



TENNESSEE MATHEMATICS TEACHER' ASSOCIATION

SIXTY-FOURTH ANNUAL MATHEMATICS CONTEST

2022

Calculus and Advanced Topics

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Scoring formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

2022 TMTA Calculus and Advanced Topics

1. What is the period of $4\cot(2x-3\pi)+12$?

a. $\frac{\pi}{4}$

b. $\frac{\pi}{2}$

c. π

d. 2π

e. 4π

2. What is the equation of the tangent line for $f(x) = \frac{\cos x}{1+x^2}$ at $x = \frac{\pi}{2}$?

a. $y = \frac{(\pi-2x)}{2+\pi^2}$

b. $y = \frac{2(\pi-2x)}{4+\pi^2}$

c. $y = \frac{2(\pi-2x)}{(4+\pi^2)^2}$

d. $y = \frac{2(2x-\pi)}{(4+\pi^2)^2}$

e. $y = \frac{2(2x-\pi)}{4+\pi^2}$

3. What is $f(x)$, given that $f'(x) = 8x^3 - 6x + 2$ and $f(1) = 5$?

- a. $2x^4 - 3x^2 + 2x$
- b. $2x^4 - 3x^2 + 2x + 5$
- c. $2x^4 - 3x^2 + 2x + 4$
- d. $8x^4 - 6x^2 + 2x$
- e. $8x^4 - 6x^2 + 2x + 1$

4. Given the table on the right, what is $(f^{-1})'(5)$ assuming f is invertible?

x	$f(x)$	$f'(x)$
1	1	5
2	4	7
3	5	8
4	7	11
5	9	12

- a. $\frac{1}{8}$
- b. $\frac{1}{3}$
- c. 3
- d. 9
- e. 12

5. What are all local maximums for $f(x) = \sin\left(\frac{1}{x}\right)$ on the interval $(0, \infty)$?

- a. $\frac{1}{\pi k}$ for all nonnegative integer values of k
- b. $\frac{1}{2\pi k}$ for all nonnegative integer values of k
- c. $\frac{1}{4\pi k}$ for all nonnegative integer values of k
- d. $\frac{2}{\pi(1+2k)}$ for all nonnegative integer values of k
- e. $\frac{2}{\pi(1+4k)}$ for all nonnegative integer values of k

6. What is the distance from the point $(0, -15)$ to the line $3x - 5y = 7$?
- a. 11.31
 - b. 11.66
 - c. 14.06
 - d. 16.00
 - e. 16.41
7. Let $f(x) = \frac{1}{\sqrt[3]{x}}$. Using the tangent line at $x = 8$ what is the approximate value of $f(9)$?
- a. 0.4599
 - b. 0.4629
 - c. 0.4792
 - d. 0.4808
 - e. .4822
8. What is the determinant of the matrix $A = \begin{bmatrix} 1 & -1 & 4 \\ 0 & a & -2 \\ 3 & 0 & 2 \end{bmatrix}$?
- a. $-6 + 14a$
 - b. $-6 - 14a$
 - c. $6 + 14a$
 - d. $6 - 10a$
 - e. $-6 - 10a$

9. What is the second derivative of $f(x) = x \cos x$?

- a. $x \sin x + \cos x$
- b. $-x \sin x + \cos x$
- c. $-x \cos x - 2 \sin x$
- d. $(2 - x) \sin x$
- e. $(2 + x) \sin x$

10. What is the coefficient of $x^3 y^8$ in the expansion of $\left(-4x + \frac{y}{2}\right)^{11}$?

- a. $\frac{1}{4}$
- b. $-\frac{1}{4}$
- c. -64
- d. $-\frac{165}{4}$
- e. $\frac{165}{4}$

11. What is the absolute maximum value for the function $f(x) = 2x^3 - 9x^2 + 12x - 1$ on the interval $[0, 3]$?

- a. 1
- b. 2
- c. 3
- d. 4
- e. 8

12. Let $a > 0$. What is the value of $\int_a^\infty \frac{e^{-\sqrt{x}}}{\sqrt{x}} dx$?

- a. $-2e^{\sqrt{a}}$
- b. $2e^{\sqrt{a}}$
- c. $\frac{2}{e^{\sqrt{a}}}$
- d. $-\frac{2}{e^{\sqrt{a}}}$
- e. $\frac{2}{e^a}$

13. Let $f(x) = -(x-1)^2 + 5$ and let $c \in [0, 3]$. The Intermediate Value Theorem can be used to justify that there exists a $c \in [0, 3]$ such that

- a. $f(c) = 0$
- b. $f(c) = 5$
- c. $f'(c) = -3$
- d. $f'(c) = 3$
- e. $f''(c) = 0$

14. What is $\lim_{x \rightarrow 0} \frac{e^{2x} - e^{-2x}}{\sin x}$?

- a. 1
- b. 2
- c. 3
- d. 4
- e. Undefined

15. What is $f'(x)$ given that $f(x) = \int_{x^2}^{4x} \cos(t) dt$?

- a. $4 \cos(4x) - 2x \cos(x^2)$
- b. $4 \cos(4x) - 2 \cos(x^2)$
- c. $\cos(4x) - \cos(x^2)$
- d. $\sin(4x) - \sin(x^2)$
- e. $4 \sin(4x) + 2 \sin(x^2)$

16. What is $\int \sin x \cos^4 x dx$?

- a. $-\frac{1}{4} \cos^4 x + C$
- b. $\frac{1}{5} \sin^5 x + C$
- c. $-\frac{1}{5} \cos^5 x + C$
- d. $-4 \cos^4 x + C$
- e. $4 \sin^4 x + C$

17. For what values of x do the functions $f(x) = x^3 - x$ and $g(x) = \ln \sqrt{x}$ have perpendicular tangent lines?

- a. -1
- b. $\frac{1}{3}$
- c. 1
- d. 1 and $\frac{1}{3}$
- e. -1 and $\frac{1}{3}$

18. A bag contains 10 red, 12 blue, and 8 green marbles. What is the probability that if you pull out 5 marbles, you get 3 red and 2 blue?

- a. 0.006
- b. 0.056
- c. 0.069
- d. 0.151
- e. 0.185

19. A radioactive substance decay exponentially. If we start with 1000g, what is the average quantity present over the first half life?

- a. 721.3
- b. 734.7
- c. 743.9
- d. 750.0
- e. 761.2

20. How many critical points does $f(x) = \frac{1}{x+1} - \frac{1}{x-2}$ have?

- a. 0
- b. 1
- c. 2
- d. 3
- e. 4

21. What is the value of $\int_1^2 4x \ln(x) dx$?

- a. $\ln(4) - 3$
- b. $\ln(4) + 3$
- c. $\ln(16) + 3$
- d. $\ln(256) - 3$
- e. $\ln(256) + 3$

22. Suppose that $f(x)$ is continuous. If $\int_2^3 f(x) dx = 2$, $\int_0^6 f(x) dx = 3$, $\int_0^9 f(x) dx = 7$, what is $\int_2^3 f(3x) dx$?

- a. $\frac{4}{3}$
- b. $\frac{10}{3}$
- c. 4
- d. 6
- e. 12

23. Let $f(x) = \arccos(\sqrt{ax})$ where a is a constant. Find the derivative $f'(x)$.

- a. $-\frac{1}{\sqrt{(1-ax)}}$
- b. $\frac{1}{\sqrt{(1-ax)}}$
- c. $\frac{1}{\sqrt{ax(1-ax)}}$
- d. $\frac{a}{2\sqrt{ax(1-ax)}}$
- e. $\frac{a}{2\sqrt{ax(1-ax)}}$

24. At time $t = 0$, ship A is traveling due south at 16 mi/hr. Ship B, which is initially 32 miles south of A, is traveling due east at 12 mi/hr. Assuming that they stay on these courses, what is the closest they will ever be to each other?

- a. 18.9 mi
- b. 19.2 mi
- c. 19.5 mi
- d. 19.8 mi
- e. 20.1 mi

25. What is the volume of the solid generated by taking the region bounded by $y = x^3$, $x = 0$, $y = 8$ and rotating it about $x = 2$?

- a. $\frac{52\pi}{5}$
- b. $\frac{72\pi}{5}$
- c. $\frac{104\pi}{5}$
- d. $\frac{144\pi}{5}$
- e. $\frac{208\pi}{5}$

26. What is the average value $f(x) = \sin^2 x$ on the interval $\left[0, \frac{\pi}{3}\right]$?

- a. 0.29
- b. 0.36
- c. 0.50
- d. 0.59
- e. 0.88

27. What is the area of the region bounded by the parabola $y^2 = 4x$ and the line $y = 2x - 4$?

a. $\frac{9}{2}$

b. $\frac{19}{3}$

c. $\frac{20}{3}$

d. $\frac{32}{3}$

e. 9

28. Given that $\tan(xy) = xy$, what is $\frac{dy}{dx}$?

a. $\sec^2(xy)$

b. $\frac{y}{x}$

c. $-\frac{y}{x}$

d. $-\frac{x}{y}$

e. $\frac{x}{y}$

29. What is the length of the path along $f(x) = \ln(\sec x)$ from $x = 0$ to $x = \frac{\pi}{4}$?

- a. $\ln(\sqrt{2})$
- b. $\ln(\sqrt{2}-1)$
- c. $\ln(\sqrt{2}+1)$
- d. $\ln\left(\frac{\sqrt{2}}{2}-1\right)$
- e. $\ln\left(\frac{\sqrt{2}}{2}+1\right)$

30. What is the interval of convergence for $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n}$?

- a. $(-1,1)$
- b. $(1,3)$
- c. $[1,3]$
- d. $(1,3]$
- e. $[1,3)$

31. What is the area enclosed by the curve $r = a\sqrt{\cos 2\theta}$ from $\theta = -\frac{\pi}{4}$ to $\theta = \frac{\pi}{4}$?

a. $\frac{\pi}{2}$

b. $\frac{a}{2}$

c. $\frac{\pi^2}{2}$

d. $\frac{a^2}{2}$

e. $\frac{a\pi^2}{2}$

32. What is the Maclaurin series for the function $f(x) = xe^x$?

a. $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

b. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{n!}$

c. $\sum_{n=0}^{\infty} \frac{x^n}{(n+1)!}$

d. $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

e. $\sum_{n=0}^{\infty} \frac{x^{n+1}}{(n+1)!}$

33. If the position of a particle is given by $r(t) = \sin t \vec{i} + \cos t \vec{j} + 3t \vec{k}$, what is its speed at time t , $|v(t)|$?

- a. 2
- b. $2t$
- c. $\sqrt{10}$
- d. $t\sqrt{10}$
- e. $\sqrt{1+9t^2}$

34. Newton's Law of Cooling states that the rate of change of an object's temperature is proportional to the difference between its temperature and the ambient temperature. A bowl of soup with a temperature of 80° is placed in a refrigerator that is kept at a temperature of 45° . If it takes 15 minutes for the temperature of the soup to drop to 70° , approximately how many more minutes will it take for the temperature to drop to 60° ?

- a. 23
- b. 28
- c. 31
- d. 33
- e. 38

35. Let $f(x) = \begin{cases} -2x-3 & x < -1 \\ g(x) & x \geq -1 \end{cases}$. Which of the following choices for $g(x)$ will make $f(x)$ differentiable?

- a. $x^2 + 1$
- b. $(x+1)^2 - 1$
- c. $-(x+1)^2 - 1$
- d. $-(x+2)^2$
- e. $(x-1)^2 - 5$

36. The velocity of a particle moving along the number line is given by $v(t) = t^2 - 5t + 6$. What is the total distance traveled by the particle between $t = 0$ and $t = 5$?

- a. 9
- b. $\frac{55}{6}$
- c. $\frac{37}{3}$
- d. $\frac{19}{2}$
- e. $\frac{37}{2}$

37. An 8-foot high fence is located 1 foot from a building. What is the length of the shortest ladder that can be placed on the ground and leaned against the building such that it also touches the top of the fence?

- a. $4\sqrt{5}$
- b. 10
- c. $5\sqrt{5}$
- d. $5\sqrt{10}$
- e. $8\sqrt{5}$

38. What is the value of $\int_0^1 \frac{-4-x}{(x+2)(x+1)} dx$?

- a. $-\log\left(\frac{32}{9}\right)$
- b. $-\log\left(\frac{9}{32}\right)$
- c. $-\log\left(\frac{5}{3}\right)$
- d. $\log\left(\frac{5}{3}\right)$
- e. $\log\left(\frac{3}{5}\right)$

39. Which of these is an equation for y that goes through the point $(1, 4)$ given that

$$\frac{dy}{dx} = \frac{2xy}{1+x^2} ?$$

- a. $2+2x^2$
- b. $1+x^2$
- c. $3+x^2$
- d. $\frac{1}{2}(1+x^2)$
- e. $\frac{1}{17}(1+x^2)$

40. For the surface $z = 3x^2 + 2y^2 - 11$, what is the equation of the tangent plane at the point $(2, 1, 3)$.

- a. $12x + 4y - z = 0$
- b. $12x + 4y + z = 0$
- c. $12x + 4y - z = 25$
- d. $12x + 4y + z = 31$
- e. $12x + 4y + z = -31$