TWENTY-FIFTH ANNUAL MATHEMATICS CONTEST
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ADVANCED TOPICS TEST, 1981

Scoring Formula: 4R - W + 40

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This test was prepared from a list of Advanced Topics questions submitted by University of Tennessee, Knoxville.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. The limit of $\frac{x^2 - 25}{x - 5}$ as x approaches 5 is

   (a) 0

   (b) 1

   (c) undefined

   (d) 10

   (e) 5

2. An equation of the circle concentric with $x^2 + y^2 + 2x - 4y = 0$ and passing through (5, 3) is

   (a) $(x - 1)^2 + (y + 2)^2 = 41$

   (b) $(x + 1)^2 + (y - 2)^2 = 7$

   (c) $(x - 2)^2 + (y + 1)^2 = 25$

   (d) $(x + 1)^2 + (y - 2)^2 = 37$

   (e) $(x - 2)^2 + (y + 4)^2 = 58$

3. A committee of four is selected randomly from among six people: A, B, C, D, E, F. The probability that A and B are members of the committee is

   (a) $\frac{1}{30}$

   (b) $\frac{1}{15}$

   (c) $\frac{1}{3}$

   (d) $\frac{2}{5}$

   (e) $\frac{2}{3}$

4. The complete solution set for $|x^2 - 7| > 2$

   (a) is empty

   (b) contains all real numbers

   (c) is $\{x|x > 3\}$

   (d) is $\{x|x > 3 \text{ or } x < 5\}$

   (e) is none of the above

5. Three thermometers A, B, and C have different scales. If A rises 16°, B rises 48°. If B falls 15°, C falls 25°. Then if A falls 7°, C falls

   (a) $\frac{525^{\circ}}{2}$

   (b) $\frac{16^{\circ}}{5}$

   (c) $\frac{35^{\circ}}{3}$

   (d) 35°

   (e) 21°

6. The ratio of the volume of a cylinder to the volume of the regular quadrangular prism circumscribed about it is

   (a) $\pi/2$

   (b) $\pi/4$

   (c) $4/\pi$

   (d) $2/\pi$

   (e) π

7. The volume of a sphere which is inscribed in a regular hexahedron with edges of length e is

   (a) $\frac{\pi}{6} e^3$

   (b) $\frac{3\pi}{2} e^3$

   (c) $\sqrt{2}\,\pi e^3$

   (d) $\frac{4}{3}\pi e^3$

   (e) $\frac{\sqrt{3}}{2}\pi e^3$

8. $\sin 75^{\circ}$ is equal to

   (a) $\cos 75^{\circ}$

   (b) $\frac{\sqrt{3}}{2}$

   (c) $\frac{1}{4}(\sqrt{6} + \sqrt{2})$

   (d) $\frac{1}{2}(\sqrt{6} - \sqrt{2})$

   (e) $\frac{1}{2}(\sqrt{2} + 1$

9. If $x \neq k\frac{\pi}{2}$, $k = 0, \pm 1, \pm 2, \cdots$, then $\frac{1 + \sec x}{\sin x + \tan x}$ is equal to

(a) 1

(b) $\sec x$

(c) $\csc x$

(d) $\frac{\sin x + \cos x}{\tan^2 x - 1}$

(e) $\sin x$

10. The number of real numbers x such that $\sqrt{x + 1} = \sqrt{x - 1}$ is

(a) 0

(b) 1

(c) 2

(d) 3

(e) 6

11. If $\theta \neq k\pi$, $k = 0, \pm 1, \pm 2$, then $\tan \frac{1}{2}\theta$ is equal to

(a) $\frac{1}{2} \tan \theta$

(b) $2 \tan \theta \cot \theta$

(c) $\csc \theta - \cot \theta$

(d) $\sec \theta - \tan \theta$

(e) $\frac{2 \tan \theta}{1 - \tan^2 \theta}$

12. In the figure, FD is a diameter and FE and DE are equal cords of a circle with center O. Arc AB is a quarter-circle, with midpoint C. Which of the following is the ratio of the area of $\triangle FED$ to the area of $\triangle ABC$?

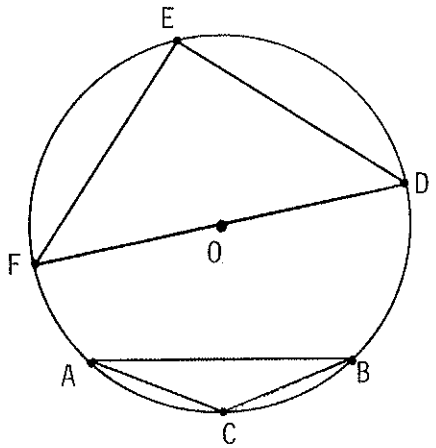
(a) $(4\sqrt{2} - 1)$: 1

(b) 4:1

(c) $(2\sqrt{2} - 1)$: 1

(d) $(2\sqrt{2} + 2)$: 1

(e) $(4\sqrt{2} + 2)$: 1



13. A point moves along a straight line so that its distance from the origin at time t is given by the formula $s(t) = t^2 - 6t + 8$. At what time is the velocity 0?

(a) 8

(b) -6

(c) 2

(d) 3

(e) 6

14. The product of the slopes of the lines connecting point P to (2, 4) and to (4, 4) is -c where $c > 0$. The locus of P is

(a) a straight line

(b) a parabola

(c) a hyperbola

(d) an ellipse

(e) a single point

15. The solution set for $\log_{10} (\log_{10} x) = 3$ is

(a) $\{\log_{10} 3\}$

(b) $\{\ \}$

(c) $\{e\}$

(d) $\{10^{10^3}\}$

(e) $\{100^3\}$

16. Which of the following is a $4^{\underline{th}}$ root of -1?

(a) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} i$

(b) i

(c) $-i$

(d) $\sqrt{2} + \sqrt{2} i$

(e) none of the above

17. The solution set, using principal value ranges only, for $-\arccos(x) = \arcsin(x)$ is

(a) $\{0\}$

(b) $\left\{\frac{\sqrt{2}}{2}\right\}$

(c) $\left\{-\frac{\sqrt{2}}{2}\right\}$

(d) $\{\ \}$

(e) none of the above

18. The equation of the tangent line to the curve $y = 6 - x^2$ at the point whose abscissa is 3 is

(a) $y - 3 = -6(x + 3)$

(b) $y + 3 = -6(x - 3)$

(c) $y = 0$

(d) $y = -6(x - 3)$

(e) $y = -2x(x - 3)$

19. If a matrix B satisfies the equation $B^3 + 3B^2 - 2B - 4I = 0$, then B^{-1} can be written as

(a) $2B + 4I$

(b) 0

(c) $B^2 - \frac{2}{3}B - \frac{4}{3}I$

(d) $\frac{1}{4}B^2 + \frac{3}{4}B - \frac{1}{2}I$

(e) -1

20. The standard deviation of the five scores: 3, 4, 5, 6, 7 is

(a) 5

(b) $\frac{2\sqrt{5}}{2}$

(c) 2

(d) 4

(e) none of the above

21. Let $f(x) = x^3 + x^2 + x + 2$. Observe that $f(2) > 0$, $f(-2) < 0$. An interval of length one in which a root of the equation $f(x) = 0$ lies is

(a) $[1, 2]$

(b) $[0, 1]$

(c) $[-1, 0]$

(d) $[-2, -1]$

(e) none of the above

22. The output of the FORTRAN program

```
      J = 1
      DO 10 I = 1, 10
      J = J * I + 1
      IF (J . GT . 10) GO TO 20
   10 CONTINUE
   20 PRINT, J
      STOP
      END
```

is

(a) 3

(b) 4

(c) 16

(d) 10

(e) 11

23. The trapezoidal approximation to the area above the x-axis, beneath the curve $y = x^2 + x + 1$ and between $x = 0$ and $x = 2$ is

(a) 4

(b) 8

(c) 6

(d) 20/3

(e) none of the above

24. A spherical balloon is being blown up so that its radius is changing at the rate of 2 ft./min. The rate of change of the volume of the balloon when the radius is 3 ft. is

(a) 36π ft.^{3}/min.

(b) 12π ft.^{3}/min.

(c) 9π ft.^{3}/min.

(d) 6 ft.^{3}/min.

(e) 72π ft.^{3}/min.

25. The vertices of a triangle are (-6, 9), (2, 3) and (0, b) where $b > 0$. If the area of the triangle is 30, then b is equal to

(a) 12

(b) 3

(c) 7

(d) 10

(e) 15

26. $10^{10^{10}}$ is equal (in decimal notation) to 1 followed by

(a) 100 zeros

(b) a million zeros

(c) a billion zeros

(d) ten billion zeros

(e) a trillion zeros

27. $\sqrt[4]{x^3\sqrt{x^5}}$ simplifies to

(a) $x^{\frac{15}{2}}$

(b) $x^{\frac{17}{8}}$

(c) $x^{\frac{9}{2}}$

(d) $x^{\frac{11}{8}}$

(e) none of the above

28. Given $A = \begin{bmatrix} 1 & 1 & 2 & 2 & 4 \\ 2 & 2 & 4 & 4 & 8 \\ 3 & 3 & 7 & 8 & 9 \end{bmatrix}$, the rank of matrix A is

(a) 1

(b) 2

(c) 3

(d) 4

(e) 5

29. If a polyhedron has seven vertices and eight faces, how many edges does it have?

(a) 15

(b) 13

(c) 17

(d) 56

(e) 21

30. A solution of the equation $2 \sin^2(t) - \cos(t) - 1 = 0$ is

(a) $t = 0$

(b) $t = \frac{\pi}{2}$

(c) $t = \frac{\pi}{4}$

(d) $t = \frac{\pi}{3}$

(e) none of the above

31. The equation $2x^2 + 2y^2 + cx + dy + e = 0$ is the equation of a circle with positive radius if and only if

(a) $e > 0$

(b) $c = d$

(c) $c = d$ and $e > 0$

(d) $c^2 + d^2 > 8e$

(e) $e < 0$

40. The area bounded by the parabola $y = 5x - x^2$ and the x-axis is

(a) $\frac{125}{6}$

(b) -5

(c) 5

(d) $\frac{25}{4}$

(e) 30