FORTY-FOURTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics II 2000

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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Advanced Topics II

1. Identify the graph of $\begin{vmatrix} 2 & y \\ 6 & x \end{vmatrix} = \begin{vmatrix} x & -y \\ y & x \end{vmatrix}$.

A) ellipse B) circle C) hyperbola D) parabola E) line

2. How many different "words"(arrangements of letters) can be formed using all the letters of the word *deserve*?

A) 840 B) 5040 C) 120 D) 2520 E) none of these

3. Find the y-coordinate of the point of inflection of the graph of $y = x^3 - 3x^2 - 9x + 2$.

A) 7 B) -12 C) 1 D) -9 E) 2

4. The first three terms of an arithmetic progression are x, $2x + 1$ and $5x-4$. Find the value of x.

A) 1 B) -1 C) 2 D) 3 E) -2

5. A pendulum on a clock is 10 inches long and swings 60°. How far does a point on the tip travel?

A) 600 in. B) 24.5 in. C) $\frac{10\pi}{3}$ in. D) $\frac{\pi}{3}$ in. E) 6 in.

6. Solve the equation $3^{3x-2} = 4^{2x+1}$ for x.

A) $\frac{\ln 36}{\ln\left(\frac{27}{16}\right)}$ B) $\frac{\ln(36 \cdot 16)}{\ln 27}$ C) $\frac{\ln 13}{\ln 11}$ D) $\frac{3}{5}$ E) $\frac{11}{6}$

7. Solve for x. $\begin{bmatrix} 2 & x+y \\ 2y & -1 \end{bmatrix} = \begin{bmatrix} 2 & 8+3y \\ 4-x & -1 \end{bmatrix}$

A) 2 B) 3 C) 4 D) 6 E) 5

8. Find the y-coordinate(s) of the point(s) that are 5 units from the point (4,2) and have an x-coordinate of 7.

A) -6 B) 2 C) 6 and 2 D) 6 and -2 E) -6 and 2

9. Solve $\log\left(\frac{1}{100}\right) = \sqrt[5]{2x+1}$ for x.

A) -16 B) $-\frac{33}{2}$ C) 1 D) 8 E) No real solutions.

10. If f(x) = 3x – 2, then f(f(a + 2)) is equal to

A) 9a + 2 B) 3a + 4 C) 0 D) 3a + 10 E) 9a + 10

11. The number of real solutions of $\sqrt{x-1}+12=x+11$ in [0,5] is

A) 2 B) 0 C) 1 D) 3 E) more than 3

12. Let f be a continuous odd function on the interval [-*a*, *a*], with $a>0$. Which of the following must be true?

A) $\int_{-a}^{a} f(x)dx = 0$ B) $\int_{-a}^{0} f(x)dx = \int_{0}^{a} f(x)dx$ C) $\int_{-a}^{0} f(x)dx < 0$

D) $\int_{-a}^{a} f(x)dx = 2\int_{0}^{a} f(x)dx$ E) $\int_{-a}^{a} f(x)dx = f(a) - f(-a)$

13. Given that $\int_{4}^{9} f(x)dx = 10$, which of the following must also true?

A) $\int_{4}^{9} [f(x)+3]dx = 13$ B) $\int_{4}^{6} f(x)dx \le 10$ C) $\int_{2}^{3} x\, f(x^2)dx = 10$ D) $\int_{2}^{7} [3f(x+2)+1]dx = 35$

E) $\int_{4}^{9} f\left(\frac{x}{2}\right)dx = 5$

14. Find the equation of the normal line to the curve defined by the equation $x^3y^4 - 5 = x^3 - x^2 + y$ at the point (2, -1).

A) $33x + 4y - 62 = 0$ B) $33x + 4y + 62 = 0$ C) $4x + 33y - 41 = 0$ D) $33x - 4y - 62 = 0$

E) $4x + 33y + 41 = 0$

15. How many inflection points does the graph of $y = \frac{x^3}{x^3 - 1}$ have?

A) 0 B) 1 C) 2 D) 3 E) more than 3

16. From 1970 to 1980, the rate of potato consumption in a certain country was $c(t) = 2.2 + 1.1^t$ millions of bushels per year, with t representing the number of years since the beginning of 1970. How many bushels were consumed from the beginning of 1972 to the end of 1973?

A) 3.4695391 million bushels B) 7.0660321 million bushels C) 3.3541265 million bushels

D) .121 million bushels E) .2541 million bushels

17. Which of the following functions satisfies the equation $f'''(x) = f'(x)$?

A) $f(x) = \ln x$ B) $f(x) = \sin x$ C) $f(x) = e^{2x}$ D) $f(x) = 2e^x + 1$ E) $f(x) = e^x + x$

18. If the substitution u = x/2 is made, the integral $\int_2^4 \frac{1-(x/2)^2}{x}\,dx$ becomes

A) $\int_1^2 \frac{1-u^2}{u}\,du$ B) $\int_1^2 \frac{1-u^2}{4u}\,du$ C) $\int_2^4 \frac{1-u^2}{u}\,du$ D) $\int_2^4 \frac{1-u^2}{2u}\,du$ E) $\int_1^2 \frac{1-u^2}{2u}\,du$

19. If $A = \begin{bmatrix} 1 & 0 & -5t \\ 2 & -3 & 8 \\ 0 & 2t & 0 \end{bmatrix}$, find $\frac{d}{dt}[\det(A)]$.

A) -16 – 40t B) -16 + 40t C) -36 D) 36 E) -10

20. A candy jar contains 10 red jelly beans and 20 green jelly beans. If two jelly beans are drawn out at random without replacement, what is the probability of one red and one green jelly bean (in any order)?

A) $\frac{20}{87}$ B) $\frac{23}{64}$ C) $\frac{4}{9}$ D) $\frac{2}{9}$ E) $\frac{40}{87}$

21. $\lim_{x\to 10} \frac{x^3-7x^2-28x-20}{x^2-9x-10}$ equals

A) 0 B) 1 C) 10 D) 12 E) does not exist

22. How many different integers less than 1000 can be formed from the digits 1,2,3,4 and 5 if no digit may be repeated in any one number?

A) 60 B) 20 C) 85 D) 125 E) 120

23. Given the equation $x^3 - 2x^2 + x - 3 = 0$, write the equation whose roots are each 2 less than the roots of the given equation.

A) $x^3 - 8x^2 + 21x + 21 = 0$ B) $x^3 - 4x^2 - x - 5 = 0$ C) $x^3 - 4x^2 + 2x - 6 = 0$

D) $x^3 + 4x^2 + 5x - 1 = 0$ E) $x^3 + 4x^2 - 2x + 6 = 0$

24. Two of the roots of the equation $2x^3 - 3x^2 + px + q = 0$ are 3 and -2. The third root is

A) ½ B) 2 C) -3 D) 1/3 E) 1

25. Let $y = \int_0^{x^2} \sin t\,dt$. Then $\frac{dy}{dx} =$

A) $\sin x^2$ B) $2x \sin x^2$ C) $\cos x^2$ D) $2x \cos x^2$ E) $\cos 2x$

26. If the operation $x \circledast y$ is defined by $x \circledast y = (x + 1)(y + 1) - 1$, where + and − are the usual real number operations, which of the following is false?

A) $x \circledast y = y \circledast x$ for all real x and y

B) $x \circledast (y + z) = (x \circledast y) + (x \circledast z)$ for all real x, y and z

C) $(x - 1) \circledast (x + 1) = (x \circledast x) - 1$ for all real x

D) $x \circledast 0 = 0$ for all real x

E) $x \circledast (y \circledast z) = (x \circledast y) \circledast z$ for all real x, y and z

27. If $a > 1$, $b > 1$ and $p = \dfrac{\log_b(\log_b a)}{\log_b a}$, then $a^p =$

A) 1 B) b C) $\log_a b$ D) $\log_b a$ E) $a^{\log_b a}$

28. Evaluate this limit. $\lim\limits_{x\to\infty}\left(1-\dfrac{3}{x}\right)^x$

A) 1 B) ∞ C) 0 D) e^3 E) e^{-3}

29. $\displaystyle\int \frac{1}{1+1/x}\,dx =$

A) $x - \ln|x+1| + C$ B) $x + 1 + \ln|x+1| + C$ C) $x + \dfrac{x^2}{2} + C$ D) $\ln\left|\dfrac{x+1}{x}\right| + C$

E) $x - x^{-2} + C$

30. $\displaystyle\int \frac{dx}{1-e^{-x}} =$

A) $\ln|1-e^{-x}| + C$ B) $\ln|e^x - 1| + C$ C) $x + e^{-x} + C$ D) $x - e^{-x} + C$ E) $-\ln\left(e^{-x}\right) + C$

31. If $y = (x^2+2)^{2x}$, then $\dfrac{dy}{dx} =$

A) $4x^2(x^2+2)^{2x-1}$ B) $(x^2+2)^{2x}\left[\dfrac{4x^2}{x^2+2} + \ln(x^2+2)^2\right]$ C) $2x\ln(x^2+2)$

D) $4 + 4x^2 + 2\ln(x^2+2)$ E) $6x^2 + 4$

32. If $f(5) = 6$, $f'(-4) = -3$, $g(5) = -4$ and $g'(5) = 3$, then $\frac{d}{dx}[(f \circ g)(5)] =$

A) 12 B) -9 C) -12 D) -24 E) 8

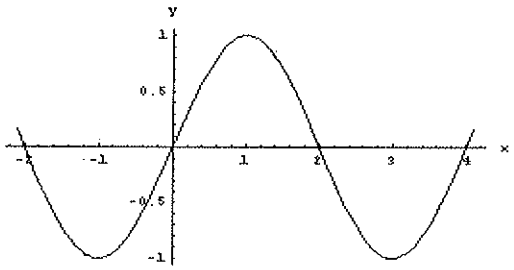
33. If $\tan^{-1}\left(-\frac{2}{3}\right) = A$, then $\sin 2A =$

A) $-\frac{12}{13}$ B) $-\frac{4}{\sqrt{13}}$ C) $-\frac{6}{13}$ D) $\frac{4}{13}$ E) $\frac{6}{\sqrt{13}}$

34. The length of the major axis of the ellipse $4x^2 + 9y^2 - 8x + 54y + 49 = 0$ is

A) 6 B) 9 C) 4 D) 36 E) 1

35. Let $F(x) = \int_{-2}^{x} f(t)\,dt$, where the graph of $y = f(t)$ is given below. On what interval(s) is F(x) increasing?



A) (0,2) B) (-1,1) and (3,4) C) (-2,-1) and (1,3) D) (-2,0) and (2,4)

E) Not possible to determine from the given information

36. A can is to be made to hold one liter of oil. Determine the ratio of the radius to the height (radius:height) to minimize the surface area of the can.

A) 1:4 B) 4:1 C) 2:1 D) 1:2 E) 1:1

37. $\lim_{x \to \infty} (1 - 2^{1/x})x =$

A) ln 2 B) - ln 2 C) - 0.6931 D) 0.6931 E) 0

38. Eliminate the parameter from the equations $x = \sin t$, $y = (\sin t)^3$.

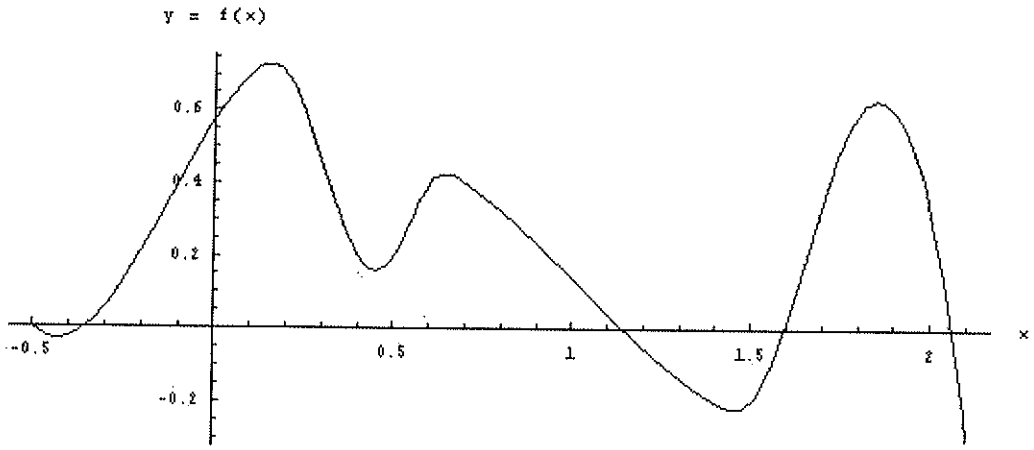
A) $y = x^3$, $0 \le x \le 1$ B) $y = x^3$, $-1 \le x \le 1$ C) $y = x^3$, $-1 \le x \le 0$ D) $y = \sqrt[3]{x}$, $0 \le x \le 1$

E) $y = \sqrt[3]{x}$, $-1 \le x \le 1$

39. Let $f'(x) = \sin(\ln x)$, $x > 0$. The number of horizontal tangents of the graph of f in (0,1) is

A) 0 B) 2 C) 4 D) 8 E) infinitely many

40. Below is the graph of a function f which is twice differentiable in the interval [0,2]. Which of the following is a true statement?



A) $\int_0^2 f''(x)dx > 0$ and $\int_0^2 f'(x)dx > 0$ and $\int_0^2 f(x)dx > 0$

B) $\int_0^2 f''(x)dx < 0$ and $\int_0^2 f'(x)dx > 0$ and $\int_0^2 f(x)dx > 0$

C) $\int_0^2 f''(x)dx < 0$ and $\int_0^2 f'(x)dx < 0$ and $\int_0^2 f(x)dx > 0$

D) $\int_0^2 f''(x)dx > 0$ and $\int_0^2 f'(x)dx > 0$ and $\int_0^2 f(x)dx < 0$

E) $\int_0^2 f''(x)dx > 0$ and $\int_0^2 f'(x)dx < 0$ and $\int_0^2 f(x)dx < 0$