EIGHTEENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHER'S ASSOCIATION

COMPREHENSIVE II TEST

1974

Scoring Formula: 4R - W

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This test was prepared from a list of questions submitted by David Lipscomb College.

DIRECTIONS:

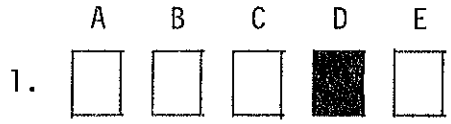
To not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with soft lead (No. 2 or softer). A sample problem follows:

1. If $2x = 3$, then x equals

(a). 2/3. (b). 3. (c). 6.

(d). 3/2. (e). none of these



The correct answer for the sample problem is 3/2, which is answer (D); so you would answer this problem by making a heavy black mark under space D as indicated above.

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet.

Figures given on this test are not necessarily drawn to scale.

The answer sheets will be used for a statewide statistical compilation and will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

1. If two sides of a non-degenerate triangle are six and ten, the third side may be

   (a). 1.
   (b). 4.
   (c). 8.
   (d). 16.
   (e). 20.

2. If set $M = \{1, 3, 6\}$, which of the following is <u>not</u> an element of the set $M \times M$?

   (a). (1, 6)
   (b). (1, 2)
   (c). (6, 1)
   (d). (3, 3)
   (e). (1, 3)

3. $\sum_{i=1}^{5} i^2$ is equal to

   (a). 55.
   (b). 45.
   (c). 36.
   (d). 15.
   (e). 25.

4. The solution set of $\log_3(x + 2) + \log_3(x + 4) = 1$ is

   (a). $\{-5\}$.
   (b). $\{-1, -5\}$.
   (c). $\{-3 + \sqrt{2}, -3 - \sqrt{2}\}$.
   (d). $\{-1\}$.
   (e). $\{-2, -4\}$.

5. The sum of the infinite series $\frac{1}{3} - \frac{1}{9} + \frac{1}{27} - \frac{1}{81} + \cdots$ is

   (a). 1/4.
   (b). 1/2.
   (c). 3/4.
   (d). 21/81.
   (e). 3/2.

6. Given: All registered voters are eighteen years old or older. Which statement expresses a true logical conclusion to the given statement?

   (a). If a person is not a registered voter, then he is not eighteen years old or older.
   (b). If a person is eighteen years old or older, then he is not a registered voter.
   (c). All persons who are eighteen years old or older are registered voters.
   (d). If a person is not eighteen years old or older, then he is not a registered voter.
   (e). If a person is not eighteen years old or older, then he may or may not be a registered voter.

7. A number pair which is a solution to the system $\begin{cases} 3x + 4y = 23 \\ x - 3y = -1 \end{cases}$ is

   (a). (1, 5).
   (b). (4, 11/4).
   (c). (5, -2).
   (d). (-7, -2).
   (e). none of these.

8. An airplane is traveling at 150 mph and is heading downward at an angle of 30°. How fast does the ground appear to be rising?

   (a). 150 mph.
   (b). 50 mph.
   (c). 300 mph.
   (d). 75 mph.
   (e). 124.95 mph.

9. $\log_2(1/16)$ is equal to

(a). 8.
(b). 4.
(c). (-1/4).
(d). -4.
(e). (1/4).

10. The solution set of $\sqrt{5x - 1} - \sqrt{x + 6} = 3$ is

(a). {1/4, 10}.
(b). {1/4}.
(c). {2/3}.
(d). {4}.
(e). {10}.

11. $f(x) = 3x + 7$ and $f(a + 3) = 7$. a is equal to

(a). 4.
(b). -1.
(c). 10.
(d). 3.
(e). -3.

12. For what value(s) of k will the quadratic equation $kx^2 - 4x - k + 5 = 0$ have equal roots?

(a). k = 1
(b). k = -1 or k = 4
(c). k = 1 or k = -4
(d). k = 1 or k = 4
(e). k = -1 or k = -4

13. Right triangles ABC and A'B'C' are similar, $m(\overline{BC}) = 3$, $m(\overline{AC}) = 4$, and $m(\overline{B'C'}) = 6$. What is $m(\overline{A'B'})$?

(a). 5
(b). $2\sqrt{7}$
(c). $\sqrt{7}$
(d). $\sqrt{14}$
(e). None of the above.

14. The solution set of the inequality $(2 - x)/(x - 3) \geq 0$ is

(a). $\{x \mid x \leq 2\}$.
(b). $\{x \mid x \geq 3\}$.
(c). $\{x \mid 2 < x < 3\}$.
(d). $\{x \mid 2 \leq x \leq 3\}$.
(e). none of the above sets.

15. If $x^{17} - 2$ is divided by $x - 1$, the remainder is

(a). -3.
(b). 1.
(c). -1.
(d). 0.
(e). $\log_{10}17$.

16. $(\sec^2 A)/(1 + \cot^2 A)$ is identical to

(a). cot A.
(b). sec A.
(c). tan A.
(d). $\tan^2 A$.
(e). $\cot^2 A$.

17. 37^{o} is how many radians?

(a). 37/360
(b). 37/180
(c). $37\pi/100$
(d). $37\pi/360$
(e). $37\pi/180$

18. Express the number 343(base 5) in base 6 notation.

(a). 232
(b). 242
(c). 231
(d). 241
(e). 98

19. The equation of the perpendicular bisector of the segment connecting the points (-3, 2) and (5, -4) is

(a). $4x - 3y = -7$.
(b). $3x - 4y = 7$.
(c). $4x + 3y = 7$.
(d). $3x + 4y = 7$.
(e). none of these.

20. If an unbiased coin is tossed five times, what is the probability that "heads" will appear at least twice?

(a). 3/16
(b). 5/16
(c). 1/32
(d). 13/16
(e). 1/2

21. The statement $\sin^2 x - \cos^2 x = 1$ is

(a). always false.
(b). never true.
(c). never false.
(d). equivalent to $\sin^2 2x - \cos^2 2x = 2x$.
(e). sometimes false.

22. Every triangle is divided into two triangles having equal areas by

(a). an altitude.
(b). a median.
(c). any line from a vertex drawn to the opposite side.
(d). the bisector of one of its angles.
(e). the perpendicular bisector of a side.

23. The equation $2^{x^2-x} = 64$ has

(a). no solutions.
(b). one real solution.
(c). two real solutions.
(d). two imaginary solutions.
(e). one real and one imaginary solution.

24. If $f(x) = (x + 1)/x$, $x \neq 0$, and if $f[g(x)] = x$, then $g(x)$ is equal to

(a). $x^2/(x + 1)$.
(b). $x(x - 1)$.
(c). $1/(x - 1)$.
(d). $1(1 - x)$.
(e). $x(x + 1)$.

25. The value of the determinant, $\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 0 & 0 \\ 3 & 2 & -1 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 & 1 \end{vmatrix}$, is

(a). 35.
(b). -1.
(c). 0.
(d). 33.
(e). 1.

26. The term involving x^{12} in the expansion of $(x^2 - 2y)^9$ is

(a). $84x^{12}y^3$.
(b). $-84x^{12}y^3$.
(c). $672x^{12}y^3$.
(d). $-672x^{12}y^3$.
(e). none of these.

27. (12)(3) = 40 in which one of the following number bases?

(a). 4
(b). 5
(c). 6
(d). 9
(e). 10

28. The minor determinant of the element 3 in the determinant, $\begin{vmatrix} 1 & -1 & -2 \\ 3 & 1 & 2 \\ -2 & 0 & 4 \end{vmatrix}$, is equal to

(a). 3.
(b). -4.
(c). 4.
(d). -24.
(e). 16.

29. If $i^2 = -1$, the cube roots of $-i$ are

(a). $-i$.
(b). $\sqrt{3}/2 + i/2, \quad -\sqrt{3}/2 + i/2, \quad -i$.
(c). $i, \quad -\sqrt{3}/2 - i/2, \quad \sqrt{3}/2 + i/2$.
(d). $1/2 + \sqrt{3}i/2, \quad -1, \quad 1/2 - \sqrt{3}i/2$.
(e). none of these.

30. Which of the following implications is <u>not</u> true?

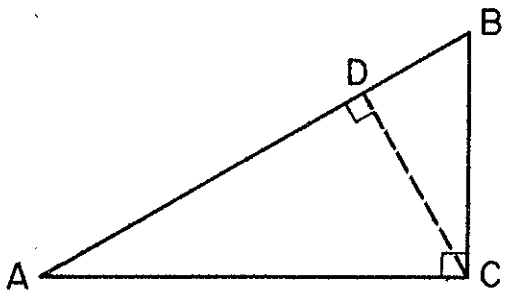
(a). If $2 = 3$, then $4 = 5$.
(b). If $2 = 3$, then $4 = 4$.
(c). If $4 = 4$, then $3 = 3$.
(d). If $4 = 4$, then $2 = 3$.
(e). If $4 = 5$, then $2 = 3$.

31. Which of the following sets forms a group under ordinary multiplication?

(a). $\{x|x$ is an integer$\} - \{0\}$
(b). $\{x|x$ is a rational number$\}$
(c). $\{x|x$ is a rational number$\} - \{0\}$
(d). $\{x|x$ is a positive integer$\}$
(e). $\{x|x$ is a negative integer$\}$

32. In the right triangle ABC, CD is perpendicular to AB and $m(\overline{AC}) = 4$, $m(\overline{AB}) = 5$, $m(\overline{BC}) = 3$. What is $m(\overline{BD})$?

(a). 12/5
(b). 3
(c). 16/5
(d). 7/5
(e). 9/5



33. The solution set of the inequality $|x^2 + 5x + 2| < 2$ is

(a). $\{x|-5 < x < 0\}$.
(b). $\{x|x < -4\} \cup \{x|x > -1\}$.
(c). all real numbers.
(d). $\{x|-1 < x < 0\} \cup \{x|-5 < x < -4\}$.
(e). $\{x|x > -4\} \cap \{x|x < -1\}$.

34. From three Republicans and three Democrats, a committee of three is chosen at random. What is the probability that the committee will consist of one Republican and two Democrats?

(a). 3/20
(b). 1/6
(c). 1/3
(d). 9/20
(e). 1/2

35. The center of the conic section $x^2 + y^2 = 4x + 6y + 3$ is

(a). (0, 0).
(b). (2, 3).
(c). (-2, -3).
(d). (3, 2).
(e). (-3, -2).

36. The solution set of the inequality $|2 - x| - |2x - 3| < 1$ is

(a). the set of all real numbers.
(b). $\{x|x < 2\}$.
(c). $\{x|3/2 < x < 2\}$.
(d). the set of real numbers - $\{2\}$.
(e). none of the above.

37. If $(\log_k x)(\log_5 k) = 3$, then x is equal to

(a). k^2.

(b). 15.

(c). $5k^3$.

(d). $(2k)/125$.

(e). $25^{3/2}$.

38. The figure at the right is a rectangle of length b and width a. Angle ABC is a right angle. What is the area of triangle ABC?

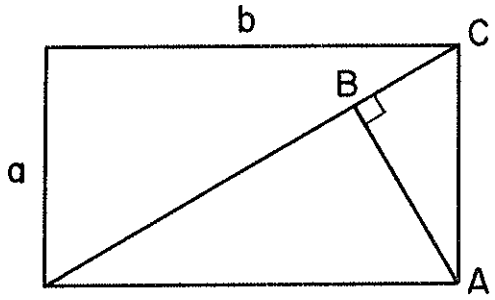
(a). $(ab)/(a^2 + b^2)$

(b). $(ab)/2(a^2 + b^2)$

(c). $(a^3b)/(a^2 + b^2)$

(d). $(a^3b)/2(a^2 + b^2)$

(e). $ab/5$



39. If $f(x) = x - 1/x$, which of the following are correct?
I. $f(-x) = -f(x)$; II. $f(1/x) = -f(x)$; III. $f(-1/x) = f(x)$;
IV. $f(x^2) = -f(x)\ f(-x)$.

(a). only I and II

(b). only I and III

(c). only I, II, and III

(d). only I and IV

(e). all are correct

40. If we define $a \equiv b \pmod 6$ to mean that there exists an integer n such that $a - b = 6n$, which of the following is not necessarily true?

(a). $23 \equiv 17 \pmod 6$

(b). $1 \equiv 19 \pmod 6$

(c). If $a \equiv b \pmod 6$, then $a^2 \equiv b^2 \pmod 6$

(d). If $a \equiv b \pmod 6$, then $3a \equiv 3b \pmod 6$

(e). If $3a \equiv 3b \pmod 6$, then $a \equiv b \pmod 6$