is false.

(5) (a) and (c) are true and (b) is false.

31. 

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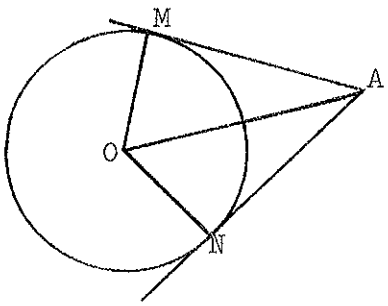
In the triangle ABC, the angle C equals the sum of the other two angles. D is a point on AB such that AD = DB. Then

(1) DC bisects angle C.
(2) $\angle$ DCB = $\angle$ B
(3) $\angle$ DCB = $\angle$ A
(4) $\angle$ DCB = $\angle$ BDC
(5) None of the relations is true.

32. Tangents AM and AN are drawn to a circle with center O from a point A outside the circle. The radius of the circle is 1 in. and $\angle$ MAN = $\frac{1}{2}\angle$ NOM.



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The length of AO is:

(1) $\sqrt{3}$ in.
(2) $(1+\sqrt{2})$ in.
(3) $\sqrt{5}$ in.
(4) 2 in.
(5) $1+\frac{1}{2}\sqrt{3}$ in.

33. One diagonal of a rhombus is 8 inches, and the area of the rhombus is 24 square inches. The length of the other diagonal is:

(1) 3 in. (2) $2\sqrt{6}$ in. (3) 6 in. (4) 8 in.
(5) not determined.

34. The diagonal of a rectangle is 13 inches and the perimeter is 34 inches. The area of the rectangle is:

(1) 60 sq. in. (2) $55\frac{1}{4}$ sq. in. (3) $72\frac{1}{4}$ sq. in.

(4) $110\frac{1}{2}$ sq. in. (5) $73\frac{2}{3}$ sq. in.

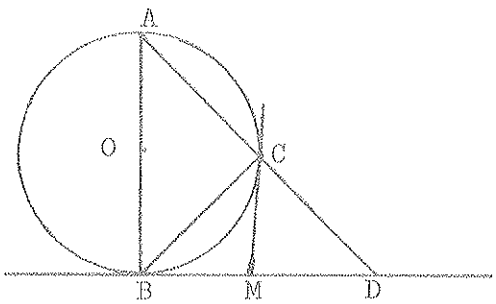
35. If x is an integer and the relations $2x - \sqrt{2} < 6$, and $1 - 2x < -4$ hold, then the value of x is:

(1) -3 (2) 4 (3) $3\frac{1}{2}$ (4) 3 (5) not determined

36. $\frac{1}{8^{\frac{2}{3}}} - \sqrt{27} + 3 \cdot 2^{-2} + (3\sqrt{3})^0$ equals:

(1) 0 (2) 1 (3) $\frac{1}{4} + 3\sqrt{2}$ (4) $2\sqrt{2} - 3\sqrt{3} + 1$ (5) $2 - 3\sqrt{3}$

37.



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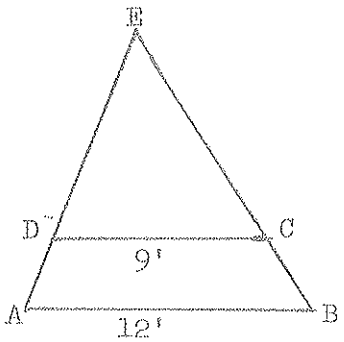
AB is a diameter of a circle with center O. A line through A intersects the circle at C and intersects the tangent drawn at B at the point D. The tangent at C intersects BD at M. Then the length of MC is the same as:

(1) The radius of the circle.
(2) The line segment CD.
(3) $\frac{2}{3}$ of BC
(4) AB = AC
(5) MD

38.



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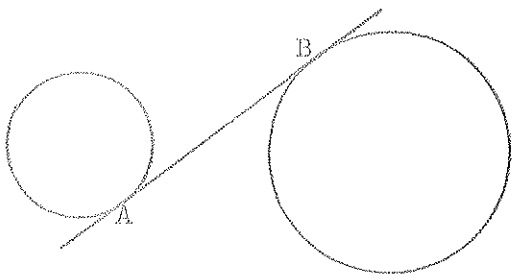
The bases AB and DC of a trapezoid ABCD are 12 ft. and 9 ft. respectively. The sides AD and BC are extended to meet at E. The area of the triangle DCE is equal to:

(1) $\frac{3}{4}$ the area of $\triangle$ ABE
(2) $\frac{1}{2}$ the area of $\triangle$ ABE
(3) $\frac{9}{16}$ the area of $\triangle$ ABE
(4) $\frac{3}{8}$ the area of $\triangle$ ABE
(5) $\frac{1}{3}$ the area of $\triangle$ ABE

39. Two circles of radii 2 inches and 4 inches have their centers 10 inches apart. The length of their common internal tangent is equal to:



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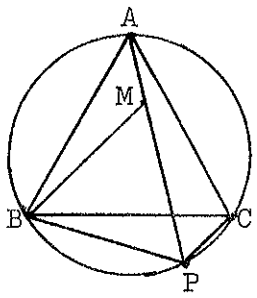
(1) 7 in. (2) 8 in.
(3) $\sqrt{120}$ in. (4) 10 inches
(5) none of these lengths.

40.



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Triangle ABC is an equilateral inscribed triangle. P is any point on the arc BC, and PM = PB. Then PB + PC is the same as:

(1) BM + MA

(2) AB

(3) AM + MC

(4) BM + MC

(5) PA