1. The derivative of $x^2(x+1)^2$ is

   A. $4x^3+6x^2+2x$ B. $4x^3+1$ C. $x^4+2x^3+x^2$ D. $4x^3+4x^2$ E. $4x^2+4x$

2. Evaluate $\int_0^1 \sqrt{x^2-x^4}\,dx$.

   A. 1/6 B. 1/3 C. 1/2 D. 2/3 E. 1

3. Evaluate $\lim_{x\to\infty} \dfrac{\sqrt{4x^2+x-2019\cos x^4}}{x-2019\sin x}$.

   A. $-1/1009$ B. 0 C. 1 D. 2 E. 2019

4. Find the 2019^{th} derivative of $\sin x+\cos x$.

   A. $\sin x\cos x$

   B. $-\sin x-\cos x$

   C. $-\sin x+\cos x$

   D. $\sin x+\cos x$

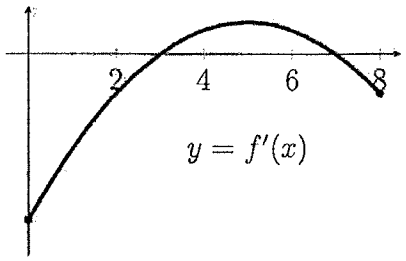
   E. $\sin x-\cos x$

5. The velocity function in meters per second for a particle moving along a line is $v(t)=t^2-4$. Find the distance traveled by the particle during the interval $0\le t\le 5$.

   A. $\frac{65}{3}$ meters B. $\frac{97}{3}$ meters C. 81 meters D. 27 meters E. $\frac{16}{3}$ meters

6. A function $f(x)$ is defined on $[0,8]$. On the right is a graph of its derivative $f'(x)$. If the maximum of $f(x)$ is at $x=a$ and the minimum is at $x=b$, then $a+b$ is

   A. 3 B. 7 C. 8 D. 11 E. 15



7. Oil leaked out from a tank at a rate of $r(t)$ liters per hour. The rate decreased as time passed and values at two hour intervals are shown in the table. Use five subintervals to approximate the upper-sum estimate for the amount of oil that leaked.

t	0	2	4	6	8	10
$r(t)$	8.7	7.6	6.5	6.2	5.5	5.0

A. 61.6 B. 67.0 C. 68.0 D. 69.0 E. 71.6

8. Evaluate $h'(1)$ if $h(x) = f(x)g(x)$ and $f(1) = -3$, $f'(1) = 4$, $f'(-1) = 2$, $g(1) = -1$, $g'(1) = 5$.

A. 10 B. 20 C. -19 D. -17 E. none of these

9. What is the average value of $f(x) = \arccos\cos x - \arcsin\sin x$ on the interval $0 \leq x \leq 2\pi$?

A. 0 B. $1/2$ C. $\pi/4$ D. $\pi/2$ E. $\pi/3$

10. Check if the function $f(x) = x^3 + x - 1$ satisfies the hypotheses of the Mean Value Theorem on $[0, 2]$. If it does, then find all numbers c that satisfy the conclusion of the Mean Value Theorem on that interval.

A. none

B. $\dfrac{\sqrt{3}}{3}$

C. $\dfrac{2\sqrt{3}}{3}$

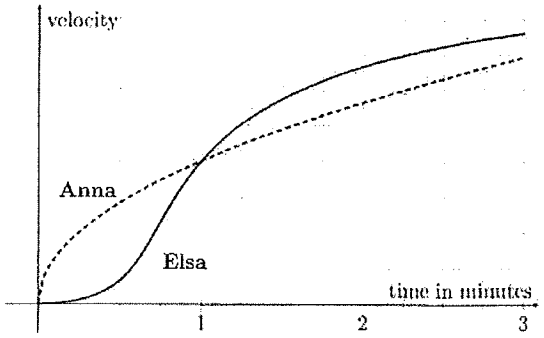
D. $\dfrac{2\sqrt{3}}{3}, -\dfrac{2\sqrt{3}}{3}$

E. The function does not satisfy the hypotheses of the Mean Value Theorem for the given interval.

11. What is the range of the real valued function $\dfrac{\sqrt{1-\cos^2\theta}}{\sin\theta} + \dfrac{\sqrt{1-\sin^2\theta}}{\cos\theta}$?

A. all reals B. $\{-2, 2\}$ C. $\{-2, 0, 2\}$ D. $(-2, 2)$ E. $[-2, 2]$

12. Two cars, Anna and Elsa, start side by side and accelerate from rest. The figure on the right shows the graphs of their velocity functions. Which car is ahead after one minute? then after two minutes?



A. Anna, then Anna

B. Anna, then Elsa

C. Elsa, then Elsa

D. tied, then Anna

E. tied, then Elsa

13. The derivative of x^3e^x is

A. $(x+2)x^2e^x$

B. $(x+3)x^2e^x$

C. $(x+3)x^3e^x$

D. $(x+3)^3e^x$

E. $3x^2e^x$

14. The radius of a sphere is increasing at 2 m/sec. How fast is the volume increasing when the surface area is 7 m^2?

A. 14 m^3/sec B. 9 m^3/sec C. 9π m^3/sec D. π m^3/sec E. $\frac{7\pi}{2}$ m^3/sec

15. Which of the following sums is the largest?

A. $\sum_{n=1}^{\infty} \frac{2019}{n^3}$ B. $\sum_{n=1}^{\infty} \left(\frac{2019}{n^2}\right)^n$ C. $\sum_{n=0}^{\infty} \left(\frac{1}{2019}\right)^n$ D. $\sum_{n=0}^{2019} 2019^n$ E. $\sum_{n=2019}^{\infty} \frac{1}{n \ln n}$

16. Find the volume of the solid generated when the region bounded by $x = 2 + y^2$, $x = 0$, $y = 1$, and $y = 3$ is rotated about the x-axis.

A. 56π B. 61π C. 54π D. 91.07π E. 27π

17. Sometimes an approximation of an integral is sufficient. The value of the integral

$$\int_{-100}^{102} (x-1)^{2019} 2019^{(1-x)^4}\, dx$$

is in which of the following intervals?

A. $(-\infty, -2019]$

B. $(-2019, 0]$

C. $(0, 2019]$

D. $(2019, 2019^{2019}]$

E. $(2019^{2019}, \infty)$

18. Find the area bounded by the two curves $x = y^2$ and $x - 2y = 3$.

A. $\frac{29}{3}$ B. $\frac{32}{3}$ C. $\frac{38}{3}$ D. $\frac{41}{3}$ E. undefined

19. Use implicit differentiation to find the equation of the tangent line to $x^4 + y^4 = 17$ at the point $(1, 2)$ in the form $y = ax + b$. What is $a + b$?

A. 1 B. 2 C. 9/4 D. 17/8 E. 33/16

20. Evaluate the integral $\int_0^1 \frac{1}{x^{99/100}}\, dx$.

A. Undefined B. 1 C. 10 D. 100 E. 1000

21. Which of the following are true statements?

I. There are functions continuous at a point but not differentiable at that point.

II. There are differentiable functions which are not continuous.

III. Every continuous function satisfies the Intermediate Value Property.

The true statements are:

A. I B. II and III C. I and II D. I and III E. I, II and III

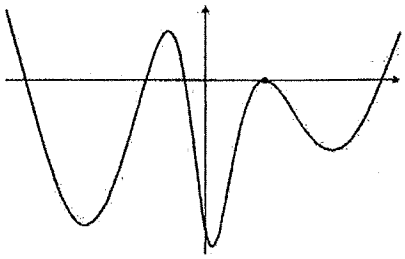
22. Define a function on the reals by $f(x) = \begin{cases} x^2 & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational.} \end{cases}$ What is $\lim_{x\to 0} f(x)$?

A. undefined B. 1 C. ∞ D. 0 E. -1

23. Find the average value of $\cos x$ on the interval from 0 to $\frac{\pi}{3}$.

A. 0 B. $\frac{5}{19}$ C. $\frac{2}{\pi}$ D. $\frac{3\sqrt{3}}{2\pi}$ E. 1

24. On the right is a graph of the second derivative of a function f. How many points of inflection does the original function f have?



A. 3 or less B. 4 C. 5 D. 6 E. 7 or more

25. Five students evaluated the integral $\int \sin\theta\cos\theta\, d\theta$ and got the following five answers. Which one is not correct?

A. $\frac{1}{2}\sin^2\theta + C$

B. $-\frac{1}{2}\cos^2\theta + C$

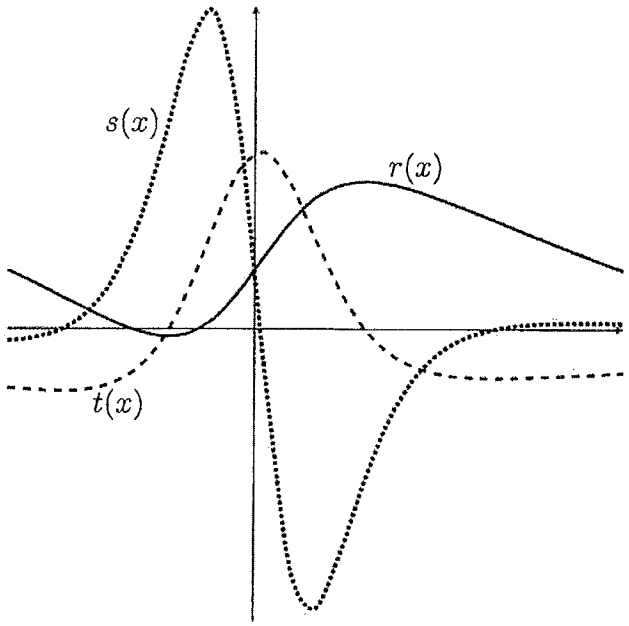
C. $\frac{1}{4}\sin 2\theta + C$

D. $-\frac{1}{4}\cos 2\theta + C$

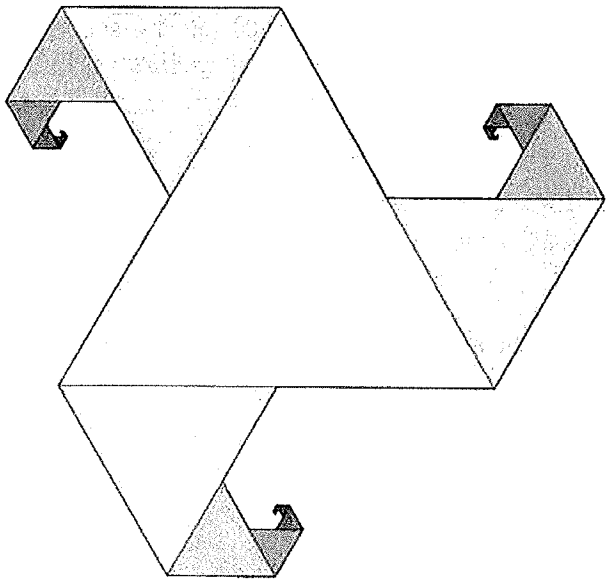
E. $\frac{1}{4}(\sin^2\theta - \cos^2\theta) + C$

26. Below are the graphs of $f(x)$, $f'(x)$ and $f''(x)$. The graphs labelled $r(x)$ and $s(x)$ are (in order):

A. $f(x), f'(x)$ B. $f(x), f''(x)$ C. $f'(x), f''(x)$ D. $f''(x), f'(x)$ E. $f''(x), f(x)$



27. Sofia started with an equilateral triangle of area A. To each side she added an equilateral triangles with half the side length. To each of those she added equilateral triangles with half the side length, and she kept this up creating the shape shown on the right. What is the total area of all of her triangles?



A. $7A/3$

B. $2A$

C. $5A/3$

D. $\sqrt{3}A$

E. $8A/3$

28. Which of these is not equal to the others?

A. $0.9999\ldots$ B. 1 C. $\frac{1}{2}+\frac{1}{4}+\frac{1}{7}+\frac{1}{14}+\frac{1}{28}$ D. $(0.0001)^0$ E. $\frac{\sqrt{10^{100}+1}}{10^{50}+1}$

29. Which of the following is the smallest? (Here x is measured in degrees.)

A. $(0.0001)^0$ B. $0.9999\ldots$ C. 1 D. $\lim_{x\to 0}\frac{\sin x}{x}$ E. $\int_1^e \frac{dx}{x}$

30. How many positive integers satisfy $\log_4 x + \log_x 4 < 4$?

A. 4 B. 37 C. 64 D. 175 E. none of these

31. Which of the following is the smallest when $x = 2019$?

A. $\cos(-10^{-x})$ B. e^{-x^2} C. $(-e^{-x})^2$ D. 10^{-x} E. $\sin(-e^{-x})$

32. Let f be a continuous function defined on $[0,3]$ such that $f(x) + f(3-x) = 2$ for all $x \in [0,3]$. What is $\int_0^3 f(x)\,dx$?

A. -1 B. 0 C. 1 D. 2 E. 3

33. Against her Calculus teacher's advice, Sally measures her variable θ only in degrees. If θ is measured in degrees, what is the derivative of $\cos\theta$?

A. $-\pi\sin\theta$ B. $-180\sin\theta$ C. $-\sin\theta$ D. $-\frac{180}{\pi}\sin\theta$ E. $-\frac{\pi}{180}\sin\theta$

34. For $0 < \theta < \frac{\pi}{2}$, what does $\dfrac{\sqrt{1-\cos^2\theta}}{\sin\theta} + \dfrac{\sqrt{1-\sin^2\theta}}{\cos\theta}$ equal?

A. $\csc\theta + \sec\theta$ B. $\tan\theta$ C. 0 D. 1 E. 2

35. If $b > a > 0$ and $\theta = \sin^{-1}(a/b)$, then what is $\tan\theta$?

A. $\dfrac{a}{\sqrt{a^2+b^2}}$ B. $\dfrac{\sqrt{a^2+b^2}}{b}$ C. $\dfrac{b}{\sqrt{b^2-a^2}}$ D. $\dfrac{a}{\sqrt{b^2-a^2}}$ E. $a+b$

36. Which of the following approximations is closest to $\ln(\ln(e^{2019}+1)) - \ln(\ln(e^{2019}))$?

A. 0 B. $\frac{e^{-2019}}{2019}$ C. e^{-2019} D. $\frac{2019}{e^{2019}}$ E. $\frac{2019^2}{e^{2019}}$

37. What is the sum of the following?

$$\frac{1}{1} + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \cdots + \frac{1}{1+2+3+\cdots+2019}$$

A. 2019/1011
B. 2019/1010
C. 4039/2019
D. 4037/2019
E. none of the above

38. Suppose we define a function $d(n)$ on the set of integers by letting $d(p)$ be one for every prime and extend to the other integers using the "product rule" $d(mn) = d(m)\,n + m\,d(n)$. (This is function is called the arithmetic derivative.) What is $d(36)$?

A. 4 B. 9 C. 24 D. 27 E. 60

39. What positive value of a is a solution of the equation $\int_e^{a^e} \frac{dx}{x\int_a^{ax^2} \frac{dy}{y}} = 1$.

A. e^e B. $\sqrt{e}$ C. e D. $e/2$ E. $2e$

40. Let $f(x) = x^{x^{x^{\cdots}}}$ for $e^{-e} < x < e^{1/e}$. What is $f'(x)$ on this interval?

A. $f(x)$

B. $\dfrac{xf(x)}{x(1-\ln f(x))}$

C. $\dfrac{f(x)^2}{x(1-\ln x)}$

D. $\dfrac{f(x)^2}{x(1-\ln f(x))}$

E. $\dfrac{f(x)^2}{x}(1-\ln f(x))$