TWENTY-FOURTH ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS TEST 1980

Scoring Formula: 4R - W + 40

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This test was prepared from a list of Advanced Topics questions submitted by Chattanooga State Technical Community College.

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers; one and only one is correct. You are to work each problem, determine the correct answer, and indicate your choice by making a heavy black mark in the correct place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do much wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

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1. If Bob can beat Jim by one-tenth of a mile in a two-mile race, and Jim can beat Henry by one-fifth of a mile in a two-mile race, by what distance could Bob beat Henry in a two-mile race?

   (a) .1 mi.

   (b) .33 mi.

   (c) .29 mi.

   (d) .15 mi.

   (e) .31 mi.

2. A, B, and C are thermometers with different scales. When A reads 12^{o} and 36^{o}, B reads 13^{o} and 29^{o}, respectively. When B reads 20^{o} and 32^{o}, C reads 57^{o} and 84^{o}, respectively. If the temperature drops 18^{o} using A's scale, how many degrees does it drop using C's scale?

   (a) 12^{o}

   (b) 27^{o}

   (c) 15^{o}

   (d) 18^{o}

   (e) 21^{o}

3. Some hikers start on a walk at 3:00 p.m. and return by the same route at 9:00 p.m. If their speed is 4 mph on level land, 3 mph uphill and 6 mph downhill, the total distance they walked is

   (a) 12 mi.

   (b) 18 mi.

   (c) 21 mi.

   (d) 24 mi.

   (e) 27 mi.

4. The sum of three integers that form a geometric progression is 21 and the sun of their reciprocals is $\frac{7}{12}$. Find the common ratio r.

   (a) 3

   (b) $\frac{1}{3}$

   (c) $\frac{2}{3}$

   (d) $\frac{2}{3}$

   (e) 2

5. The inscribed circle of right triangle ABC is tangent to hypotenuse AB at D. Find the area of the triangle if $\overline{AD} = 2$ and $\overline{DB} = 3$.

   (a) 4

   (b) 6

   (c) 8

   (d) 12

   (e) 9

6. A diameter $\overline{AB}$ of circle O is extended through A to C so that AC = 4. At B, $\overline{BD}$ is drawn perpendicular to $\overline{AB}$ so that BD = 12. Determine length AB so that $\overline{DC}$ shall be tangent to the circle.

   (a) 6

   (b) 12

   (c) 8

   (d) 5

   (e) $\sqrt{34}$

7. P is a point inside square ABCD such that PA = 5, PB = 3, and PC = 7. Find the length of the side of the square.

   (a) $\sqrt{34}$

   (b) $\sqrt{40}$

   (c) $\sqrt{58}$

   (d) 8

   (e) $\sqrt{65}$

8. The limit of $\frac{x^2 + x - 2}{x - 1}$ as x approaches 1 is:

   (a) 0

   (b) 3

   (c) -1

   (d) 2

   (e) indeterminate

9. If $\begin{vmatrix} 2x & 5 \\ x & x \end{vmatrix} = 3$, then the equation

   (a) is satisfied by only 1 value of x.

   (b) is satisfied for 2 values of x.

   (c) is satisfied for no values of x.

   (d) is satisfied for an infinite number of values of x.

   (e) none of these

10. In a circle of radius 6.00 m, the length of the arc of a sector is 10.00 m. What is the area of the sector?

   (a) 30.00 m^2

   (b) 180.00 m^2

   (c) 120.00 m^2

   (d) 60.00 m^2

   (e) none of these

11. A real root of $x^4 - 3x^3 + 2x^2 - 5x + 1 = 0$ is located between

   (a) -2 and -1

   (b) -1 and 0

   (c) 1 and 2

   (d) 2 and 3

   (e) 3 and 4

12. An equation of the straight line through the point (3, -4) and perpendicular to the line $2y + 3x = 1$ is

   (a) $x + y = -1$

   (b) $x + 2y = 5$

   (c) $3y = 2x - 6$

   (d) $x - 7 = 7$

   (e) none of these

13. Find all values of x in the interval $[0, 2\pi]$ for which $2 \sin^2 2x - 1 = 0$.

(a) $\{\frac{\pi}{4}, \frac{5\pi}{4}\}$

(b) $\{\frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}\}$

(c) $\{\frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{11\pi}{8}, \frac{13\pi}{8}, \frac{15\pi}{8}\}$

(d) $\{\frac{\pi}{2}, \frac{3\pi}{2}\}$

(e) none of the above

14. The measure of the angle between vector A = -2i + 3j - 1K and B = 1i + 1j + 1k in degrees is

(a) 90

(b) 0

(c) $\text{Cos}^{-1} \frac{1}{\sqrt{42}}$

(d) $\text{Cos}^{-1} \frac{5}{\sqrt{42}}$

(e) none of the above

15. Three pennies are tossed. What is the probability of obtaining at least one head?

(a) $\frac{1}{2}$

(b) $\frac{7}{8}$

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$

(e) none of these

16. Two angles of a triangle are 30^0 and 45^0, respectively. The length of the side opposite the 30^0 angles is 10. What is the length of the side opposite the 45^0 angle?

(a) $10\sqrt{2}$

(b) $10\sqrt{3}$

(c) 15

(d) 12

(e) $8\sqrt{6}$

17. A man is driving directly toward a vertical cliff. At one point the angle of elevation of the top of the cliff is 30°. Four miles further down the road the angle of elevation of the top of the cliff is 45°. How high is the cliff above the road?

(a) $\frac{3}{\sqrt{2}+3}$ miles

(b) $\frac{3}{1+\sqrt{2}}$ miles

(c) $\frac{4}{1-\sqrt{2}}$ miles

(d) $\frac{4}{\sqrt{3}-1}$ miles

(e) $\frac{4}{1-\sqrt{3}}$ miles

18. Find the equation of the tangent line to $x^2y^2 = x^2 + y^2$ at the point $(2, \frac{2\sqrt{3}}{3})$.

(a) $y = \frac{\sqrt{3}}{3}x$

(b) $y = \frac{\sqrt{3}}{3}x + \sqrt{3}$

(c) $y = \frac{\sqrt{3x}}{3} - \sqrt{3}$

(d) $y = \frac{\sqrt{3}}{3}x$

(e) none of the above

19. If $A = \begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix}$ then $\left[A^{-1} + A\right]A =$

(a) $\begin{bmatrix} 0 & 12 \\ 24 & 0 \end{bmatrix}$

(b) $\begin{bmatrix} 18 & 0 \\ 0 & 18 \end{bmatrix}$

(c) $\begin{bmatrix} 18 & 12 \\ 24 & 18 \end{bmatrix}$

(d) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

(e) none of the above

20. Let $f(x) = x^4 - x^3 - 2x^2 + 7x - 5$. Which of the following polynomials is a factor of $f(x)$?

(a) $x^3 - 2x + 5$

(b) $x^3 - 2x^2 + 7$

(c) $x^3 - x^2 - 2x + 7$

(d) $x^3 - 2x^2 + x - 5$

(e) none of the above

21. Sin (4) is equal to which one of the following?

(a) $\sin(2\pi - 4)$

(b) $\sin(4 - \pi)$

(c) $\sin(0.96)$

(d) $\sin(7.14)$

(e) $\sin(\pi - 4)$

22. Let A, B, C denote 3 x 3 matrices with real numbers as entries. Which of the following properties always holds?

(a) $AB = BA$

(b) If $AB = CA$ then $B = C$

(c) $(AB)^2 = A^2B^2$

(d) $A(B + C) = AB + AC$

(e) none of the above

23. For the matrix $A = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ 0 & A_{22} & A_{23} \\ 0 & 0 & A_{33} \end{pmatrix}$ to possess an inverse it is necessary and sufficient that

(a) $A_{11} \cdot A_{22} \cdot A_{33} \neq 0$

(b) $A^2_{11} + A_{22}^2 + A_{33}^2 \neq 0$

(c) the diagonal elements are all positive

(d) the diagonal elements are all negative

(e) Any of these

24. If $\log_{10}(x^2 - 3x + 6) = 1$, the value of x is

(a) 4 or -1

(b) 2 or -1

(c) 2 or 5

(d) 4 or -2

(e) 2 or -3

25. If the larger base of an isosceles trapezoid equals a diagonal and the small base equals the altitude, then the ratio of the smaller base to the larger base is

(a) $\frac{2}{5}$

(b) $\frac{3}{5}$

(c) $\frac{2}{3}$

(d) $\frac{1}{2}$

(e) $\frac{3}{4}$

26. The points (2, -3), (4, 3) and (5, $\frac{k}{2}$) are on the same straight line. The value of k is

(a) 12

(b) -12

(c) 6

(d) -6

(e) none of the above

27. The complete factorization of $x^4 - 16$ over the field of complex numbers where i is the imaginary unit is

(a) $(x^2 + 4)(x^2 - 4)$

(b) $(x^2 + 4)(x - 2)(x + 2)$

(c) $(x - 2i)(x + 2i)(x + 2)(x - 2)$

(d) $4(x - i)(x + i)(x + 1)(x - 1)$

(e) none of the above

28. The vertex of the parabola given by $y = x^2 - 4x + 3$ has coordinates

(a) $(2, 15)$

(b) $(\frac{1}{2}, \frac{5}{4})$

(c) $(2, -1)$

(d) $(\frac{1}{2}, 2\frac{1}{4})$

(e) $(2, 15)$

29. The expression $\sin(\frac{5\pi}{7})$ represents the same number as the expression

(a) $\cos(\frac{5\pi}{7})$

(b) $\cos(\frac{-5\pi}{7})$

(c) $\cos(\frac{2\pi}{7})$

(d) $\cos(\frac{-2\pi}{7})$

(e) $\cos(\frac{-3\pi}{14})$

30. If $f(x) = -x + 1$ and $g(x) = 2x^2 - 3$, then $\frac{f(g(1)) + g(f(1))}{f(1) + g(1)}$ is

(a) 0

(b) undefined

(c) -1

(d) 1

(e) none of the above

31. Find the cartesian equation which is equivalent to the polar equation $r = \frac{4}{1 - \cos\theta}$.

(a) $x^2 + y^2 = 4^2$

(b) $y^2 = 8x + 16$

(c) $y^2 = 4(x - 4)^2$

(d) $x^2 = y + 8$

(e) $y^2 = x^2 + 4$

32. The hyptoenuse of a right triangle is 25 in. The altitude on the hypotenuse is 12 in. What are the lengths of the other two sides of the triangle?

(a) 16 and 20

(b) $12\sqrt{3}$ and 16

(c) $15\sqrt{2}$ and $5\sqrt{7}$

(d) 15 and 18

(e) none of the above

33. Find the lengths of both diagonals of a parallelogram, two of whose sides are 5 m. and 8 m., their included angle being 60°.

(a) $\sqrt{89}$ and 7

(b) 9 and $\sqrt{39}$

(c) 7 and $\sqrt{129}$

(d) 13 and $\sqrt{129}$

(e) none of the above

34. Let x, y, and a be positive real numbers. Then the statement

If $x > y$, then $\log_a x > \log_a y$.

is true if and only if a is a number of which of the following sets of real numbers.

(a) $\{a \mid 0 < a < 1\}$

(b) $\{a \mid a > 0\}$

(c) $\{a \mid a > 1\}$

(d) $\{a \mid a > 0, a < x, \text{ and } a < y\}$

(e) none of the above

35. Which of the following defines a function f for which $f(-x) = -f(x)$?

(a) $f(x) = x^2$

(b) $f(x) = \cos(x)$

(c) $f(x) = \ln x$

(d) $f(x) = \sin x$

(e) $f(x) = e$

36. A particle moves in a straight line with velocity $V(t) = t^2$. How far does the particle move between times $t = 1$ and $t = 3$?

(a) $\frac{1}{3}$

(b) $\frac{26}{3}$

(c) 8

(d) 26

(e) 27

37. In a factory, machine A produces 30% of the ouptut, machine B 25%, and Machine C produces the remaining 45%. One percent of the output of machine A is defective, as is 1.2% of B's output, and 2% of C's. In a day's run, the three machines produce 10,000 items. An item drawn at random from a day's output is defective. What is the probability that it was produced by Machine A?

(a) .3000

(b) .010

(c) .003

(d) .700

(e) .200

38. Given the points (1, 2), (2, 6), (3, 16), determine the 2nd degree equation of the form $y = ax^2 + bx + c$ whose graph passes through these points.

(a) $y = 2x^2 - 2x + 2$

(b) $y = 2x^2 + 2x - 2$

(c) $y = 3x^2 - 5x + 4$

(d) $y = 4x^2 - 2$

(e) none of the above

39. An irregular tetrahedron is tossed into the air. The four faces, numbered 1, 2, 3, 4 have corresponding probabilities 0.1, 0.2, 0.3, 0.4 of being on the bottom. Given that face 2 or 3 is down, what is the probability that it is face 2?

(a) .4
(b) .2
(c) .5
(d) .1
(e) .6

40. If the area of a square inscribed in a circle is 15 sq. inches, what is the area of the square inscribed in a semicircle of the same circle?

(a) 7.5 sq. in.

(b) $\sqrt{15}$ sq. in.

(c) 6 sq. in.

(d) 10 sq. in.

(e) none of the above