TMTA Calculus and Advanced Topics

1. Find the limit:
$$\lim_{x \to 0} \frac{\tan(2x)}{7x}.$$

(a) $\dfrac{2}{7}$
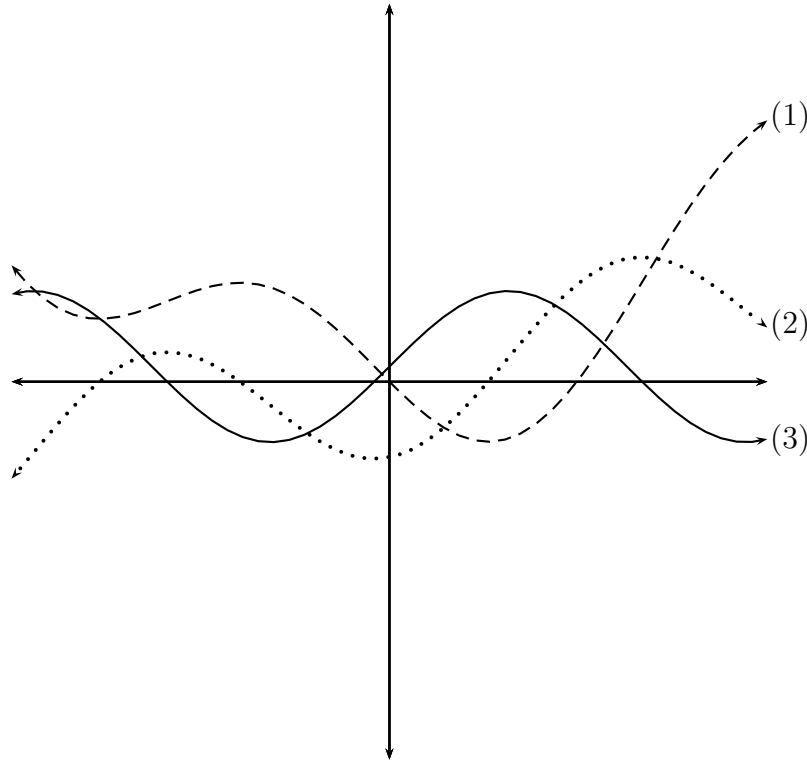
(b) 0

(c) $\dfrac{7}{2}$

(d) ∞

(e) 1

2. Find the equation of the tangent line to the curve $y = \dfrac{x}{x^2 + 1}$ at the point $(2, 0.4)$.

(a) $y = 0.25x - 0.1$

(b) $y = -0.12x + 0.64$

(c) $y = -0.25x + 0.9$

(d) $y = -0.1x + 0.6$

(e) $y = 0.1x + 0.2$

3. Let $f(x) = \displaystyle\int_0^{x^3} \sqrt{t^3 + 1}\, dt$. Find $f'(x)$.

(a) $\sqrt{x^6 + 1}$

(b) $3x^2\sqrt{x^9 + 1}$

(c) $\sqrt{x^9 + 1}$

(d) $\sqrt{x^3 + 1}$

(e) $x^3\sqrt{x^3 + 1}$

4. The following is the graph of the functions $f(x)$, $f'(x)$, and $f''(x)$. Indicate which curves correspond to which functions.

(a) $f(x)$ is curve (1), $f'(x)$ is curve (2), $f''(x)$ is curve (3).

(b) $f(x)$ is curve (3), $f'(x)$ is curve (1), $f''(x)$ is curve (2).

(c) $f(x)$ is curve (2), $f'(x)$ is curve (3), $f''(x)$ is curve (1).

(d) $f(x)$ is curve (2), $f'(x)$ is curve (1), $f''(x)$ is curve (3).

(e) $f(x)$ is curve (1), $f'(x)$ is curve (3), $f''(x)$ is curve (2).

5. Which of the following functions is continuous but not differentiable at $x = 0$?

(a) $\dfrac{1}{x}$

(b) $\sqrt[3]{x}$

(c) $\ln(x^2 + 1)$

(d) $\dfrac{|x|}{x}$

(e) None of the above

6. Find the average value of the function $f(x) = \dfrac{1}{x^2 + 1}$ on the interval $[-1, 1]$.

(a) $\dfrac{\pi}{4}$

(b) $\dfrac{3}{4}$

(c) $\dfrac{5}{6}$

(d) $\dfrac{\pi}{5}$

(e) None of the above

7. Which of the following is NOT an antiderivative of $\sin(2x)$?

(a) $\dfrac{1}{2}(\sin^2 x - \cos^2 x)$

(b) $\sin^2 x$

(c) $-\dfrac{1}{2}\cos(2x)$

(d) $-\cos^2 x$

(e) None of the above

8. Find the area of the region bounded between the curves $y = 3 - x$, $y = 2^x$, and the y-axis.

(a) $\dfrac{3}{2} + \ln 2$

(b) $\dfrac{3}{2}$

(c) $\dfrac{5}{2} - \ln 2$

(d) $\dfrac{3}{2} + \dfrac{1}{\ln 2}$

(e) $\dfrac{5}{2} - \dfrac{1}{\ln 2}$

9. Let $y = e^x \cos x$. Find y'.

(a) $-e^x \sin x$

(b) $-xe^x \sin x$

(c) $e^x(\cos x + \sin x)$

(d) $e^x \sin x$

(e) $e^x(\cos x - \sin x)$

10. Find all critical numbers of the function $f(x) = \dfrac{1}{x^2 - 4}$.

(a) 0

(b) -2 and 2

(c) There are no critical numbers.

(d) 0 and 2

(e) -2, 0, and 2

11. The velocity of a particle moving along the number line is given by $v(t) = 4t - t^2$. Find the total distance traveled by the particle between $t = 1$ and $t = 5$ seconds.

(a) 8 units

(b) $\dfrac{20}{3}$ units

(c) $\dfrac{14}{3}$ units

(d) $\dfrac{34}{3}$ units

(e) None of the above

12. Find the limit:
$$\lim_{x \to 1} \frac{1 - x^2}{x^{11} - 1}.$$

(a) $\dfrac{2}{11}$

(b) The limit does not exist.

(c) ∞

(d) $-\infty$

(e) $-\dfrac{2}{11}$

13. Let $f(x)$ be a differentiable function where $f'(1) = 5$ and $f(1) = -2$. Find the derivative of $[f(x)]^3$ evaluated at $x = 1$.

(a) -60

(b) 60

(c) 0

(d) -150

(e) 125

14. Consider the polynomial function $f(x) = x^4 + x^3 + cx^2 + 5$. Find all the values of c where f has no inflection points.

(a) $c = \dfrac{3}{8}$

(b) $c > \dfrac{3}{8}$

(c) $c \geq \dfrac{3}{8}$

(d) $c \leq \dfrac{3}{8}$

(e) $c < \dfrac{3}{8}$

15. Find the sum:

$$\sum_{n=1}^{500} \binom{1001}{n}.$$

(a) $2^{1000} - 1$

(b) 2^{1001}

(c) 2^{500}

(d) $2^{1001} - 1$

(e) 2^{1000}

16. Find the derivative $y = x^x$.

(a) $y' = x^x e^x$

(b) $y' = x^x(1 + e^x)$

(c) $y' = x^x(1 + \ln x)$

(d) $y' = x^x \ln x$

(e) $y' = x^x$

17. Find the limit:

$$\lim_{n \to \infty} \left(\frac{2n + 1}{2n} \right)^n$$

(a) e

(b) ∞

(c) $\sqrt{e}$

(d) 1

(e) e^2

18. Find the measure of the acute angle, to the nearest degree, formed by the line with equation $y = \dfrac{1}{4}x + 2$ and the x-axis.

(a) $15°$

(b) $25°$

(c) $18°$

(d) $11°$

(e) $14°$

19. Let $y = \sqrt[5]{x}$. Use the tangent line to the curve at the point $(32, 2)$ to approximate the value of $\sqrt[5]{33}$. Round to the nearest ten thousandth.

(a) 2.0124

(b) 2.0122

(c) 2.0126

(d) 2.0125

(e) 2.0123

20. The master code on burglar alarm consists of 5 digits. How many different codes are possible if consecutive digits in the code may not be identical? For example, 12121 is a valid code, but 12212 is not.

(a) 15120

(b) 59049

(c) 30240

(d) 100000

(e) 65610

21. The amount (in milligrams) of an active ingredient in a pain reliever medication is given by the equation $y(t) = me^{-0.012t}$ in a patient's body. The value t is the number of minutes after an m milligram dose. How long does it take for there to be 15% of the initial dose remaining in the patient's body?

(a) 158 minutes

(b) 71 minutes

(c) 16 minutes

(d) 142 minutes

(e) None of the above

22. Approximate the value of $\displaystyle\int_1^2 \sqrt[3]{x}\,dx$ with 4 subintervals using left endpoints.

    (a) 1.107

    (b) 1.166

    (c) 1.172

    (d) 1.140

    (e) 1.218

23. Which of the following functions is a solution to the differential equation $f''(x) = f(x)$?

    (a) $f(x) = e^x \cos x$

    (b) $f(x) = \sin x$

    (c) $f(x) = \dfrac{1}{e^x}$

    (d) $f(x) = e^x \sin x$

    (e) $f(x) = \cos x$

24. Find all of the inflection points of the curve $y = e^{-x^2}$.

    (a) $(0, 1)$

    (b) $\left(-1, \dfrac{1}{e}\right)$ and $\left(1, \dfrac{1}{e}\right)$

    (c) $\left(-\sqrt{2}, \dfrac{1}{e^2}\right)$ and $\left(\sqrt{2}, \dfrac{1}{e^2}\right)$

    (d) $\left(-\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{e}}\right)$ and $\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{e}}\right)$

    (e) The curve has no inflection points.

25. Let $f(x) = \dfrac{e^x + e^{-x}}{2}$ where $x > 0$. Find $f^{-1}(x)$.

    (a) $\ln(x + \sqrt{x^2 - 1})$

    (b) $\ln(\sqrt{x^2 + 1} + x)$

    (c) $\ln(x - \sqrt{x^2 - 1})$

    (d) $\dfrac{2}{e^x + e^{-x}}$

    (e) $\ln(\sqrt{x^2 + 1} - x)$

26. Let $f(x) = \sqrt[3]{x^2}$ on the interval $[-8, 8]$. Find all the values c guaranteed by the Mean Value Theorem.

(a) $\sqrt[3]{2}$

(b) -2 and 2

(c) 0

(d) $-\sqrt{2}$ and $\sqrt{2}$

(e) The Mean Value Theorem does not apply.

27. Let $f(x) = \int_x^1 t^n \, dt$ where n is a positive integer. Find $f^{(n)}(x)$.

(a) $-n!$

(b) 0

(c) $-n!x$

(d) $n!$

(e) $n!x$

28. Find the area under the curve $y = x + \sin x$ where $0 \le x \le \pi$.

(a) $\dfrac{23114}{3333}$

(b) $\dfrac{19}{5} + \pi$

(c) $2 + \dfrac{\pi^2}{2}$

(d) $\dfrac{2}{3} + 2\pi$

(e) $\dfrac{12\pi}{5}$

29. Let $y = \sin(2x)$. Find $y^{(9)}$.

(a) $512 \cos(2x)$

(b) $-512 \cos(2x)$

(c) $\sin^9(2x)$

(d) $512 \sin(2x)$

(e) $-512 \sin(2x)$

30. How many points of inflection does the curve $y = \sin x + \cos(2x)$ have on the interval $(0, 2\pi)$?

(a) 5

(b) 1

(c) 4

(d) 3

(e) 2

31. A cylindrical can is required to have a volume of π cubic units. Find the height of such a can that minimizes the total surface area.

(a) $2\sqrt[3]{2}$

(b) $\dfrac{1}{\sqrt[3]{4}}$

(c) $\sqrt[3]{4}$

(d) $\dfrac{1}{\sqrt[3]{2}}$

(e) 1

32. Consider the region contained within the first quadrant that is bounded by the line $x = 1$ and the curve $y = \sqrt{1 - x^2} + 1$. Find the volume of the solid obtained by rotating the region about the x-axis.

(a) $\dfrac{5\pi}{3}$

(b) $\dfrac{\pi^2}{4} + \pi$

(c) $\dfrac{5\pi}{3} + \dfrac{\pi^2}{2}$

(d) $1 + \dfrac{\pi}{4}$

(e) $\dfrac{5\pi}{3} - \dfrac{\pi^2}{2}$

33. Find the coefficient of $x^7 y^5$ in the expansion of $(x - 2y)^{12}$.

(a) -792

(b) -4096

(c) 792

(d) -25344

(e) 25344

34. If $x^y = e^x$, find y'.

(a) $\dfrac{\ln x}{1 + e^x}$

(b) $\dfrac{1}{1 + \ln x}$

(c) $\dfrac{\ln(x) - 1}{(\ln x)^2}$

(d) $\dfrac{\ln x}{1 + \ln x}$

(e) $\dfrac{e^x}{1 + \ln x}$

35. Find the value of $\sqrt{3 + \sqrt{3 + \sqrt{3 + \sqrt{3 + \cdots}}}}$.

(a) $\dfrac{4\sqrt{3}}{3}$

(b) $\dfrac{23}{10}$

(c) $\dfrac{1 + \sqrt{13}}{2}$

(d) $\dfrac{7\sqrt{7}}{8}$

(e) $\dfrac{3\pi}{4}$

36. Find the limit:
$$\lim_{x \to 0} \frac{x^5 + x^3 - 3x + 3\tan^{-1} x}{x^5}.$$

(a) $-\infty$

(b) ∞

(c) $\dfrac{8}{5}$

(d) $\dfrac{\pi}{2}$

(e) 0

37. When an oven is set at 300°F, its actual temperature as a function of time is

$$T(t) = 300 + 25 \cos\left(\frac{t\pi}{20}\right)$$

where t is the number of minutes since the oven finished preheating. What is the average temperature of the oven between times $t = 0$ and $t = 30$?

(a) $300 + \dfrac{25}{3\pi}$

(b) 300

(c) $300 + \dfrac{50}{3\pi}$

(d) $300 - \dfrac{25}{3\pi}$

(e) $300 - \dfrac{50}{3\pi}$

38. The graph below depicts the curve $y = 4x - x^2$ and the line through the origin and the point P. Find the x-coordinate of the point P if the regions R_1 and R_2 have equal area.

(a) π
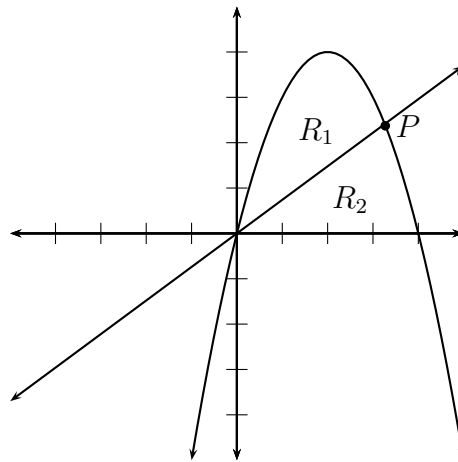
(b) $\sqrt{10}$

(c) $\dfrac{16}{5}$

(d) $\dfrac{127}{40}$

(e) $\sqrt[3]{32}$

39. Find the limit:
$$\lim_{n\to\infty} \frac{2}{n} \sum_{k=1}^{n} e^{\frac{4k}{n}}$$

(a) $2e^4$

(b) $\frac{1}{2}(e^4 - 1)$

(c) $2(e^4 - 1)$

(d) e^4

(e) $e^4 - 1$

40. The size of a population of squirrels in a park can be modeled by the equation
$$y = \frac{2000}{1 + 199e^{-0.21t}}$$

where t is the number of years since 2000. For which value of t is the population of squirrels growing the fastest?

(a) 25.3

(b) 24.9

(c) 25.2

(d) 25.0

(e) 25.1