

TWENTY-SEVENTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS TEST

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Scoring Formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

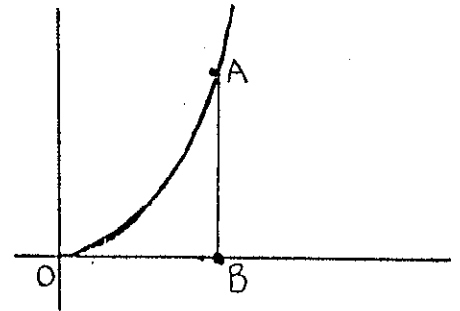
Acme Boot Company, Clarksville, Tennessee
Anderson Ford, Inc., Kingsport, Tennessee
Beasley Distributing Company, Inc., Chattanooga, Tennessee
Berklene Company, Morristown, Tennessee
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee
Coca-Cola Bottling Company of Memphis, Memphis, Tennessee
Commercial and Industrial Bank, Memphis, Tennessee
Department of Mathematics, Shelby State Community College, Memphis, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
Exxon Company, U.S.A., Memphis, Tennessee
First National Bank of Sullivan County, Kingsport, Tennessee
First National Bank, Jefferson City, Tennessee
First People's Bank, Jefferson City, Tennessee
Fulton Siphon Division, Robertshaw Controls, Knoxville, Tennessee
Great Lakes Research Corporation, Elizabethton, Tennessee
Harris Foundation, Johnson City, Tennessee
Holiday Inns, Inc., Memphis, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
Home Federal Savings and Loan Association, Knoxville, Tennessee
IBM Corporation, Chattanooga, Tennessee
IBM Corporation, Kingsport, Tennessee
IBM Corporation, Nashville, Tennessee
Jefferson County Bank, Dandridge, Tennessee
Johnson City Spring and Bedding Company, Johnson City, Tennessee
Klopman Mills, Incorporated, Johnson City, Tennessee
McDonald's Restaurants, Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-OTM), Memphis, Tennessee
Morristown Rotary Club, Morristown, Tennessee
New Jersey Zinc Company, Jefferson City, Tennessee
Provident Life and Accident Insurance Company, Chattanooga, Tennessee
Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Sears, Madison, Tennessee
Shoney's Inc., Nashville, Tennessee
Tennessee Eastman Company, Kingsport, Tennessee
Tennessee Handbag Company, Dandridge, Tennessee
Tri-State Container Corporation, Elizabethton, Tennessee
IRW, Ross Gear Division, Lebanon, Tennessee
Mr. Heeks B. Vaughan, Kingsport, Tennessee

1. A subset is chosen at random from a set of 12 elements, each subset being equally likely. What is the probability that the number of elements in the subset is odd?

- a) $5/11$
- b) $2147/4096$
- c) $1/2$
- d) $2149/4096$
- e) $6/11$

2. What is the area of the region OAB? Curve OA is $y = x^2$, point A is $(3,9)$ and point B is $(3,0)$.

- a) 9
- b) $27/2$
- c) 27
- d) $9\pi/4$
- e) $27 - 9\pi/4$



3. What is $\lim_{x \rightarrow 0} \frac{\tan(2x)}{x}$?

- a) $1/4$
- b) $1/2$
- c) 1
- d) 2
- e) ∞

4. The system of linear equations $4x + 2y + 3z = 7$ has exactly
 $3x + 3y + 4z = 7$
 $x + 5y + 6z = 7$

how many solutions?

- a) None
- b) One
- c) Two
- d) Three
- e) Four

5. What is the value of x in the solution of

$$2x + 3y + 4z = 1$$

$$3x + 4y + 5z = 1$$

$$4x + 5y + 6z = 1$$

- a) -2
- b) -1
- c) 0
- d) 1
- e) All values of x are possible.

6. Evaluate the determinant $\begin{vmatrix} 4 & 2 & 3 \\ 3 & 3 & 4 \\ 1 & 5 & 6 \end{vmatrix}$

- a) 0
- b) 10
- c) 63
- d) 125
- e) 250

7. Let r, s, t and u be the roots of $x^4 - 10x^3 + 35x^2 - 50x + 25 = 0$. Evaluate $r^2 + s^2 + t^2 + u^2$.

- a) 15
- b) 25
- c) 30
- d) 35
- e) 50

8. Let r, s and t be the roots of $x^3 + 2x^2 + 3x + 4 = 0$. Let A, B and C be such that the roots of $x^3 + Ax^2 + Bx + C = 0$ are

$r - 2, s - 2$ and $t - 2$. Then $B =$

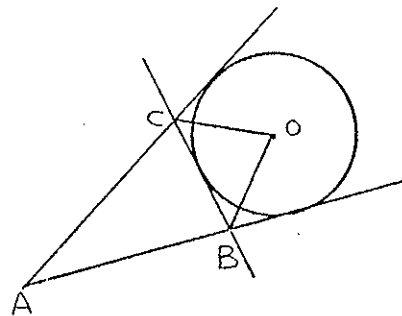
- a) -5
- b) -1
- c) 23
- d) 31
- e) 34

9. What is the sum of the roots of the equation $\sqrt{x-1} = x-7$
- 5
 - 7
 - 10
 - 15
 - 50
10. If $\frac{3x^2 - 6x - 1}{(x-1)(x^2-1)} = \frac{A}{(x-1)^2} + \frac{B}{(x-1)} + \frac{C}{x+1}$ then B =
- 2
 - 1
 - 0
 - 1
 - 2
11. For a set A, let $|A|$ denote the number of elements in A. If A, B and C are sets such that $|A| = 9$, $|B| = 7$, $|C| = 7$, $|A \cup B| = 13$, $|A \cup C| = 12$, $|B \cup C| = 11$, $|A \cup B \cup C| = 15$, what is $|A \cap B \cap C| =$
- 0
 - 1
 - 2
 - 3
 - 4
12. A group of 8 people is to be divided into two committees of three and six members respectively. The chairman of the group is to serve on both committees and is the only member who serves on both committees. In how many ways can the committee assignments be made?
- | | |
|-------|-------|
| a) 19 | d) 42 |
| b) 21 | e) 84 |
| c) 36 | |

13. Let A be the matrix $\begin{bmatrix} 0 & 1 \\ -2 & 2 \end{bmatrix}$; let I denote the identity matrix. What is the value of x such that $A^2 + xA + 2I = 0$.
- a) -2
 - b) -1
 - c) 0
 - d) 1
 - e) 2
14. Let A denote an invertible 2×2 matrix and let A^{-1} be its inverse. If $\begin{bmatrix} a & b \\ c & d \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} A^{-1}$, which of the following must be true?
- a) $a + d = 0$
 - b) $ad + bc = -1$
 - c) $b + c = 0$
 - d) $b - c = 0$
 - e) $a = 1$
15. Let $f(x)$ be a function of one variable and define the function $g(u,v) = f(u - v)$. Then for all u, v
- a) $g(u,v) - g(v,u) = 0$
 - b) $g(u,v) + g(v,u) = 0$
 - c) $2g(u,v) = 0$
 - d) $g(u,u) > 0$
 - e) $g(u,v) = 0$
16. If x and z are positive integers such that $x^2 + 900 = z^2$, then a possible value of $x + z$ is
- a) 18
 - b) 36
 - c) 45
 - d) 50
 - e) 75

17. The curves $x^2 + y = 10$ and $x + y = 8$ intersect in two points. What is the distance between these points?
- 3
 - $3\sqrt{2}$
 - 6
 - 9
 - 18
18. From a point within a triangle, line segments are drawn to the vertices. A necessary and sufficient condition that the three triangles thus found have equal areas is that the point be
- the center of the inscribed circle
 - the center of the circumscribed circle
 - the intersection of the altitudes
 - the intersection of the medians
 - the center of the nine-point circle
19. Triangle ABC is formed by three tangents to circle O and $\angle CAB = 30^\circ$. Angle BOC = ?

- 70°
- 75°
- 85°
- 100°
- 150°



20. If $i^2 = -1$, then $\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i\right)^{-10} =$
- | | |
|--|---------|
| a) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$ | d) $-i$ |
| b) $2^{-10} \left(\frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}i\right)$ | e) -1 |
| c) i | |

21. The nine entries of the 3×3 matrix A consist of three 1's and six 0's placed at random. What is the probability that $\det(A) > 0$?
- a) $1/28$
 - b) $3/28$
 - c) $255/512$
 - d) $1/2$
 - e) 1
22. A particle moves with velocity $1 + 4t$ from $t = 1$ to $t = 3$. What is the total distance travelled?
- a) 4
 - b) 8
 - c) 13
 - d) 18
 - e) 21
23. The median of the set of numbers 35, 40, 45, 50, 60, 90, 100 is
- a) 35
 - b) 50
 - c) 60
 - d) 67.5
 - e) 100
24. The line ℓ is tangent to the curve $y = 2\sqrt{1+x}$ at the point $(3,4)$. What is the y intercept of this line?
- a) $-11/2$
 - b) $1/2$
 - c) $5/2$
 - d) 3
 - e) 4

33. The limit of the sum $\frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \frac{1}{81} + \dots + \frac{1}{3^n}$ as n increases without limit is
- a) $4/9$
 - b) $1/2$
 - c) $2/3$
 - d) 1
 - e) 3
34. A 25 foot ladder is placed against a vertical wall of a building. The foot of the ladder is 7 feet from the building. If the top of the ladder slips 4 feet, the base of the ladder will move
- a) 4 feet
 - b) 5 feet
 - c) 8 feet
 - d) 9 feet
 - e) 13 feet
35. Which of the following is largest?
- a) $2(3^4)$
 - b) $4(3^2)$
 - c) $(2^3)^4$
 - d) $(4^3)^2$
 - e) $2(4^3)$
36. The area of the base of a rectangular box is 2 in^2 ; the area of the front is 4 in^2 ; and the area of the side is 8 in^2 . What is the volume?
- a) 4 in^3
 - b) 8 in^3
 - c) 16 in^3
 - d) 32 in^3
 - e) 64 in^3

