

1. If  $\tan x = -\frac{5}{4}$  where  $\frac{\pi}{2} < x < \pi$ , find  $\sec x$ .

a.  $\frac{\sqrt{41}}{4}$

d.  $\frac{\sqrt{41}}{5}$

b.  $-\frac{\sqrt{41}}{4}$

e.  $\frac{5\sqrt{41}}{41}$

c.  $-\frac{4}{5}$

2. If  $f(x) = \frac{x}{\cot x}$ , then  $f'\left(\frac{\pi}{4}\right) =$

a.  $1 - \frac{\pi}{2}$

d. 2

b.  $1 + \frac{\pi}{2}$

e.  $\frac{3}{4}$

c.  $\frac{\pi}{2} - 1$

3. Find an equation of the circle with center at (4, -2) and a radius equal to 5.

a.  $x^2 + y^2 - 4x + 2y - 5 = 0$

d.  $x^2 + y^2 + 8x + 4y - 5 = 0$

b.  $x^2 + y^2 - 8x - 4y - 5 = 0$

e.  $x^2 + y^2 + 8x + 4y + 5 = 0$

c.  $x^2 + y^2 - 8x + 4y - 5 = 0$

4. Find  $\lim_{x \rightarrow +\infty} \frac{ax^2 - bx + c}{dx^2 + ex - f}$ ,  $d \neq 0$

a.  $-\frac{c}{f}$

d.  $\frac{d}{a}$

b.  $\frac{a}{d}$

e.  $a$

c.  $-\frac{b}{e}$

5. Find  $\lim_{x \rightarrow 0} \frac{\sin ax}{\sin bx}$ ,  $b \neq 0$

a.  $\frac{a}{b}$

d.  $\frac{b}{a}$

b.  $+\infty$

e.  $-\infty$

c. 0

6. A ball is thrown vertically upward from the ground with an initial velocity of 64 ft/sec. If the positive direction of the distance from the starting point is up, the equation of motion is:  $s = -16t^2 + 64t$  What is the instantaneous velocity of the ball at the end of 1 second?
- a. -48 ft/sec
  - b. 48 ft/sec<sup>2</sup>
  - c. 32 ft/sec
  - d. 32 ft/sec<sup>2</sup>
  - e. 64 ft/sec
7. Given  $f(x) = 4\sqrt[3]{x^2}$  Find  $f'(x)$
- a.  $6\sqrt{x}$
  - b.  $\frac{12}{\sqrt[3]{x}}$
  - c.  $\frac{8}{3\sqrt[3]{x}}$
  - d.  $6\sqrt[3]{x}$
  - e.  $\frac{6}{\sqrt[3]{x}}$
8. Find the area above the x-axis and under the curve  $y = (x-2)(4-x)$ .
- a. 16/3
  - b. 4/3
  - c. 12
  - d. -12
  - e. none of the above
9. On what interval/s is the following function concave up?  
 $y = x^4 - 2x^3 - 12x^2 - 6x + 7$
- a.  $x < -1$  or  $2 < x$
  - b.  $x > 1$  or  $x < -2$
  - c.  $-2 < x < 1$
  - d.  $-1 < x < 2$
  - e.  $y$  is never concave up
10. Find  $\frac{dy}{dx}$  for  $y = \left(\frac{x-7}{x+2}\right)^{\frac{1}{3}}$ .
- a.  $\frac{dy}{dx} = \frac{1}{3}\left(\frac{x-7}{x+2}\right)^{-\frac{2}{3}}$
  - b.  $\frac{dy}{dx} = \frac{3}{4}\left(\frac{x-7}{x+2}\right)^{\frac{4}{3}}$
  - c.  $\frac{dy}{dx} = \frac{1}{3}\left[\frac{9}{(x+2)^2}\right]^{\frac{-2}{3}}$
  - d.  $\frac{dy}{dx} = \frac{1}{3}\left(\frac{x-7}{x+2}\right)^{-\frac{2}{3}} \cdot \frac{9}{(x+2)^2}$
  - e.  $\frac{dy}{dx} = \frac{1}{3}\left(\frac{x-7}{x+2}\right)^{\frac{-2}{3}} \cdot \frac{9}{(x+2)}$

11. Find the exact value of  $\int_0^a xe^{x^2} dx$ .

a.  $\frac{e^a}{2}$

d.  $\frac{e^{a^2}}{2} - \frac{1}{2}$

b.  $\frac{e^a}{2} - \frac{1}{2}$

e.  $\frac{1}{2} - \frac{e^{a^2}}{2}$

c.  $\frac{1}{2} - \frac{e^a}{2}$

12. Using the table of values given below, determine  $(f \circ g)'(1)$ .

$x$	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	1	2	3	7
2	5	0	10	9
3	4	8	11	6

a. 8

d. 28

b. 13

e. 56

c. 14

13. If  $\int_0^7 g(x) dx = 8$ ,  $\int_5^7 g(x) dx = 6$ , and  $\int_5^{10} g(x) dx = 9$ , determine the value of  $\int_0^{10} g(x) dx$ .

a. 5

d. 18

b. 7

e. 23

c. 11

14. Let  $p$  be a polynomial function of degree three that has zeros of -1, 1, and 5. If the graph of  $y = p(x)$  passes through the point  $(0, 15)$ , then determine the slope of the tangent line to the graph at that point.

a. -3

d. 1

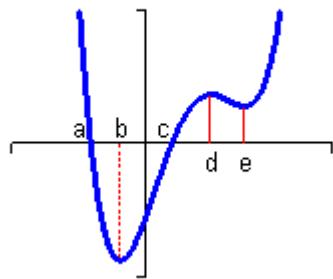
b. -1

e. 3

c. 0

15. The graph of  $y = f'(x)$  is given below. At what value of  $x$  does the graph of  $y = f(x)$  have a local maximum?

Graph of the first derivative of  $f$



- |  |   |
|--|---|
| <p>a. <math>x = a</math><br/>           b. <math>x = b</math><br/>           c. <math>x = c</math></p> | <p>d. <math>x = d</math><br/>           e. <math>x = e</math></p> |
|--|---|
16. If  $f(x) = h(3x) \cdot g(\cos x)$ , then  $f'(x) =$
- $3h(x)g'(\cos x) - (\sin x)h'(3x)g(x)$
  - $3h'(x)g(\cos x) - (\sin x)h(3x)g'(x)$
  - $3h'(3x)g'(\cos x) - (\sin x)h'(3x)g'(\cos x)$
  - $3h'(3x)g(\cos x) - (\sin x)h(3x)g'(\cos x)$
  - $3h(3x)g'(\cos x) - (\sin x)h'(3x)g(\cos x)$
17.  $\sum_{k=1}^{400} k$
- |                                   |                            |
|-----------------------------------|----------------------------|
| a. 400<br>b. 80,200<br>c. 160,400 | d. 21,413,400<br>e. 80,300 |
|-----------------------------------|----------------------------|
18. Four wires (red, green, blue, and yellow) need to be attached to a circuit board. A robotic device will attach the wires. The four wires can be attached in any of 10 slots that are available on the circuit board (one wire per slot). The circuit board will perform different functions depending on which color wire is attached to which slot. How many different ways can these wires be attached (or how many different functions are possible)?
- 5040
  - 210
  - 40
  - 24
  - $10!$

19. A class consists of 12 girls and 10 boys. The teacher is going to select a group of 3 students from the class. This group must contain at least one girl and one boy. How many outcomes are possible?

- |           |         |
|-----------|---------|
| a. 10,648 | d. 2400 |
| b. 9240   | e. 1200 |
| c. 1540   |         |

20. If  $\int f(x)e^x dx = f(x)e^x - \int 2xe^x dx$ , then  $f(x)$  could be:

- |           |          |
|-----------|----------|
| a. $2x$   | d. $e^x$ |
| b. $x^2$  | e. 2     |
| c. $-x^2$ |          |

21. The average value of a continuous function  $y = f(x)$  over the interval  $[a,b]$  is given by the formula:

- |                               |                               |
|-------------------------------|-------------------------------|
| a. $\int_a^b ydx$             | d. $\frac{\int_a^b ydx}{2}$   |
| b. $\frac{\int_a^b ydx}{b-a}$ | e. $\frac{\int_a^b xdx}{b-a}$ |
| c. $\frac{\int_a^b ydx}{b-a}$ |                               |

22. If  $g(x) = \int_0^{3x} t^2 \cos t dt$ , determine  $g'(\frac{\pi}{3})$ .

- |                                |                       |
|--------------------------------|-----------------------|
| a. $\frac{\pi^2 \sqrt{3}}{18}$ | d. $\frac{\pi^2}{18}$ |
| b. $-3\pi^2$                   | e. 0                  |
| c. $-\pi^2$                    |                       |

23. Which of the following statements is/are true:

- |  |                    |
|--|--------------------|
| I. If $f$ is continuous everywhere, then $f$ is differentiable everywhere.                               |                    |
| II. If $f$ is differentiable everywhere, then $f$ is continuous everywhere.                              |                    |
| III. If $f$ is continuous and $f(x) \geq 2$ for every $x$ in $[3, 7]$ , then $\int_3^7 f(x) dx \geq 8$ . |                    |
| a. I only  | d. I and III only  |
| b. II only   | e. II and III only |
| c. III only  |                    |

24. If  $f$  and  $g$  are twice differentiable functions such that  $g(x) = \ln(f(x))$  and

$$g''(x) = h(x)/(f(x))^2, \text{ then } h(x) =$$

- |                             |                             |
|-----------------------------|-----------------------------|
| a. $f(x)f''(x) - 2f'(x)$    | d. $f(x)f''(x) - [f'(x)]^2$ |
| b. $f(x)f''(x) - f'(x)$     | e. $[f(x)f''(x) - f'(x)]^2$ |
| c. $f(x)[f''(x)]^2 - f'(x)$ |                             |

25. A funnel is in the shape of a circular cone with the height equal to the diameter (both 6 inches). Liquid is being poured through it at the rate of 2 cubic inches per minute when it becomes clogged. How fast is the level of liquid rising when it is 2 inches deep? (The

volume of a cone is  $\frac{1}{3}\pi \cdot r^2 h$ .)

- |                           |                           |
|---------------------------|---------------------------|
| a. $\frac{1}{\pi}$ in/min | d. $\frac{\pi}{2}$ in/min |
| b. $\frac{2}{\pi}$ in/min | e. $\pi$ in/min           |
| c. 1 in/min               |                           |

26. Determine the slope of the tangent line to the curve  $3x^2 + 4xy - y^3 = 5$  at the point

$$(2, -1).$$

- |                    |                   |
|--------------------|-------------------|
| a. -12             | d. $-\frac{8}{5}$ |
| b. -8              |                   |
| c. $-\frac{12}{5}$ | e. $\frac{16}{3}$ |

27. If  $y = (2x+1)^{3x}$ , determine  $\frac{dy}{dx}$ .

- |  |  |
|--|--|
| a. $\frac{dy}{dx} = 3x(2x+1)^{3x-1}$   |  |
| b. $\frac{dy}{dx} = 6x(2x+1)^{3x-1}$   |  |
| c. $\frac{dy}{dx} = \frac{3x}{2x+1} + 3\ln(2x+1)$                            |  |
| d. $\frac{dy}{dx} = (2x+1)^{3x} \left[ \frac{6x}{2x+1} + 3\ln(2x+1) \right]$ |  |
| e. $\frac{dy}{dx} = (2x+1)^{3x} \left[ \frac{3x}{2x+1} + 3\ln(2x+1) \right]$ |  |

28.  $\lim_{n \rightarrow \infty} \frac{3}{n} \sum_{i=1}^n \left( \frac{3i}{n} \right)^2 =$

- a. 0
- b. 3
- c.  $\frac{9}{2}$
- d. 9
- e. 27

29. If  $f'(x) < 0$  for all  $0 \leq x \leq 2$  and  $f(0) = 5$ , then which of the following cannot be true?

I. $\int_0^2 f(x) dx = 12$	II. $\int_0^2 f(x) dx = 7$	III. $\int_0^2 f(x) dx = 0$
a. only I		d. I and II
b. only II		e. II and III
c. only III		

30.  $\lim_{h \rightarrow 0} \frac{3^{x+h} - 3^x}{h} =$

- a.  $3^x$
- b.  $\frac{3^x}{\ln 3}$
- c.  $3^x \ln 3$
- d.  $x3^{x-1}$
- e. does not exist

31. Let  $f$  be a function that is continuous on the interval  $[1, 3]$ , differentiable on the interval  $(1, 3)$ , and such that  $f(1) = -5$  and  $f(3) = 5$ . Which of the following are true?

- I. There must be a number  $x_1$  in the interval  $(1, 3)$  such that  $f(x_1) = 0$ .
  - II. There must be a number  $x_2$  in the interval  $(1, 3)$  such that  $f'(x_2) = 5$ .
  - III. There must be a number  $x_3$  in the interval  $(1, 3)$  such that  $f'(x_3) = 0$ .
- a. I only
  - b. II only
  - c. III only
  - d. I and II
  - e. I and III

32.  $\int 2x^3 (x^2 + 6)^5 dx =$

- a.  $\frac{1}{7} (x^2 + 6)^7 - (x^2 + 6)^6 + C$
- b.  $\frac{1}{6} (x^2 + 6)^6 + C$
- c.  $\frac{1}{6} x^2 (x^2 + 6)^6 + C$
- d.  $\frac{1}{12} x^4 (x^2 + 6)^6 + C$
- e.  $\frac{1}{6} x^{12} + 6x^{10} + 90x^8 + 720x^6 + 3240x^4 + 7776x^2 + C$

33. Determine the area bounded between the curves  $x = y^2 - 4$  and  $y = x - 2$ .

- a.  $\frac{5}{6}$   
b.  $\frac{49}{6}$   
c.  $\frac{61}{6}$

- d.  $\frac{27}{2}$   
e.  $\frac{125}{6}$

34. A certain movie theater can accommodate up to 225 guests per show. If the price per ticket is set at \$3.25, then all 225 tickets will be sold. But each \$0.50 increase in ticket price will result in 10 fewer tickets sold. What is the maximum possible revenue for each show?

- a. \$731.25  
b. \$806.25  
c. \$971.25

- d. \$1020.00  
e. \$1051.25

35. For what values of  $p$  and  $q$  is the function  $f(x) = \begin{cases} x^2 + px + q, & \text{if } x \leq 1 \\ 2qx + 3, & \text{if } x > 1 \end{cases}$  differentiable

at  $x = 1$ ?

- a.  $p = 4$  and  $q = 3$   
b.  $p = 8$  and  $q = 6$   
c.  $p = 6$  and  $q = 4$

- d.  $p = 0$  and  $q = 1$   
e.  $p = 2$  and  $q = 0$

36. The coefficient of  $(x-1)^3$  in the Taylor series for  $f(x) = \ln x$  about  $x = 1$  is:

- a.  $\frac{1}{6}$   
b.  $\frac{2}{3}$   
c.  $\frac{1}{2}$

- d.  $\frac{1}{3}$   
e.  $\frac{1}{4}$

37. Let  $x(t) = \frac{1}{2}(e^t + e^{-t})$ ,  $y(t) = \frac{1}{2}(e^t - e^{-t})$  generate a curve from  $t = -1$  to  $t = 1$ . What is the length of this arc?

- a.  $\sqrt{2} \int_0^1 \sqrt{e^{2t} + e^{-2t}} dt$   
b.  $\int_{-1}^1 \sqrt{e^{2t} - e^{-2t}} dt$   
c.  $\sqrt{2} \int_0^1 \sqrt{e^{2t} - 1} dt$

- d.  $\sqrt{2} \int_0^1 \sqrt{e^{2t} - e^{-2t}} dt$   
e.  $\sqrt{2} \int_{-2}^2 \sqrt{e^t + e^{-t}} dt$

38. Using Newton's Method to approximate the root of the equation  $x^3 - x^2 + 2x + 5 = 0$ , with initial approximation  $x_1 = 1$ , determine the next approximation to the root,  $x_2$ .

- |                     |                   |
|---------------------|-------------------|
| a. $-\frac{17}{15}$ | d. $\frac{4}{7}$  |
| b. $-\frac{4}{3}$   | e. $\frac{10}{3}$ |
| c. $-1$             |                   |

39. If  $A$  and  $B$  are both  $2 \times 2$  matrices such that  $\det(A) = 2$  and  $\det(B) = 3$ , then

$$\det(5A - B) = .$$

- a. 7
- b. 8
- c. 47
- d. 53
- e. cannot be determined from the given information

40. Which of the following is an eigenvector of the matrix  $A = \begin{bmatrix} 5 & -2 \\ 2 & 1 \end{bmatrix}$ ?

- |   |  |
|---|--|
| a. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ | d. $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$ |
| b. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ | e. $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$  |
| c. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$ |  |