TWENTY-SIXTH ANNUAL MATHEMATICS CONTEST Sponsored by THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS TEST 1982

Edited by: The University of

Tennessee at Martin

Scoring Formula: 4R - W + 40

This test was prepared from a list of Advanced Topics questions submitted by David Lipscomb College and The University of the South

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

Acme Boot Company, Clarksville, Tennessee
Anderson Ford, Inc., Kingsport, Tennessee
Berkline Company, Morristown, Tennessee
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee
Coca-Cola Bottling Company of Memphis, Memphis, Tennessee
Commercial and Industrial Bank, Memphis, Tennessee
Department of Mathematics, Shelby State Community College, Memphis, TN
Exxon Company, U.S.A., Memphis, Tennessee
First National Bank of Sullivan County, Kingsport, Tennessee
First National Bank, Jefferson City, Tennessee
Harris Foundation, Johnson City, Tennessee
Harris Foundation, Johnson City, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
Home Federal Savings and Loan Association, Knoxville, Tennessee
IBM Corporation, Chattanooga, Tennessee
IBM Corporation, Nashville, Tennessee
Jefferson County Bank, Dandridge, Tennessee
Jefferson County Bank, Dandridge, Tennessee
McDonald's Restaurants, Memphis, Tennessee
McDonald's Restaurants, Memphis

- 1. Which of the following defines a function of two variables for which f(-x, -y) = -f(x, y)?
 - a) xy

d) x + y

b) $x^2 + y^2$

e) none of the above

- c) x + y + 1
- 2. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 3A + 2I =$
 - a) $\begin{bmatrix} -1 & 8 \\ 2 & 1 \end{bmatrix}$

 $\begin{array}{ccc} d) & \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$

e) none of the above

- c) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- 3. A class of 20 students will meet in a classroom with 32 student desks. How many different seating assignments are possible?
 - a) $\frac{32!}{12!20!}$

- d) 32 · 20
- e) 20^{32}

- b) $\frac{32!}{12!}$
- c) $\frac{32!}{20!}$
- 4. Let L be a line and Q be a point not on L. Let P denote the plane containing L and Q. The set of points in P that are equidistant from L and Q is:
 - a) A circle whose radius is half the distance between L and ${\tt Q}$.
 - b) A pair of lines parallel to L.
 - c) A parabola whose vertex lies midway between L and Q.
 - d) Two lines through Q, one parallel to L, the other perpendicular to L.
 - e) An ellipse whose major axis lies on L and whose minor axis is perpendicular to L, passing through Q.

- 5. Let f be the function from the real numbers to the real numbers such that $f(x) = \frac{1}{3} \cdot x^3 - \frac{1}{2} \cdot x^2 - 6x$. The point on the graph of f where the slope of the tangent line has its least value is

- a) $(-2, \frac{22}{3})$ b) $(3, \frac{-27}{2})$ c) (0, 0) d) $(\frac{1}{2}, \frac{-25}{4})$ e) $(\frac{1}{2}, \frac{-37}{12})$
- Suppose that each of a, b, c, d is a real number and that ad bc = 2. The inverse of the matrix a b is
 - a) 2a 2b 2c 2d
- c) $\begin{bmatrix} 2d & -2b \\ -2c & 2a \end{bmatrix}$ e) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$

- b) $\begin{bmatrix} \frac{1}{a} & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{d} \end{bmatrix}$
 - $d) \begin{bmatrix} \frac{d}{2} \frac{b}{2} \\ \frac{-c}{2} & \frac{a}{2} \end{bmatrix}$
- Ten fair coins are tossed and the number of heads recorded. This procedure is done 1000 times, resulting in a list of 1000 numbers. The average of these numbers is likely to be near
 - a) 5
- b) 500
- c) 100
- d) 2.5
- e) none of these
- Consider a coordinate plane. Let R be the reflection of the plane in the x-axis. Let S be the reflection of the plane in the line y = x that bisects quadrants I and III. Let T be the motion of the plane that consists of doing R, followed by S. Where does T send the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$?

 - a) $(\frac{1}{2}, \frac{-\sqrt{3}}{2})$ b) $(\frac{-\sqrt{3}}{2}, \frac{-1}{2})$ c) $(\frac{-\sqrt{3}}{2}, \frac{1}{2})$ d) $(\frac{\sqrt{3}}{2}, \frac{-1}{2})$

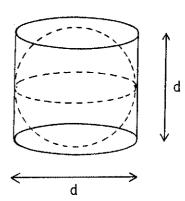
- e) $(\frac{-1}{2}, \frac{-\sqrt{3}}{2})$
- Let f be the function from the real numbers to the real numbers such that $f(x) = x^3 + 3$. The tangent line to the graph of f at (1, 4), also intersects the graph of f at the point

- a) (-2, 11) b) (-2, -5) c) (-1, 2) d) $(\frac{-1}{2}, \frac{23}{8})$
- e) (-3, -24)

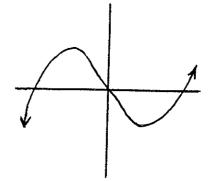
- 10. What is the median of this set of scores: {17, 20, 31, 22, 25}?
 - a) 24
 - b) 17
 - c) 23
 - d) 31
 - e) 22
- 11. Consider these statements about the algebra of n x n matrices:
 - (I) AB is always equal to BA.
 - (II) A(B + C) is always equal to AB + AC.
 - (III) If A is not the zero matrix, then A must have an inverse.
 - (IV) A(BC) is always equal to (AB)C.

The following is the set of true statements:

- a) {I, III}
- b) {I, II, III, IV}
- c) {II, III, IV}
- d) {II, IV}
- e) {I, II, IV}
- 12. Consider the sphere inscribed inside a right circular cylinder. The ratio of the volume of the cylinder to the sphere is
 - a) 2:1
 - b) 4:3
 - c) 3:2
 - d) 3:1
 - e) $\pi:2$

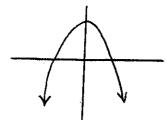


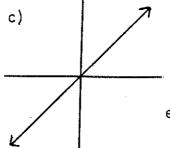
- 13. In a group of 4 randomly chosen people, the probability that at least two were born on the same day of the week is
 - a) $\frac{4}{7}$
 - b) $\frac{4}{2401}$
 - c) $\frac{24}{5040}$
 - d) $\frac{120}{343}$
 - e) $\frac{223}{343}$
- 14. If the graph of f(x) looks like this,



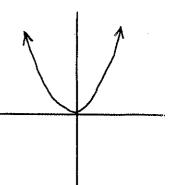
then the graph of f'(x) looks like:

a)

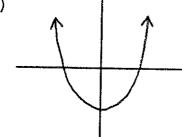


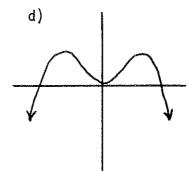


e)



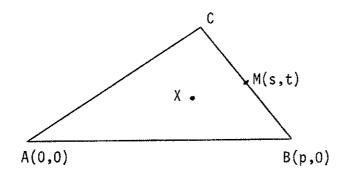
b)





15.		equation $f(x + y)$ isfied by	= f	(x) f(y) for all rea	al n	umbers x and y is
	a)	$f(x) = 2^X$	c)	$f(x) = 3^{2x}$	e)	all of these
	b)	$f(x) = 5^X$	d)	$f(x) = 2^{3x}$		
16.	and is cho	6 black balls. A chosen. Otherwise,	fair Urn	coin is tossed twice II is chosen. Then	e. one	II contains 4 red balls If HH occurs, then Urn I ball is drawn from the t is the probability it
	a)	<u>48</u> 73	c)	73 160	e)	41 80
	b)	$\frac{1}{2}$	d)	<u>32</u> 41		
17.		_		o t sec. after blast t is the ship moving		
	a)	5500 ft./sec.	c)	500 ft./sec.	e)	none of these
	b)	3000 ft./sec.	d)	1500 ft./sec.		
18.	som	e x in the domain o	f f}	_	tion	set {y:y = f(x) for from the real numbers . What is the range of f?
	a)	{y:y ≥ 13	c)	$\{y:y \ge 12\}$	e)	${y:0 < y \le 13}$
•	b)	${y:y \ge 12.5}$	d)	${y:y \ge 0}$		
19.	Let	the determinant d	b e h	$\begin{vmatrix} c \\ f \\ i \end{vmatrix} = 1$. The solution	tion	set of the system of
	thre	ee equations, dx + e	y +	cz = 0 fz = 0, has iz = 0		member(s). The
	cor	rect number or phras	e fo	or the blank is		
	a)	no	c)	infinitely many	e)	none of the above
	b)	three	d)	one		•

20. Consider triangle ABC in the figure below. A = (0, 0) and B = (p, 0). M = (s, t) is the mid-point of side BC. Let X be the point $(\frac{2}{3}s, \frac{2}{3}t)$.



The line CX meets AB in a point Y. Which of the following statements is true about the first coordinate of Y?

- a) There is not enough information to say anything.
- b) It equals (s + t)/2.
- c) It equals (s + t + p)/3.
- d) It equals $\frac{1}{3}p + (s + t)/2$.
- e) It equals p/2.
- 21. If x is a real number other than zero, we can infer that
 - a) $x^2 > x$
 - b) $x^n > x$ for sufficiently large n
 - c) $x^2 > x$ if x > 0
 - d) $x^2 < x \text{ if } x < 1$
 - e) none of the above
- 22. Which of the following arrangements of men comprise a chronological list?
 - a) Riemann, Archimedes, Euclid, Newton
 - b) Archimedes, Riemann, Leibniz, Euclid
 - c) Euclid, Gauss, Leibniz, Newton
 - d) Dedekind, Gauss, Euclid, Newton
 - e) Pythagoras, Newton, Gauss, Dedekind

23.	Wha	at is the greatest value assur	ned	by the function $f(x) = 2 - x - 2 $?
	a)	1	d)	
	ь)	2	e)) none of the above
	c)	3		·
				1
24.	The	e solution set for the inequal	lity	$\sqrt{\frac{2}{x^2}} > 1$ is
	a)	$\{x \mid x < -1 \text{ or } x > 1\}$	d)	$\{x \mid -1 \leq x \leq 1\}$
	b)	$\{x \mid x < -1 \text{ and } x > 1\}$	e)	none of the above
	c)	$\{x \mid -1 < x < 1\}$		
25.	In the	any triangle, the point that intersection of:	is e	equidistant from the three vertices is
	a)	the angle bisectors		
	b)	the incircle and the circumc	irc	le
	c)	the medians		
	d)	the perpendicular bisectors	of t	the sides
	e)	the symmedians		
26.	and	could do a task in 3 hours. Sam in 1½ hours. How long w ether?	Dic oulc	ck could do the same task in 2 hours, d it take Tom and Sam to do it working
	a)	2 hours	d)	1 hour
	b)	2 1/6 hours	e)	none of the above
	c)	2/3 hours		
27.	Fine	_		ollowing equation is an identity:
		$(\cos x + \sin x)^2 + K \sin x$	cos	x - 1 = 0
	a)	1	d)	2
	b)	-1	e)	none of the above
	c)	-2		

28. For what values of t is the following inequality true?

$$t + \frac{1}{t} \le -2$$

a) $t \leq -1$

d) t > 0

b) t < 0

e) $t \leq 0$

c) $t \ge -1$

29. The area of a rhombus is 50, and the length of one diagonal is four times that of the other. Find the length of the shorter diagonal.

a) $2\sqrt{5}$

d) $5\sqrt{2}$

b) 4

e) 6

c) 5

30. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$ is

a) 0

b) 1

d) 2

c) -1

e) -2

31. The angle of elevation to the top of a building for a pedestrian who is 6 ft. tall is 30° . When he walks 100 ft. closer to the building, the angle of elevation is 60° . If he measures from eye level up, how tall is the building?

a) 200 ft.

b) $50\sqrt{3}$ ft.

c) $100\sqrt{3}$ ft.

d) 150 ft.

e) none of the above

32. Evaluate: $\lim_{x \to 0} \frac{\sin(2x)}{x}$

a) 0

b) 1

c) 2

d) ∞

e) none of the above

33.	Fin	d the	term	free	of	х	in	the	expansi	on o	f	(2x ²	+	$(\frac{1}{x})^{9}$.
	a)	484							d)	596				
	b)	672							e)	778				
	c)	682												

- 34. What is the expected value (mathematical expectation) when one rolls a single die?
 - a) 3.00 d) 4.25 b) 4.00 e) none of the abov
 - o) 4.00 e) none of the above
- 35. If $f(x) = |x^2 1|$, then $\int_{1^-}^{2} f(x) dx =$
- a) 2 d) $\frac{4}{3}$ b) 4 e) $\frac{8}{3}$
- 36. What is the area of the largest rectangle that can be inscribed in a semicircle of radius r if one side of the rectangle lies along the diameter of the semicircle?
 - a) $\frac{4}{5} r^2$ d) r^2 b) $\frac{4}{5} \pi r^2$ e) $\frac{1}{2} \pi r^2$
 - c) $\frac{2}{5} \pi r^2$

c) 2.75

- 37. A bucket is drawn from a well by use of a rope that is pulled through an overhead pulley that is 8 inches in diameter. Find the angular velocity of the pulley if the well is 55 feet deep and the bucket is brought up in 33 seconds.
 - a) 16 radians/sec. c) 1 foot/sec. e) none of these
 - b) 5 radians/sec. d) 5 radians/sec.

- 38. Urn number one contains 2 white balls and 1 black ball. Urn number two contains 1 white ball. A ball is drawn at random from urn one and placed in urn two. Then a ball is drawn from urn two. What is the probability that the ball drawn from urn two is white?
 - a) 3/4

d) 1/2

b) 5/6

e) none of these

- c) 1
- 39. A form into which concrete is to be poured has the shape of a frustum of a regular square pyramid. The lower base edge is 12 feet. The upper base edge is 6 feet, and the altitude is 18 feet. If concrete has been poured into the form to a depth of 12 feet, find, to the nearest cubic yard, the amount of concrete needed to fill the remaining space in the form.
 - a) 10

d) 13

b) 11

e) 14

- c) 12
- 40. Find the value of the continued fraction $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$

assuming convergence.

a) $1 + \sqrt{2}$

d) $-1 + \sqrt{2}$

b) $1 - \sqrt{2}$

e) $-1 - \sqrt{2}$

c) $\frac{2}{3}$

	*		· · · · · · · · · · · · · · · · · · ·	* * * * * * * * * * * * * * * * * * *	25 1 22 2
•					
				•	
		•			
				•	
• .					
•					

the control of the co

TWENTY-SIXTH ANNUAL MATHEMATICS CONTEST Sponsored by THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS TEST 1982

Edited by: The Un

The University of Tennessee at Martin

Scoring Formula: 4R - W + 40

This test was prepared from a list of Advanced Topics questions submitted by David Lipscomb College and The University of the South

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

Acme Boot Company, Clarksville, Tennessee
Anderson Ford, Inc., Kingsport, Tennessee
Berkline Company, Morristown, Tennessee
Chattanooga Coca-Cola Bottling Company, Chattanooga, Tennessee
Coca-Cola Bottling Company of Memphis, Memphis, Tennessee
Commercial and Industrial Bank, Memphis, Tennessee
Department of Mathematics, Shelby State Community College, Memphis, TN
Exxon Company, U.S.A., Memphis, Tennessee
First National Bank of Sullivan County, Kingsport, Tennessee
First National Bank, Jefferson City, Tennessee
First People's Bank, Jefferson City, Tennessee
First People's Bank, Jefferson City, Tennessee
Fulton Sylphon Division, Robertshaw Controls, Knoxville, Tennessee
Fulton Sylphon Division, Robertshaw Controls, Knoxville, Tennessee
Harris Foundation, Johnson City, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
Home Federal Savings and Loan Association, Johnson City, Tennessee
IBM Corporation, Chattanooga, Tennessee
IBM Corporation, Kingsport, Tennessee
IBM Corporation, Nashville, Tennessee
Johnson City Spring and Bedding Company, Johnson City, Tennessee
Klopman Mills, Incorporated, Johnson City, Tennessee
McDonald's Restaurants, Memphis, Tennessee
McDonald's Restaurants, Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-0-TOM), Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-0-TOM), Memphis, Tennessee
Memphis Area Teachers of Mathematics (MAC-0-TOM), Memphis, Tennessee
Provident Life and Accident Insurance Company, Chattanooga, Tennessee
Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, TN
Sears, Madison, Tennessee
Fonnessee Handbag Company, Dandridge, Tennessee
Tennessee Handbag Company, Dandridge, Tennesse

1. Which of the following defines a function of two variables for which f(-x, -y) = -f(x, y)?

$$d) x + y$$

b)
$$x^2 + y^2$$

e) none of the above

c)
$$x + y + 1$$

2. If $A = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then $A^2 - 3A + 2I =$

a)
$$\begin{bmatrix} -1 & 8 \\ 2 & 1 \end{bmatrix}$$

 $\begin{array}{ccc} d) & \begin{bmatrix} -1 & -2 \\ 2 & 1 \end{bmatrix}$

b)
$$\begin{bmatrix} 1 & -2 \\ 2 & 1 \end{bmatrix}$$

e) none of the above

c)
$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

3. A class of 20 students will meet in a classroom with 32 student desks. How many different seating assignments are possible?

a)
$$\frac{32!}{12!20!}$$

d) 32 · 20

 $e) 20^{32}$

b)
$$\frac{32!}{12!}$$

c) $\frac{32!}{20!}$

4. Let L be a line and Q be a point not on L. Let P denote the plane containing L and Q. The set of points in P that are equidistant from L and Q is:

- a) A circle whose radius is half the distance between L and Q.
- b) A pair of lines parallel to L.
- c) A parabola whose vertex lies midway between L and Q.
- d) Two lines through Q, one parallel to L, the other perpendicular to L.
- e) An ellipse whose major axis lies on L and whose minor axis is perpendicular to L, passing through Q.

- 5. Let f be the function from the real numbers to the real numbers such that $f(x) = \frac{1}{3} \cdot x^3 - \frac{1}{2} \cdot x^2 - 6x$. The point on the graph of f where the slope of the tangent line has its least value is

- a) $(-2, \frac{22}{3})$ b) $(3, \frac{-27}{2})$ c) (0, 0) d) $(\frac{1}{2}, \frac{-25}{4})$ e) $(\frac{1}{2}, \frac{-37}{12})$
- 6. Suppose that each of a, b, c, d is a real number and that ad bc = 2. The inverse of the matrix | a b | is c d

 - a) $\begin{bmatrix} 2a & 2b \\ 2c & 2d \end{bmatrix}$ c) $\begin{bmatrix} 2d & -2b \\ -2c & 2a \end{bmatrix}$ e) $\begin{bmatrix} -a & c \\ b & -d \end{bmatrix}$

- b) $\begin{bmatrix} \frac{1}{a} & \frac{1}{b} \\ \frac{1}{2} & \frac{1}{d} \end{bmatrix}$ d) $\begin{bmatrix} \frac{d}{2} & -\frac{b}{2} \\ \frac{-c}{2} & \frac{a}{2} \end{bmatrix}$
- Ten fair coins are tossed and the number of heads recorded. This procedure is done 1000 times, resulting in a list of 1000 numbers. The average of these numbers is likely to be near
- b) 500
- c) 100
- d) 2.5 e) none of these
- 8. Consider a coordinate plane. Let R be the reflection of the plane in the x-axis. Let S be the reflection of the plane in the line y = x that bisects quadrants I and III. Let T be the motion of the plane that consists of doing R, followed by S. Where does T send the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$?

 - a) $(\frac{1}{2}, \frac{-\sqrt{3}}{2})$ b) $(\frac{-\sqrt{3}}{2}, \frac{-1}{2})$ c) $(\frac{-\sqrt{3}}{2}, \frac{1}{2})$ d) $(\frac{\sqrt{3}}{2}, \frac{-1}{2})$

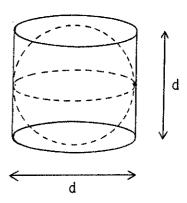
- e) $(\frac{-1}{2}, \frac{-\sqrt{3}}{2})$
- 9. Let f be the function from the real numbers to the real numbers such that $f(x) = x^3 + 3$. The tangent line to the graph of f at (1, 4), also intersects the graph of f at the point

- a) (-2, 11) b) (-2, -5) c) (-1, 2) d) $(\frac{-1}{2}, \frac{23}{8})$
- e) (-3, -24)

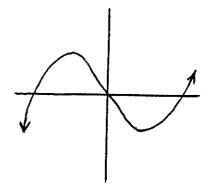
- 10. What is the median of this set of scores: {17, 20, 31, 22, 25}?
 - a) 24
 - b) 17
 - c) 23
 - d) 31
 - e) 22
- 11. Consider these statements about the algebra of $n \times n$ matrices:
 - (I) AB is always equal to BA.
 - (II) A(B + C) is always equal to AB + AC.
 - (III) If A is not the zero matrix, then A must have an inverse.
 - (IV) A(BC) is always equal to (AB)C.

The following is the set of true statements:

- a) {I, III}
- b) {I, II, III, IV}
- c) {II, III, IV}
- d) {II, IV}
- e) {I, II, IV}
- 12. Consider the sphere inscribed inside a right circular cylinder. The ratio of the volume of the cylinder to the sphere is
 - a) 2:1
 - b) 4:3
 - c) 3:2
 - d) 3:1
 - e) π:2

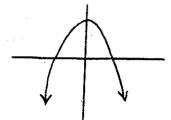


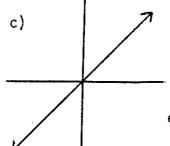
- 13. In a group of 4 randomly chosen people, the probability that at least two were born on the same day of the week is
 - a) $\frac{4}{7}$
 - $\frac{4}{2401}$
 - $\tfrac{24}{5040}$ c)
 - $\frac{120}{343}$
- 14. If the graph of f(x) looks like this,.



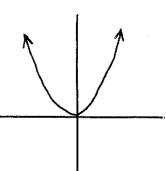
then the graph of f'(x) looks like:

a)

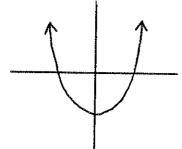


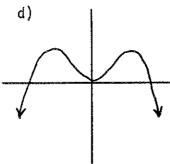


e)



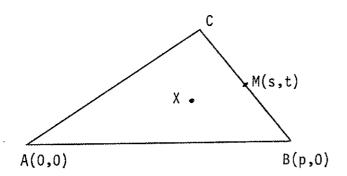
b)





15.		equation f(x + y) isfied by	= f	(x) f(y) for all rea	al n	umbers x and y is
	a)	$f(x) = 2^X$	c)	$f(x) = 3^{2x}$	e)	all of these
		$f(x) = 5^X$				
16.	and is cho	6 black balls. A t chosen. Otherwise,	fair Urn	coin is tossed twice II is chosen. Then	e. : one	II contains 4 red balls If HH occurs, then Urn I ball is drawn from the t is the probability it
	a)	<u>48</u> 73	c)	73 160	e)	$\frac{41}{80}$
	b)	$\frac{1}{2}$	d)	<u>32</u> 41		
17.				o t sec. after blast t is the ship moving		
	a)	5500 ft./sec.	c)	500 ft./sec.	e)	none of these
	b)	3000 ft./sec.	d)	1500 ft./sec.		4
18.	some	e x in the domain of	f}.	^	tion	set {y:y = f(x) for from the real numbers . What is the range of f?
	a)	{y:y ≥ 13	c)	$\{y:y \ge 12\}$	e)	${y:0 < y \le 13}$
	b)	${y:y \ge 12.5}$	d)	${y:y \ge 0}$,
19.	Let	the determinant $\begin{vmatrix} a \\ d \\ g \end{vmatrix}$	b e h	c f = 1. The solut i	tion	set of the system of
	thre	ee equations, dx + e	y +	cz = 0 fz = 0, has iz = 0		member(s). The
	cori	rect number or phras	e fo	r the blank is		
	a)	no	c)	infinitely many	e)	none of the above
	ь١	thnoo	4١	ono		

20. Consider triangle ABC in the figure below. A = (0, 0) and B = (p, 0). M = (s, t) is the mid-point of side BC. Let X be the point $(\frac{2}{3}s, \frac{2}{3}t)$.



The line CX meets AB in a point Y. Which of the following statements is true about the first coordinate of Y?

- a) There is not enough information to say anything.
- b) It equals (s + t)/2.
- c) It equals (s + t + p)/3.
- d) It equals $\frac{1}{3}p + (s + t)/2$.
- e) It equals p/2.
- 21. If x is a real number other than zero, we can infer that
 - a) $x^2 > x$
 - b) $x^n > x$ for sufficiently large n
 - c) $x^2 > x$ if x > 0
 - d) $x^2 < x \text{ if } x < 1$
 - e) none of the above
- 22. Which of the following arrangements of men comprise a chronological list?
 - a) Riemann, Archimedes, Euclid, Newton
 - b) Archimedes, Riemann, Leibniz, Euclid
 - c) Euclid, Gauss, Leibniz, Newton
 - d) Dedekind, Gauss, Euclid, Newton
 - e) Pythagoras, Newton, Gauss, Dedekind

23.	Wh	at is the greatest value assu	med	by the function $f(x) = 2 - x - 2 $?				
	a)	1	d)	4				
	ь)	2	e)	none of the above				
	c)	3						
				1				
24.	The	e solution set for the inequa	lity	$\frac{1}{x^2} > 1$ is				
	a)	$\{x \mid x < -1 \text{ or } x > 1\}$	d)	$\{x \mid -1 \leq x \leq 1\}$				
	ь)	$\{x \mid x < -1 \text{ and } x > 1\}$	e)	none of the above				
	c)	$\{x \mid -1 < x < 1\}$						
25.	In the	any triangle, the point that intersection of:	is e	equidistant from the three vertices is				
	a) the angle bisectors							
	b) the incircle and the circumcircle							
	c) the medians							
	d)	the perpendicular bisectors	of t	he sides				
	e)	the symmedians						
26.	and	could do a task in 3 hours. Sam in $1\frac{1}{2}$ hours. How long wether?	Dic ould	k could do the same task in 2 hours, it take Tom and Sam to do it working				
	a)	2 hours	d)	1 hour				
	b)	2 1/6 hours	e)	none of the above				
	c)	2/3 hours						
27.	Find	the value of K for which the $(\cos x + \sin x)^2 + K \sin x$		llowing equation is an identity: x - 1 = 0				
	a)	1	d)	2				
	b)	-1	e)	none of the above				
	c)	-2						

28. For what values of t is the following inequality true? $t+\frac{1}{t}\leq -2$ a) $t\leq -1$ d) t>0 b) t<0 e) $t\leq 0$

29. The area of a rhombus is 50, and the length of one diagonal is four times that of the other. Find the length of the shorter diagonal.

a) $2\sqrt{5}$ d) $5\sqrt{2}$ b) 4 e) 6

c) 5

30. The value of the determinant $\begin{vmatrix} 1 & 2 & 3 & 4 \\ -1 & 0 & 1 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{vmatrix}$ is

b) 1 d) 2

c) -1 e) ·

31. The angle of elevation to the top of a building for a pedestrian who is 6 ft. tall is 30° . When he walks 100 ft. closer to the building, the angle of elevation is 60° . If he measures from eye level up, how tall is the building?

a) 200 ft. b) $50\sqrt{3}$ ft. c) $100\sqrt{3}$ ft. d) 150 ft.

e) none of the above

32. Evaluate: $\lim_{X \to 0} \frac{\sin(2x)}{x}$

a) 0 b) 1 c) 2 d) ∞

e) none of the above

33.	Find the term fre	e of x in the expans	ion of $(2x^2 + \frac{1}{x})^9$.
	a) 484	d)	596
	b) 672	e)	778
	c) 682		
34.	What is the expec single die?	ted value (mathemati	cal expectation) when one rolls a
	a) 3.00	d)	4.25
	b) 4.00	e)	none of the above
	c) 2.75		
35.	If $f(x) = x^2 - 1 $, then $\int_{1^{-}}^{2} f(x) dx$	=
,	a) 2	d)	<u>4</u> <u>3</u>
	b) 4	e)	8
	c) $\frac{2}{3}$	e)	<u>3</u>
36.		ius r if one side of	ngle that can be inscribed in a the rectangle lies along the
	a) $\frac{4}{5}$ r ²	d)	r ²
	b) $\frac{4}{5} \pi r^2$	e)	$\frac{1}{2} \pi r^2$
	c) $\frac{2}{5} \pi r^2$		
37.	overhead pulley th	at is 8 inches in di	f a rope that is pulled through an ameter. Find the angular velocity eep and the bucket is brought up in
	a) 16 radians/sec	. c) 1 foo	t/sec. e) none of these
	b) 5 radians/sec.	d) 5 rad	ians/sec.

- 38. Urn number one contains 2 white balls and 1 black ball. Urn number two contains 1 white ball. A ball is drawn at random from urn one and placed in urn two. Then a ball is drawn from urn two. What is the probability that the ball drawn from urn two is white?
 - a) 3/4

d) 1/2

b) 5/6

e) none of these

- c) 1
- 39. A form into which concrete is to be poured has the shape of a frustum of a regular square pyramid. The lower base edge is 12 feet. The upper base edge is 6 feet, and the altitude is 18 feet. If concrete has been poured into the form to a depth of 12 feet, find, to the nearest cubic yard, the amount of concrete needed to fill the remaining space in the form.
 - a) 10

d) 13

b) 11

e) 14

- c) 12
- 40. Find the value of the continued fraction $\frac{1}{2 + \frac{1}{2 + \frac{1}{2 + \dots}}}$

assuming convergence.

a) $1 + \sqrt{2}$

d) $-1 + \sqrt{2}$

b) $1 - \sqrt{2}$

e) $-1 - \sqrt{2}$

c) $\frac{2}{3}$

en programme de la composição de la compos Programme