Calculus and Advanced Topics 2009

1. $\sec\left(\arctan\left(5x\right)\right) =$

a. $25x^2+1$ b. $\dfrac{5x}{\sqrt{25x^2+1}}$ c. $\sqrt{25x^2+1}$ d. $\dfrac{1}{5x}$ e. $\dfrac{\sqrt{25x^2+1}}{5x}$

2. Suppose that the U.S. factory sales of electronics (in billion of dollars) from 2000 to 2005 can be modeled by the function $S(t) = 20-8(2^{-t})$ where t is the number of years since 2000. Find the average rate of change of S from 2000 to 2003.

a. \$2.33 billion/yr
b. \$7 billion/yr
c. \$6.33 billion/yr
d. \$2.67 billion/yr
e. \$1.55 billion/yr

3. Let $c\left(-3\right)=13$ and $c\left(2\right)=-2$. If c is linear, find $c\left(\dfrac{4}{3}\right)$.

a. 4/3 b. -16/9 c. -20/3 d. 58/9 e. 0

4. Let $h^{-1}(x)$ denote the inverse function of $h(x)=x^3+3x-5$. Find $h^{-1}(7)$ accurate to 3 decimal places.

a. 0.003 b. 1.859 c. 359.000 d. -359.000 e. -4.569

5. A spherical balloon is inflated with gas at the rate of 800 cm^3/min. If the volume of a sphere is $V=\frac{4}{3}\pi r^3$, how fast is the radius of the balloon increasing at the instant the radius is 30 cm?

a. $\dfrac{2}{9\pi}\,cm/\min$
b. $36000\pi\,cm/\min$
c. 26.67 cm/min
d. 0.0375cm/min
e. $24000\,cm/\min$

6. On what interval(s) is $f(x)=\dfrac{5x}{x^2+1}$ concave down?

I. $(-\infty,-1)$ IV. $\left(-\sqrt{3},0\right)$

II. $(1,\infty)$ V. $\left(0,\sqrt{3}\right)$

III. $\left(-\infty,-\sqrt{3}\right)$ VI. $\left(\sqrt{3},\infty\right)$

a. I only b. I and II c. IV and V d. III and V e. IV and VI

7. The position of a particle is given by $s(t)=t^3-9t^2+15t+10,\ t\geq 0$. When is the velocity at a minimum?

a. t = 0 b. t = 1 c. t = 3 d. t = 5 e. t = 10

8. $\lim\limits_{x\to 0}\dfrac{xe^{-2x+1}}{x^2+2x}=$

a. 0 b. e/2 c. ∞ d. -2e e. -1.25e

9. The derivative of $y=e^{2x}x^3$ is:

a. $e^{2x}6x^2$

b. $y\left(2+\dfrac{3}{x}\right)$

c. $y=e^{2x}3x^2$

d. $x^2e^{2x}\left(\dfrac{2x^2}{e}+3\right)$

e. $x^2e^{2x}(3-2x)$

10. Let $f(x)=x^3$ and $g(x)=\dfrac{x}{x^2-2}$. Find $(f\circ g)'(x)$.

a. $\left(\dfrac{x}{x^2-2}\right)^3$ b. $\dfrac{x}{2\left(x^2-2\right)^2}$ c. $\dfrac{x^3}{x^6-2}$ d. $\dfrac{-3x^2\left(x^2+2\right)}{\left(x^2-2\right)^4}$ e. $\dfrac{1}{2x^3}$

11. Given the graphs, find $g'(2)$.

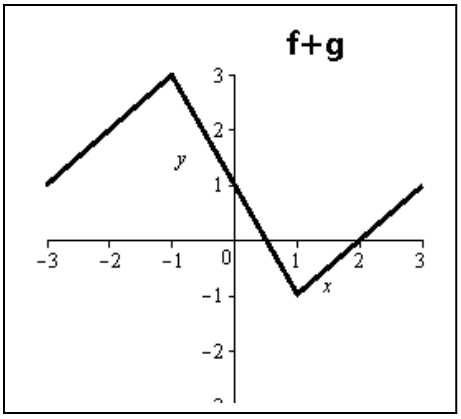
a. -1

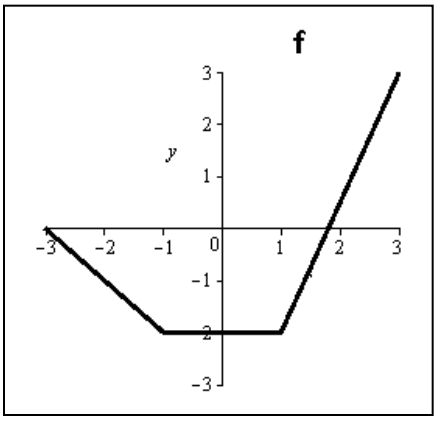
b. 1

c. -3/2

d. 3/2

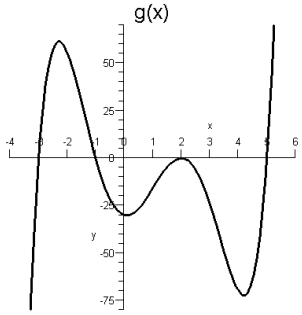
e. 5/2





12. Given the graph, how many inflection points does g have?

a. 2 b. 3 c. 4 d. 5 e. 6



13. Find a polynomial function with real coefficients with zeros 2, -3, 4*i*.

a. $p(x) = (x-2)(x+3)(x-4)$
b. $p(x) = (x+2)(x-3)(x^2+16)$
c. $p(x) = (x-2)(x+3)(x-4i)$
d. $p(x) = (x+2)(x-3)(x+4i)$
e. $p(x) = (x-2)(x+3)(x^2+16)$

14. $\lim_{h\to 0} \dfrac{(x+h)\sin(x+h) - x\sin(x)}{h} =$

a. $x\cos x + \sin x$ b. 0 c. $x\sin x$ d. does not exist e. $x\sin x + \cos x$

15. Find the slope of $f(x) = e^{\cos x}$ at $\left(\dfrac{\pi}{2}, 1\right)$.

a. -1 b. 1 c. e d. e^{-1} e. $e^{-\sin 1}$

16. $\frac{d}{dx}\int_{x^2}^{3}\left(\sqrt{t^2+4}\right)dt =$

a. $\sqrt{x^4+4}$ b. $3-\sqrt{x^2+4}$ c. $-\sqrt{x^4+4}$ d. $-2x\sqrt{x^4+4}$ e. $\frac{x}{\sqrt{x^2+4}}$

17. $\lim_{x\to 2}\left(\frac{x^3-8}{2\ln\left(x^2-3\right)}\right)=$

a. 0 b. 1 c. ∞ d. $-\infty$ e. 3/2

18. Find the equation of the line tangent to $f(x)=\frac{-3x}{5x+1}$ at x = 1.

a. $x+12y=-5$
b. $6x+10y=1$
c. $x-12y=7$
d. $x+24y=23$
e. $6x+10y=7$

19. Using four subintervals, find the difference in estimating $\int_0^2 x^2\,dx$ with a right-hand sum and a left-hand sum.

a. 1.750 b. 2.000 c. 2.625 d. 2.667 e. 3.750

20. The slope of a curve is *k* times the y-value at every point. Find the curve if it passes through (0, b).

a. $f(x)=be^{kx}$
b. $f(x)=kx^2+kx+b$
c. $f(x)=k^x+b$
d. $f(x)=x^k+b$
e. $f(x)=kx^k+b$

21. $\int_0^1 [f(x) + f(x+1)]\,dx =$

a. $\int_0^1 f(2x+1)\,dx$

b. $\int_0^1 [2f(x)+1]\,dx$

c. $\int_0^1 [2f(x) + f(1)]\,dx$

d. $\int_0^2 f(x)\,dx$

e. $\int_0^2 2f(x)\,dx$

22. $\lim_{x\to\frac{\pi}{4}}\left(\frac{4x-\pi}{3\cos(2x)}\right) =$

a. 0 b. 4/3 c. does not exist d. 2/3 e. -2/3

23. Given $\int_2^4 f(x)dx = 3$ and $\int_6^2 f(x)dx = -1$, $\int_4^6 5f(x)dx =$

a. -10 b. 4 c. 10 d. -2 e. 2

24. The derivative of $y = \tan\left((1-2x)^2\right)$ is:

a. $-4\tan(1-2x)$

b. $-4\tan(1-2x)\sec^2(1-2x)$

c. $-2\sec^2\left((1-2x)^2\right)$

d. $4(2x-1)\sec^2\left((1-2x)^2\right)$

e. $-2\sec\left((1-2x)^2\right)\tan\left((1-2x)^2\right)$

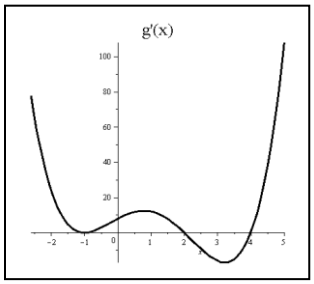
25. Let $f^{-1}(x)$ denote the inverse function of $f(x)=\dfrac{2x+1}{x-3}$. Find the derivative of $f^{-1}(x)$.

a. $\dfrac{-7}{(x-2)^2}$ b. $\dfrac{-7}{(x-3)^2}$ c. $\dfrac{7}{(2x+1)^2}$ d. $\dfrac{3x+1}{x-2}$ e. $\dfrac{7}{(x-2)^2}$

26. Assume that *y* is a differentiable function of *x* and that $x^2+2xy^2=4-ye^y$. Find the slope(s) of the curve at y = 0.

a. -4, 4 b. -2, 2 c. 2 d. -1/2 e. 1/4

27. Given the graph of the derivative of g at the right, g decreases on:



a. $(-5,-1)\cup(1,3.3)$
b. (2, 4)
c. (1, 2)
d. (-1, 2)
e. (-5, -1)

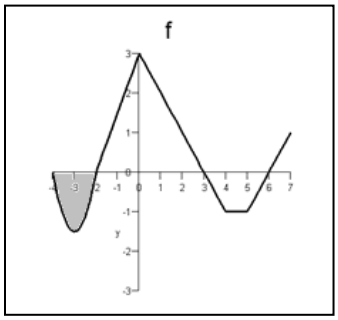
28. If you toss a fair coin 5 times, the probability of obtaining exactly 3 heads is:

a. 3/10 b. 1/32 c. 3/32 d. 5/16 e. 5/32

29. The derivative of $y=(5x+3)^{x^2+1}$ is:

a. $\left(x^2+1\right)(5x+3)^{x^2}$
b. $5\left(x^2+1\right)(5x+3)^{x^2}$
c. $2x(5x+3)^{x^2+1}$
d. $(5x+3)^{x^2}\left[5(x^2+1)+2x(5x+3)\ln(5x+3)\right]$
e. $5\left(\dfrac{x^2+1}{5x+3}\right)+2x\ln(5x+3)$

30. Find the area of the shaded region given that $\int_{-3}^{5} f(x)dx = 5$.



a. -1 b. 1 c. -2 d. 2 e. 4

31. Let k > 0, on what interval(s) does the function f(x) = e^x - kx increase?

a. $(-\infty, \infty)$ b. $(-\infty, \ln k)$ c. $(-\infty, 0)$ d. $(\ln k, \infty)$ e. $(0, e^k)$

32. $\int \left(5\sec^2 t + \frac{3}{\sqrt{1-t^2}} \right) dt$ =

a. $5\tan t + 6\sqrt{1-t^2} + C$

b. $5\tan t + 3\arcsin t + C$

c. $10\sec^2 t \tan t - \frac{3t}{\sqrt{1-t^2}} + C$

d. $5\tan^2 t + 3\arcsin t + C$

e. $5\tan^2 t + 6\sqrt{1-t^2} + C$

33. A rectangle is bounded by the x-axis and the semicircle $y = \sqrt{16 - x^2}$. What is the length of the horizontal side of the rectangle with maximum area?

a. 2.83 units b. 4 units c. 4.82 units d. 5.66 units e. 8 units

34. Find the area bounded by the graph of $f(x) = x^2 - 4$ and the x-axis on the interval $-1 \le x \le 3$.

a. -20/3 b. 20/3 c. 8 d. 34/3 e. 32/3

35. The solution of $\begin{bmatrix} 2 & 4 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$ is:

a. (-7, 5) b. (1, 2) c. (-3, -4) d. (2, 8/7) e. (2, 1)

36. $\int \left(1+\sin\left(2\theta\right)\right) d\theta =$

a. $1-\cos\left(2\theta\right)+C$

b. $1+\dfrac{\cos\left(2\theta\right)}{2}+C$

c. $\theta-\dfrac{\cos\left(2\theta\right)}{2}+C$

d. $\theta+2\cos\left(2\theta\right)+C$

e. $\theta-\cos\left(2\theta\right)+C$

37. An airplane flies at an altitude of 5 miles toward a point directly over an observer. The speed of the plane is 600 miles per hour. Find the rate, in radians per hour, at which the angle of elevation from the observer to the plane is changing when the angle is $\theta = 70^{o}$.

a. 14 rad/hr b. 70 rad/hr c. 72 rad/hr d. 106 rad/hr e. 113 rad/hr

38. Find the area bounded above by y = cos x and below by y = sin x on the interval $\dfrac{-\pi}{2} \le x \le \dfrac{\pi}{2}$.

a. 0 b. $1+\sqrt{2}$ c. $\sqrt{2}$ d. 2 e. 2π

39. $\int t^{2}\sqrt[4]{2t^{3}+5}\,dt$ =

a. $\dfrac{4}{15}t^{3}\left(2t^{3}+5\right)^{5/4}+C$

b. $\dfrac{2}{15}\left(2t^{3}+5\right)\sqrt[4]{2t^{3}+5}+C$

c. $2t\sqrt[4]{2t^{3}+5}+\dfrac{3}{2}t^{4}\left(2t^{3}+5\right)^{-3/4}+C$

d. $\dfrac{4}{5}t^{2}\left(2t^{3}+5\right)^{5/4}+C$

e. $\dfrac{12t^{3}}{\sqrt[4]{\left(2t^{3}+5\right)^{3}}}+C$

40. Find the average value of the function $f(x) = xe^{x}(2+x)$ over the interval $0 \le x \le 4$.

a. $16e^{4}$ b. $8e^{4}$ c. $12e^{4}$ d. $6e^{4}$ e. $4e^{4}$

Extra problems:

41. Using Newton's method to approximate the solution to the equation $x^5 = x - 1$ with an initial guess, x_o= -1, find the next approximation, x_1.

a. -1.25 b. -1.5 c. 1 d. 0 e. -1.2

42. $\lim_{x \to -2^+} \left(9 + \ln\left(3x + 6\right)\right) =$

a. 9 b. 10 c. 9 + ln(12) d. $-\infty$ e. 9ln(12)

43. $\int \frac{4x}{(x-1)(x+3)} dx =$

a. $\ln\left|(x-1)(x+3)^3\right| + C$

b. $\ln\left|\frac{(x-1)^4}{(x+3)^3}\right| + x + C$

c. $2\ln\left|x^2 + 2x - 3\right| + C$

d. $\frac{1}{2}\arctan(x+1) + C$

e. $\ln\left|(x-1)\right| - 3\ln\left|(x+3)\right| + C$

2009 Calculus and Advanced Topics Answer Key

1.	c
2.	a
3.	e
4.	b
5.	a
6.	d
7.	c
8.	b
9.	b
10.	d
11.	c
12.	b
13.	e
14.	a
15.	a
16.	d
17.	e
18.	a
19.	b
20.	a
21.	d
22.	e
23.	a
24.	d
25.	a
26.	a
27.	b
28.	d
29.	d
30.	d
31.	d
32.	b
33.	d
34.	d
35.	a
36.	c
37.	d
38.	b
39.	b
40.	e
41.	a
42.	d
43.	a