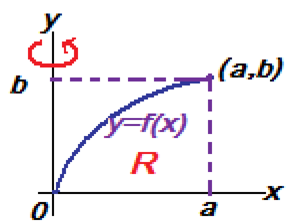
1. Consider two functions $f(x)$ and $g(x)$ defined on an interval I containing 2. $f(x)$ is continuous at $x=2$ and $g(x)$ is discontinuous at $x=2$. Which of the following is true about functions $f+g$ and $f\cdot g$, the sum and the product of f and g, respectively?
   (A) both are always discontinuous at $x=2$
   (B) both can be continuous at $x=2$
   (C) both are always continuous at $x=2$
   (D) $f+g$ can be continuous at $x=2$ but $f\cdot g$ is always discontinuous at $x=2$
   (E) $f+g$ is always discontinuous at $x=2$ but $f\cdot g$ can be continuous at $x=2$.

2. Oil spilled from a ruptured tanker spreads in a circle whose area increases at a constant rate of 6 mi^2/hr. How fast is the radius of the spill increasing when the area is 9 mi^2?
   (A) $\sqrt{\pi}\ mi/hr$ (B) $\frac{1}{\sqrt{\pi}}\ mi/hr$ (C) $\pi\ mi/hr$ (D) $\frac{1}{\pi}\ mi/hr$ (E) $\frac{1}{3\pi}\ mi/hr$

3. Region ***R*** is enclosed by the *x*-axis, the vertical line $x=a$, and a one-to-one function $y=f(x),\ 0\le x\le a$ satisfying $f(0)=0$ and $f(a)=b$. Which of the following definite integral describes the volume of the solid ***S*** obtained by rotating the region R about the *y*-axis?



$$\text{I} = \int_0^a 2\pi x f(x)dx$$

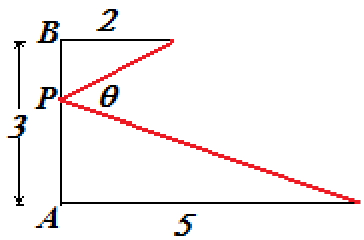
$$\text{II} = \int_0^b \pi\{a^2 - [f^{-1}(y)]^2\}\,dy$$

$$\text{III} = \int_0^b 2\pi y[f^{-1}(y)]dy$$

$$\text{IV} = \int_0^a \pi\{b^2 - [f(x)]^2\}\,dx$$

   (A) only I (B) I and II (C) I and III (D) III and IV (E) I and IV

4. Where should the point P be chosen on the line segment AB so as to maximize the angle θ?



   (A) $\frac{\sqrt{5}-2}{2}$ above A (B) $\frac{5-\sqrt{5}}{2}$ above A (C) $5+2\sqrt{5}$ above A
   (D) $5-2\sqrt{5}$ above A (E) $\frac{2+\sqrt{5}}{2}$ above A

5. Given $f'(a)$ exists, which of the following are always true?

(1) $f'(a)=\lim_{h\to a}\frac{f(h)-f(a)}{h-a}$ (2) $f'(a)=\lim_{h\to 0}\frac{f(a)-f(a-h)}{h}$

(3) $f'(a)=\lim_{h\to 0}\frac{f(a+2h)-f(a)}{h}$ (4) $f'(a)=\lim_{h\to 0}\frac{f(a+2h)-f(a+h)}{2h}$

(5) $f'(a)=\lim_{h\to 0}\frac{f(a+h)-f(a)}{h-a}$

(A) only (1) is true (B) only (1) and (2) are true (C) only (1) and (4) are true
(D) only (2) and (3) are true (E) all are true

6. Let $f(x)=\begin{cases}\frac{1}{|x|} & ,\text{if } |x|>c \\ a+bx^2 & ,\text{if } |x|\le c\end{cases}$ for $c>0$. If $f'(c)$ exists then

(A) $a\cdot b=-1$ (B) $a^3=-8b$ (C) $a^3=-(8/9)b$ (D) $a^3=-(27/4)b$
(E) none of the above

7. Let $f(x)=\begin{cases}0 & ,\text{if } x<0 \\ x & ,\text{if } 0\le x\le 2 \\ 4-x & ,\text{if } 2<x\le 4 \\ 0 & ,\text{if } x>4\end{cases}$ and $g(x)=\int_{x/2}^{x^2} f(t)\,dt$. Then $g'(\frac{\pi}{2})=$

(A) $4-\frac{\pi}{4}-\frac{\pi^2}{4}$ (B) $\frac{31\pi}{8}-\frac{\pi^3}{4}$ (C) $4\pi-\frac{\pi^3}{4}$ (D) $\frac{\pi^2}{4}-\frac{\pi}{4}$ (E) does not exist

8. The limit $\lim_{n\to\infty}[\frac{1^9+2^9+3^9+\cdots+n^9}{n^{10}}]$ is best approximated by

(A) $\frac{1}{8}$ (B) $\frac{1}{9}$ (C) $\frac{1}{10}$ (D) $\frac{1}{11}$ (E) $\frac{1}{12}$

9. Consider the equation $\mathbf{M}X=B$, where X and B are column vectors of length 3, and $\mathbf{M}$ is a 3×3 matrix. Suppose the row reduction is used to solve equation for X and the following augmented matrix is obtained.

$$\left(\begin{array}{ccc|c}1 & 4 & 5 & 7\\ 0 & 1 & 6 & 3\\ 0 & 0 & 0 & 0\end{array}\right)$$

Which of the following statement is correct?
I $\mathbf{M}X=B$ has no solution. II $\mathbf{M}X=B$ has infinitely many solutions.
III $\mathbf{M}$ does not have a multiplicative inverse.

(A) I only (B) II only (C) III only (D) both I and III (E) both II and III

10. Evaluate $\int_{-2}^{1} \frac{1}{\sqrt{|x|}} dx$.

(A) $2\sqrt{2}+2$ (B) $2\sqrt{2}-2$ (C) $2-2\sqrt{2}$ (D) $\sqrt{2}+1$
(E) It is a divergent improper integral

11. The length of the curve described by $C:\begin{cases} x = e^t \cos t \\ y = e^t \sin t \end{cases}$ $,0 \le t \le \frac{\pi}{2}$ is

(A) $e^{\pi/2} - \sqrt{2}$ (B) $\sqrt{2}(e^{\pi/2} - 1)$ (C) $\frac{e^{\pi/2}}{\sqrt{2}}$ (D) $(e-2)^{\pi/2}$ (E) $\sqrt{2}e^{\pi/2}$

12. The interval of convergence of the series $\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$ is

(A) $(-\frac{13}{3},-\frac{11}{3})$ (B) $[-\frac{13}{3},-\frac{11}{3})$ (C) $(-\frac{13}{3},-\frac{11}{3}]$ (D) $[-\frac{1}{3},\frac{1}{3})$ (E) $(-\frac{1}{3},\frac{1}{3}]$

13. Mr. Calculus has $2012 and he wants to put in a saving account to earn some interest for 2 years. Below are the options offered by the TMTA bank:

Option I. 1.38% APR (Annual Percentage Rate) , interest compounded every 4 months
Option II. 1.33%APR, interest compounded every 6 months
Option III. 1.27%APR, interest compounded continuously.

Reviewing these plans, Mr. Calculus ranks them, according to the interest earned at the end of 2 years, to be:

(A) Option II > Option I > Option III
(B) Option I > Option III > Option II
(C) Option I > Option II > Option III
(D) Option III> Option I > Option II
(E) Option II > Option III > Option I

14. The sum of the series $1-e+\frac{e^2}{2!}-\frac{e^3}{3!}+\frac{e^4}{4!}-\cdots$ is

(A) $e^{\frac{1}{e}}$ (B) $\frac{1}{1-e^{-1}}$ (C) e^{-e} (D) $\frac{1}{1+e^{-1}}$ (E) It diverges

15. Let $a_n := \frac{2n+1+\sqrt{n^2+n}}{\sqrt{n}+\sqrt{n+1}}$ for positive integers *n*. Then $a_1+a_2+a_3+\cdots+a_{2012} =$

(A) $2012\sqrt{2012}$ (B) $2013\sqrt{2013}$ (C) $2012\sqrt{2012}$ -1
(D) $2013\sqrt{2013}-1$ (E) $2012\sqrt{2012}+1$

16. Consider the following statements:

I. If $x = f(t)$ and $y = g(t)$ are differentiable, then $\dfrac{dy}{dx} = \dfrac{dy/dt}{dx/dt}$, $dx/dt \neq 0$.

II. If $x = f(t)$ and $y = g(t)$ are twice differentiable, then $\dfrac{d^2y}{dx^2} = \dfrac{d^2y/dt^2}{d^2x/dt^2}$.

III. The polar curves $r = 1 - \sin 2\theta$ and $r = \sin 2\theta - 1$ have the same graph.

IV. The parametric equations $x = t^2, y = t^4$ have the same graph as $x = t^3, y = t^6$.

(A) only I is true (B) only I and III are true (C) only II is false
(D) only IV is false (E) they are all false.

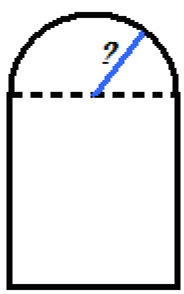
17. A function $f(x)$ has domain [0,2] and range [0,1]. What are the domain and range, respectively, of the function $g(x)$ defined by $g(x) = 1 - f(x+1)$?

(A) [-1, 1], [-1, 0] (B) [-1, 1], [0, 1] (C) [0, 2], [-1, 0] (D) [1, 3], [-1, 0]
(E) [1, 3], [0, 1]

18. The parabola $y = ax^2 + bx + c$ has vertex (p, p) and y-intercept $(0, -p)$, where $p \neq 0$. What is b?

(A) $-p$ (B) 0 (C) 2 (D) 4 (E) p

19. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the length in feet of the radius of the semicircle that will allow the maximum amount of light to enter through the window.



(A) 5 (B) $30/(2+\pi)$ (C) $15/(4+\pi)$ (D) $15/(4-\pi)$ (E) $30/(4+\pi)$

20. If $y + \tan(xy) = 0$, then $\dfrac{dy}{dx} =$

(A) $-\sec^2(xy)$ (B) $-\dfrac{y\sec^2(xy)}{1 + x\sec^2(xy)}$ (C) $-y\sec^2(xy)$ (D) $-\dfrac{x}{\sec^2(xy)}$

(E) $\dfrac{\sec^2(xy)}{1 + x\sec^2(xy)}$

21. If $f(x)$ is differentiable and never equal to 0 on $(-\infty,\infty)$, then the derivative of $\tan^{-1}(\frac{1}{f(x)})$ is equal to

(A) the derivative of $\tan^{-1}(f(x))$
(B) the reciprocal of the derivative of $\tan^{-1}(f(x))$
(C) the square of the derivative of $\tan^{-1}(f(x))$
(D) the negative of the derivative of $\tan^{-1}(f(x))$
(E) none of the above

22. The function $f(x)$ is continuous for $x \in [0,3]$ and has local (relative) minimum at x=1 and x=2. Label statements I, II, and III as **always true**, **sometimes true**, or **never true**.

I. $f'(1)=0$ II. $f(x)$ has an inflection point between x=1 and x=2   III. $f'(2)>0$
(A) I. **always true**, II. **always true**, III. **never true**
(B) I. **sometimes true**, II. **always true**, III. **never true**
(C) I. **sometimes true,** II. **always true,** III. **sometimes true**
(D) I. **sometimes true,** II. **sometimes true,** III. **never true**
(E) I. **sometimes true,** II. **sometimes true,** III. **sometimes true**

23. If $f(x)$ is an invertible function, and $g(x)=2f(x)+5$, what is $g^{-1}(x)$?

(A) $2f^{-1}(x)+5$ (B) $2f^{-1}(x)-5$ (C) $\frac{1}{2f^{-1}(x)+5}$ (D) $\frac{1}{2}f^{-1}(x)-5$ (E) $f^{-1}(\frac{x-5}{2})$

24. Let $u_n > 0$, for n = 1,2,3…. If $\sum_{n=1}^{\infty} u_n$ converges, then

(A) $\sum_{n=1}^{\infty}\sqrt{u_n}$ converges (B) $\sum_{n=1}^{\infty}\sqrt{u_n}$ diverges (C) $\sum_{n=1}^{\infty}\frac{\sqrt{u_n}}{n}$ converges
(D) $\sum_{n=1}^{\infty}\frac{\sqrt{u_n}}{n}$ diverges (E) none of the above

25. The center $C(a,b)$, in rectangular coordinates, of the conic section represented in polar form $r=\frac{3}{4-8\cos\theta}$ is at

(A) $(-\frac{1}{2},0)$ (B) $(0,0)$ (C) $(-\frac{1}{4},0)$ (D) $(-1,-\frac{1}{4})$ (E) $(-1,-\frac{1}{2})$

26. The solution of the equation $7^{x+7}=8^x$ can be expressed in the form $x=\log_b 7^7$. What is b?

(A) 7/15 (B) 7/8 (C) 8/7 (D) 15/8 (E) 15/7

27. Evaluate $\int \frac{x}{x+a}\,dx$

(A) $\frac{x^2}{x^2+2ax}+C$ (B) $x+\frac{x^2}{2a}+C$ (C) $\frac{1}{x+a\ln|x|}+C$ (D) $x-a\ln|x+a|+C$

(E) $\frac{1}{2}x^2\ln|x+a|+C$

28. A car is driving along a dark road whose shape is given by the graph of $y=x^2$. As the car travels along the road from left to right, when will its headlights be pointed directly at the point (3,8)?
(A) When the car is at (4,16) (B) The car's headlights will never point at (3,8)
(C) When the car is at (2,4) and also at (4,16) (D) When the car is at (8/3,64/9)
(E) When the car is at (2,4)

29. An observer is standing exactly 1 mile south of an east-west highway. He watches a car traveling on the highway and notes that when the angle between north and his line of sight to the car is 45°, then that angle is increasing at a rate of $1^\circ/\sec$. How fast is the car traveling at this time?
(A) 40π mile/hr (B) 120 mile/hr (C) 20π mile/hr (D) $10\sqrt{2}\,\pi$ mile/hr
(E) $20\sqrt{2}\,\pi$ mile/hr

30. If $f(x)=e^{2(x-1)}-e^{-2(x-1)}$, find $f^{-1}{}'(0)=\frac{d}{dx}f^{-1}(x)|_{x=0}$

(A) $\frac{1}{2(e^2+e^{-2})}$ (B) $\frac{1}{4}$ (C) f^{-1} is not differentiable at 0 (D) $-2(e^2+e^{-2})$

(E) $-\frac{2(e^2+e^{-2})}{(e^2-e^{-2})^{-2}}$

31. $\lim_{x\to a}\frac{2^x-2^a}{x-a}=$

(A) does not exist (B) $a2^{a-1}$ (C) $2^a\ln 2$ (D) 2^a (E) 0

32. If $f(x)$ is twice differentiable and $g(x)=f(x^2)$, what is $g''(1)$?

(A) $f''(1)+4f'(1)+2f(1)$ (B) $2f''(1)+2f'(1)$ (C) $f''(1)+2f'(2)+f(2)$

(D) $2f(1)f''(1)+2(f'(1))^2$ (E) $4f''(1)+2f'(1)$

33. If $a > 0$, $\int_0^a \frac{\sqrt{x}}{1+x}dx$ equals

(A) $2\ln(1+a)$ (B) $\frac{1}{2}\ln(1+a)$ (C) $2(\sqrt{a} - \tan^{-1}\sqrt{a})$ (D) $\frac{1}{3}a\sqrt{a}\tan^{-1} a$

(E) $2(\frac{1}{3}a\sqrt{a} + \sqrt{a})$

34. Suppose that $f(x)$ is continuous. If $\int_1^2 f(x)dx = 3$, $\int_0^4 f(x)dx = 2$, $\int_0^8 f(x)dx = 7$,

find $\int_1^2 f(4x)dx$

(A) 3/4 (B) 5/4 (C) 12 (D) 9/4 (E) 7/4

35. Suppose $a, b, c, d > 0$. Then $\lim_{x \to -\infty} \frac{\sqrt{ax^2 + b}}{cx + d}$ equals

(A) $\frac{\sqrt{b}}{d}$ (B) $\frac{\sqrt{a}}{c}$ (C) 0 (D) $-\infty$ (E) $-\frac{\sqrt{a}}{c}$

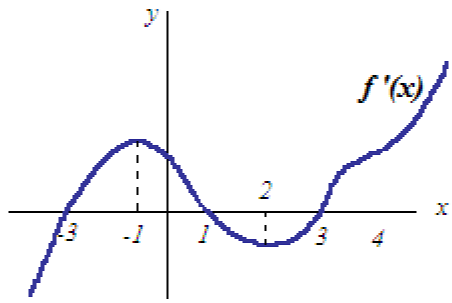
36. $\lim_{x \to \infty} xe^{\frac{a}{x}} - x$ equals

(A) 1 (B) 0 (C) a (D) the limit does not exist (E) ae^a

37. The region bounded by the graphs of $y = x(1-x)$ and $y = 0$ is rotated about the y-axis to form a solid. The volume of the solid is

(A) π (B) $\pi/4$ (C) $\pi/3$ (D) $\pi/6$ (E) $\pi/12$

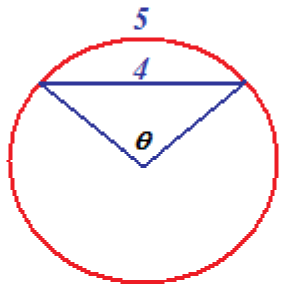
38. The graph of $f'(x)$ is as pictured. When is the graph of $f(x)$ concave up ?



(A) $(-\infty,-1)\cup(2,\infty)$ (B) $(0,3)\cup(4,\infty)$ (C) $(-3,1)\cup(3,\infty)$ (D) $(0,1)\cup(4,\infty)$

(E) $(-\infty,0)\cup(3,4)$

39. In the diagram the arc of the circle has length 5 and the chord has length 4. The measure of the angle θ at the center of the circle, in radians, is best approximated by



(A) 2.28 (B) 2.26 (C) 2.24 (D) 2.22 (E) 2.20

40. In the expansion of $(1+x+x^2+\cdots+x^{27})(1+x+x^2+\cdots+x^{14})^2$, what is the coefficient of x^{28}?

(A) 195 (B) 196 (C) 224 (D) 225 (E) 378

Key:

1	E	11	B	21	D	31	C
2	B	12	B	22	D	32	E
3	B	13	C	23	E	33	C
4	D	14	C	24	C	34	B
5	B	15	D	25	A	35	E
6	D	16	B	26	C	36	C
7	B	17	B	27	D	37	D
8	C	18	D	28	E	38	A
9	E	19	C	29	A	39	**D**
10	A	20	B	30	B	40	D

Prepared by:

Mathematics Faculty
Tennessee Technological University
Cookeville, TN

Coordinated by: Yung-Way Liu