TWENTY-EIGHTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS 1984

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Scoring Formula: 4R - W + 40

This test was prepared from a list of Advanced Topics questions submitted by the Tri-Cities State Technical Institute.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. The sum of the roots of $x^3 - 2x^2 - 13x - 11 = 0$ is

   a) 11
   b) -11
   c) -2
   d) 2
   e) 0

2. The sum of the squares of the roots of $x^3 - 2x^2 - 13x - 11 = 0$ is

   a) 4
   b) 10
   c) 13
   d) 28
   e) 30

3. The number of solutions of $\sin 2x = \cos 3x$ with $0 \leq x < 360^\circ$ is

   a) 2
   b) 3
   c) 4
   d) 5
   e) 6

4. The median of the set of numbers {1, 10, 20, 50, 99} is

   a) 10
   b) 20
   c) 30
   d) 36
   e) 50

5. Evaluate $\begin{vmatrix} 3 & -1 & 4 \\ 5 & -2 & 6 \\ 7 & 3 & -4 \end{vmatrix}$

   a) 16
   b) 1
   c) -12
   d) 24
   e) 0

6. $3 \sin x - 4 \sin^3 x$ reduces to

   a) $\frac{\cos 3x}{2}$
   b) $\cos 4x$
   c) $\sin 4x$
   d) $\cos 3x$
   e) $\sin 3x$

7. Solve for x: $|2x - 6| = |4 - 5x|$.

a) $\frac{10}{7}$

b) $-\frac{2}{3}$

c) $\frac{10}{7}, -\frac{2}{3}$

d) $3, \frac{4}{5}$

e) $\frac{1}{3}, \frac{5}{4}$

8. $\sqrt[4]{x^3 \sqrt{x^{5/4}}}$ simplifies to

a) $x^{7/8}$

b) $x^{28/33}$

c) $x^{29/32}$

d) $x^{32/29}$

e) None of the above

9. The minimum value of the function $y = 2x^2 - 3x$ occurs at the point whose coordinates are

a) $\left(-\frac{2}{3}, \frac{3}{4}\right)$

b) $\left(\frac{9}{8}, -\frac{3}{4}\right)$

c) $\left(\frac{3}{4}, -\frac{9}{8}\right)$

d) $\left(\frac{2}{3}, -\frac{3}{4}\right)$

e) The function has no minimum value.

10. A particle is moving along a horizontal line. Its position as a function of time is described by the equation

$$s = bt^2 + ct + d \quad (b, c, d \text{ constants})$$

Its velocity will be zero at time t =

a) $-2c/3b$

b) $-c/2b$

c) b/c

d) $2b/c$

e) $-c/b$

11. $\lim_{x \to 4} \frac{x - 4}{3(\sqrt{x} - 2)} =$

a) 17/29

b) 0

c) 5/4

d) 1

e) 4/3

12. The coefficient of the a^2b^{10} term in the expansion of $(a + b)^{12}$ is

a) 144

b) 121

c) 132

d) 44

e) 66

13. $\begin{bmatrix} 1 & 2 \\ 1 & 4 \end{bmatrix}^{-1} =$

a) $\begin{bmatrix} 2 & 1/2 \\ -1 & 1/2 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 1 \\ -1/2 & -1/2 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 1 \\ -1/2 & 1/2 \end{bmatrix}$

d) $\begin{bmatrix} 2 & -1 \\ -1/2 & 1/2 \end{bmatrix}$

e) Operation not possible

14. Find $\sum_{i=1}^{4} f(i)$ where $f(i) = i(i + 3)$

a) 50

b) 60

c) 70

d) 80

e) 90

15. A coin is tossed six times. What is the probability of obtaining more than one head?

a) 55/64

b) 57/64

c) 59/64

d) 61/64

e) None of the above

16. Given $A = \{1,2,\ldots,5\}$, $B = \{4,5,\ldots,10\}$, $C = \{6,7,8,\ldots,15\}$ and $U = \{x \mid x \text{ is a positive integer less than } 20\}$, find $\overline{B} \cup (\overline{A} \cap C)$.

a) $\{1,2,3,6,7,\ldots,19\}$
b) $\{1,2,3,4,6,7,\ldots,19\}$
c) $\{1,2,3,7,8,\ldots,19\}$
d) $\{1,2,3,6,11,12,\ldots,19\}$
e) $\{\emptyset\}$

17. Find the equation of the line tangent to the parabola $x^2 - 2x + y = 0$ at the point $(-3,-15)$.

a) $6x - y + 3 = 0$
b) $7x - y + 6 = 0$
c) $8x - y + 9 = 0$
d) $9x - y + 12 = 0$
e) $10x - y + 15 = 0$

18. The sum of the absolute values of the x-coordinates of the foci of the figure whose equation is $25x^2 - 9y^2 = 225$ is

a) 0
b) $2\sqrt{34}$
c) 8
d) 10
e) $2\sqrt{30}$

19. $(1 + i)^5 =$

a) $4 - 4i$
b) $-2 - 2i$
c) $-4 + 4i$
d) $-4 - 4i$
e) $2 - 2i$

20. A can do a job in 1 hour. B can do it in 2 hours, and C can do it in 3 hours. Halfway through the job with all three working together, C gets sick and must leave. A and B finish the job. How long does A work?

a) 20/33 hour
b) 7/11 hour
c) 41/66 hour
d) 2/3 hour
e) None of the above

21. In the real number system, given $f(x) = \sqrt{2 - 2x - x^2}$, what is the domain of f?

a) $-1 \leq x \leq 1$

b) $-1 - \sqrt{3} < x < -1 + \sqrt{3}$

c) $-1 - \sqrt{3} \leq x < -1 + \sqrt{3}$

d) $-1 - \sqrt{3} > x > -1 + \sqrt{3}$

e) $-1 - \sqrt{3} \leq x \leq -1 + \sqrt{3}$

22. If $f(x) = x^2$, then the difference between $f(x^2 + y^2)$ and $f[f(x)] + f[f(y)]$ must be

a) $(xy)^2$

b) xy

c) $-2xy^2$

d) $2(xy)^2$

e) $2(xy)^4$

23. The region bounded by the graphs $y = x$, $y = 1$, and $x = \frac{8}{y^2}$ lies in the

a) first quadrant

b) second quadrant

c) third quadrant

d) fourth quadrant

e) both first and second quadrant

24. The points A(0,0), B(a,0), and C(b,c) are vertices of a triangle. The coordinates of the point of intersection of the lines containing the segments formed by drawing a perpendicular from each vertex to the opposite side are

a) $\left(b, \frac{c^2 - ab}{c}\right)$

b) $\left(b, \frac{a^2 - ab}{c}\right)$

c) $\left(b, \frac{b^2 - ab}{c}\right)$

d) $\left(b, \frac{ab - b^2}{c}\right)$

e) $\left(b, \frac{b^2 - \sqrt{ab}}{c}\right)$

25. $i^{55} =$

a) 1

b) -1

c) -i

d) i

e) None of the above

26. If $\triangle ABC$ is inscribed in a semicircle with diameter AC and AB =5 and BC = 12, then the area of the semicircle minus the area of $\triangle ABC$ is

a) $(121\pi - 240)/8$
b) $(144\pi - 60)/4$
c) $(169\pi - 240)/8$
d) $(196\pi - 240)/8$
e) None of the above

27. $4\left[\cos \frac{3}{2}\pi + i \sin \frac{3}{2}\pi\right] \cdot 3\left[\cos \frac{\pi}{4} + i \sin \frac{\pi}{4}\right] =$

a) $4\sqrt{2}(1 - i)$
b) $6\sqrt{2}(1 - i)$
c) $12(1 - i)$
d) $4\sqrt{2}(1 + i)$
e) $6\sqrt{2}(-1 + i)$

28. If $5^{x-1} = 3^x$, x =

a) log 5/(log 5 - log 3)
b) log 3/(log 5 - log 3)
c) log 3/log 5
d) log 5/log 3
e) None of the above

29. In how many ways can 5 people be seated around a table if 2 of the people must be seated next to each other?

a) 10
b) 12
c) 14
d) 15
e) 16

30. Find the equation of the tangent to the curve $y = 2x^2 + 3$ at the point (2,11).

a) $y = 4x + 3$
b) $y = x + 9$
c) $y = 8x - 11$
d) $y = 6x - 1$
e) None of the above

31. Find the 58^{th} term of the arithmetic progression 10,4,-2,....

a) -314
b) -320
c) -326
d) -332
e) -338

32. Find the sum of the first 9 terms of the series $\frac{2}{3} + \frac{1}{3} + \frac{1}{6} + \cdots$

a) $\frac{513}{381}$

b) $\frac{947}{697}$

c) $\frac{417}{313}$

d) $\frac{383}{217}$

e) $\frac{511}{384}$

33. If $f(x) = \frac{1}{x-1}$ and $g(x) = x + 1$, then $f \bullet g(x) =$

a) x

b) $(x + 1)^{-1}$

c) $(x - 1)^{-1}$

d) $1/x^2$

e) None of the above

34. Find the distance from the line $x - 7y + 4 = 0$ to the point $(-4,2)$.

a) 2

b) $6\sqrt{2}/4$

c) $8\sqrt{2}/5$

d) $7\sqrt{2}/5$

e) None of the above

35. Find the equation of the line which passes through the center of the circle $x^2 + y^2 - 14x + 10y + 73 = 0$ and is parallel to the line $x + 2y = 5$.

a) $x + 2y + 3 = 0$

b) $x + 2y - 5 = 0$

c) $x + 2y + 4 = 0$

d) $x + 2y + 2 = 0$

e) $x + 2y - 1 = 0$

36. How close to 2 must x be chosen so that $5x - 1$ is within 0.01 of 9?

a) Within 0.001

b) Within 0.002

c) Within 0.004

d) Within 0.01

e) Within 0.02

37. If $\frac{x + 5}{(x - 1)^2 (x + 2)} = \frac{A}{x - 1} + \frac{B}{(x - 1)^2} + \frac{C}{x + 2}$, then A =

a) -2
b) 2
c) -1/2
d) 1/3
e) -1/3

38. The graph of $x^2 + 4xy + 3y^2 + 4x + 3y + 100 = 0$ can best be described as

a) Circle
b) Ellipse
c) Hyperbola
d) Parabola
e) No graph

39. The equation $x^4 + 4x^3 + 7x^2 + 6x + 3 = 0$ has how many distinct real roots.

a) 0
b) 1
c) 2
d) 3
e) 4

40. One possible value of i^i is

a) 0
b) -1
c) e^{π}
d) $e^{\pi/2}$
e) $e^{-\pi/2}$