FORTIETH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics II 1996

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Advanced Topics II

1. $\int_0^4 \frac{dx}{x-3}$ =

a. ln (3) b. ln(3) - 1 c. -ln(3)

d. 1 - ln(3) e. does not exist

2. Find y' if xy = ln(x + y)

a. $\frac{y-\frac{1}{x}}{\frac{1}{y}-x}$ b. $\frac{1}{x^2+xy}$ c. $\frac{1-xy-y^2}{1-x^2-xy}$

d. $\frac{xy+y^2-1}{1-x^2-xy}$ e. $\frac{1}{x+y}$

3. Find all real numbers x such that ln x < x^2.

a. $(0,\infty)$ b. $(\frac{\sqrt{2}}{2},\infty)$ c. $(0,\frac{\sqrt{2}}{2})$

d. $(-\infty,\frac{-\sqrt{2}}{2}) \cup (\frac{\sqrt{2}}{2},\infty)$ e. $(-\infty,\frac{-\sqrt{2}}{2}) \cup (0,\frac{\sqrt{2}}{2})$

4. $\lim_{m\to 0}\frac{(2+m)^3-8}{m}$ =

a. ∞ b. 8 c. 2 d. 0 e. 12

5. Find the radius of the circle $x^2 + y^2 - 2x + 4y = 4$.

a. $\sqrt{3}$ b. 4 c. 3

d. $\sqrt{5}$ e. 5

6. Find the value of: $\lim_{x \to 3^-} \frac{|x-3|}{x-3}$

a. -4 b. -2 c. -1 d. 0 e. 2

7. Find the slope of the tangent line to the curve of

$2x^3 + 2y^3 - 9xy = 0$ at the point (1,2).

a. 3 b. 9 c. 9/2 d. 1/3 e. 4/5

8. Sand is leaking out of a container at a steady rate of 2 ft.^{3} per minute. As it falls, it forms a conical pile that has a radius twice its height. At the moment when the radius is 5 feet, how fast is the radius increasing?

a. $\frac{4}{25\pi}$ *ft*/min b. $\frac{6}{5\pi}$ *ft*/min c. $(\frac{12}{\pi})^{\frac{1}{3}}$ *ft*/min

d. $\frac{12}{25\pi}$ *ft*/min e. $\frac{2}{25\pi}$ *ft*/min

9. The Mean Value Theorem for Integrals, when applied to the function $f(x) = x^2 + 2$, on the interval [0,2], predicts the existence of a number **u** where f takes on a certain value. That number **u** is:

a. $\frac{2}{\sqrt{3}}$ b. either $\frac{2}{\sqrt{3}}$ or $-\frac{2}{\sqrt{3}}$ c. $\frac{20}{3}$

d. $\frac{10}{3}$ e. 1

10. A right circular conical tank of height 1 foot and radius 1 foot at the top is full of water. Assume the water to weigh 60 pounds per cubic foot. How much work in foot-pounds is required to pump all the water up and over the top if the tank?

a. 3π b. 4π c. 5π d. 6π e. 7π

11. The solution of $4\cos^2 2x < 1, (0<x<\pi)$ is:

a. $x < \frac{\pi}{6}$ or $x > \frac{\pi}{3}$ b. $\frac{\pi}{6} < x < \frac{\pi}{3}$ c. $\frac{\pi}{6} < x < \frac{\pi}{3}$ or $\frac{2\pi}{3} < x < \frac{5\pi}{6}$

d. $\frac{2\pi}{3} < x < \frac{5\pi}{6}$ e. $x > \frac{5\pi}{6}$ or $x < \frac{\pi}{6}$

12. The length of the curve described by $\mathbf{x = e^t \cos t}$ and $\mathbf{y = e^t \sin t}$ for $0 \le t \le \frac{\pi}{2}$ is:

a. $e^{\frac{\pi}{2}} - \sqrt{2}$ b. $\sqrt{2}(e^{\frac{\pi}{2}} - 1)$ c. $\frac{e^{\frac{\pi}{2}}}{\sqrt{2}}$

d. $(e-2)^{\frac{\pi}{2}}$ e. $e^{\frac{\pi}{2}}$

13. The solution set of the inequality $6 \ge 2|2-x|$ is:

a. $[-1,5]$ b. $(-\infty,-1]$ or $[5,\infty)$ c. $(-\infty,-1]$

d. $[\frac{5}{3},\frac{7}{3}]$ e. $(-\infty,\frac{5}{3}]$ or $[\frac{7}{3},\infty)$

14. Completely simplify the expression:

$$\frac{\sin x}{1+\sin x} - \frac{\sin x}{1-\sin x}$$

a. $2\csc^2 x$ b. $2\sec^2 x$ c. $-2\tan^2 x$

d. $-2\cot^2 x$ e. $2\sin^2 x$

15. Calculate the length of an arc that subtends a central angle of measure 60° on a circle of diameter 16 meters.

a. $\frac{8\pi}{3}$ meters b. $\frac{16\pi}{3}$ meters c. 480 meters

d. $\frac{2\pi}{4}$ meters e. $\frac{\pi}{3}$ meters

16. A regular octagon is inscribed in a circle of radius 10 cm. Calculate the perimeter of the octagon.

a. $80 \sin 22.5°$ cm b. $160 \sin 45°$ cm c. $80 \sin 45°$ cm

d. $160 \sin 22.5°$ cm e. $100 \sin 45°$ cm

17. Solve $\frac{1}{2x+1} \leq \frac{1}{x-3}$ for x and express the solution in interval form.

a. $(-\infty, -4) \cup (-.5, 3)$ b. $[-4, -.5] \cup [3, \infty)$

c. $[-4, -.5) \cup (3, \infty)$ d. $(-\infty, -4) \cup [-.5, 3]$

e. $(-4, -.5) \cup (3, \infty)$

18. Determine the number of possible positive, negative, and nonreal complex solutions of:

$$5x^5 - 4x^4 + 8x^3 - 6x^2 - 8x + 3 = 0.$$

a. 4, 2, or 0 positive; 1 negative; 0, 2, or 4 nonreal complex
b. 2 or 0 positive; 3 or 1 negative; 0, 2, or 4 nonreal complex
c. 3 or 1 positive; 2 or 0 negative; 0, 2, or 4 nonreal complex
d. 1 positive; 4, 2, or 0 negative; 0, 2, or 4 nonreal complex
e. all solutions are nonreal complex

19. The fifth term of the expansion $(x^3 + \sqrt{y})^{13}$ is:

a. $130x^{13}y^4$ b. $6160x^{27}y^2$ c. $6160x^{27}y^4$

d. $715x^{36}y^2$ e. $715x^{27}y^2$

20. If $\det \begin{bmatrix} 4 & 1 \\ 3 & x \end{bmatrix} = \frac{a}{4}$ then x =

a. $16a + 12$ b. $\frac{a-12}{16}$ c. $\frac{3a}{4}$

d. $\frac{a+12}{16}$ e. $16a - 12$

21. Suppose $h = f \circ g$. If $f'(2) = 6$, $f'(1) = 5$, $f(1) = 4$, $g(1) = 2$ and $g'(1) = -2$, then $h'(1) = ?$:

a. -12 b. 8 c. 10 d. -10 e. -8

22. A car rental agency has 24 cars (identical models). The owner of the agency finds that at a price of \$18 a day, he can rent all the cars; however, for each \$1 increase in rental cost, one of the cars is not rented. What should the agency charge to maximize income?

a. \$18 b. \$21 c. \$20 d. \$22 e. \$19

23. $\lim_{x \to 0} \frac{csc2x}{cotx}$

a. 2 b. ∞ c. does not exist d. $\frac{1}{2}$ e. 0

24. Find the area of the region bounded by the graphs of the following:

$y = \sqrt{2-x}$, $y = -x$ and the positive x-axis.

a. $\frac{10}{3}$ b. $\frac{16}{3}$ c. $\frac{4\sqrt{2}}{3}$

d. $\frac{2}{3}$ e. 3

25. If $y = \sqrt{\ln\sqrt{x}}$, *find* $\frac{dy}{dx}$

a. $\frac{1}{4x}$ b. $\frac{1}{2\sqrt{x}\sqrt{\ln\sqrt{x}}}$ c. $\frac{1}{4\sqrt{x}\sqrt{\ln\sqrt{x}}}$

d. $\frac{1}{4x\sqrt{\frac{1}{2}\ln x}}$ e. $\frac{1}{x}$

26. A set S is **closed** under and operation * if and only if

$a * b \in S$ whenever $a, b \in S$.

For example the set of integers is closed under addition because the sum of any two integers is another integer. Which of the following sets are closed under the given operation?

I. Even integers, addition
II. Rational Numbers, division
III. Positive integers, subtraction

a. I, II, and III b. none of the above c. I only

d. II only e. I and II only

27. Find the position of a particle at time t = 1 sec if its acceleration a(t) is known to be a(t) = 8t - 3 m/sec/sec, its initial position is 1 meter and its initial velocity is 4 m/sec.

a. $\frac{29}{6}m$ b. 5 m c. $-\frac{1}{6}m$

d. 4 m e. 3 m

28. The area of the region bounded by $r = 1 + \sin\theta$ is:

a. $\frac{3\pi}{2}$ b. $\frac{9\pi}{4}$ c. $\frac{9\pi}{2}$ d. $\frac{3\pi}{4}$ e. $\frac{\pi}{2}$

29. Given that the graph of f(x) contains (-3,-2), find g(-3) if $g(x) = |f(x)| + 2$?

a. 0 b. 4 c. -1 d. 5 e. -2

30. Solve $\log \sqrt[4]{x+1} = \frac{1}{2}$

a. 1 b. -.9375 c. .1548 d. 99 e. .75

31. If $f(x) = \sqrt{x^2+1}$ and $g(x) = \sqrt{x^2-4}$ find the domain of $g \circ f$.

a. $(-\infty, -\sqrt{3}) \cup (\sqrt{3}, \infty)$ b. $(-\infty, \infty)$ c. $(-\infty, 2] \cup [2, \infty)$

d. $[1, \infty)$ e. $(0, 1)$

32. Solve $\frac{5^x - 5^{-x}}{2} = 3$

a. -1 b. $\frac{\ln(3 + \sqrt{10})}{\ln 5}$ c. $\frac{\log 3 + \log\sqrt{10}}{\log 5}$

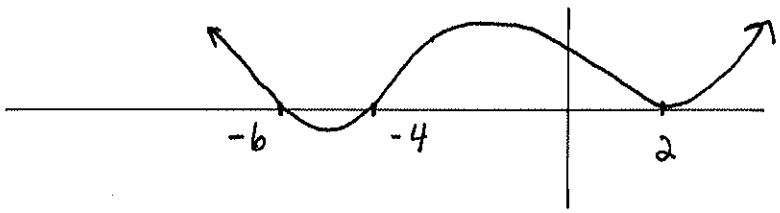
d. $\ln(\frac{3+\sqrt{10}}{5})$ e. 1

33. What is the conclusion for the following argument?

If roses are red then violets are blue.
If wishes are horses then beggars ride.
Violets are not blue and beggars do not ride.
Therefore,

a. Roses are red and wishes are horses.
b. Roses are not red or wishes are not horses.
c. It is not the case that roses are red and wishes are horses.
d. Roses are not red and wishes are not horses.
e. Roses are not red and wishes are horses.

34. The following is a graph of a polynomial function of degree 6. What is the multiplicity of the root 2?



a. 4 b. 2 c. 1

d. not possible to determine e. 3

35. A true-false test contains 9 questions. If a student guesses the answer for each question, find the probability that at least 7 answers are correct.

a. $\frac{23}{256}$ b. $\frac{5}{256}$ c. $\frac{1}{64}$

d. $\frac{41}{256}$ 3. $\frac{1}{32}$

36. Find a polar equation that has the same graph as $x^2 + y^2 = 16$.

a. $r^2 = 16 \cos 2\theta$ b. $r = 16$ c. $r = 4$

d. $r^2 = 16 \sec 2\theta$ e. $r = 2$

37. Use common logarithms to solve $2^{2x+2} = 3^{3x-2}$ for x.

a. $\log(\frac{16}{3})$ b. $\log 36$ c. $\frac{\log 36}{\log(\frac{27}{4})}$

d. $\frac{\log 13}{\log 23}$ e. $\log 4$

38. Which of the following is the equation of an asymptote for the graph of:

$$y = \frac{x-4}{(x+3)(x+1)}$$

I. $x = -3$
II. $x = 4$
III. $x = -1$

a. I only b. II only c. III only

d. I and II only e. I and III only

39. A store owner receives 12 computers: 9 are model 900s and 3 are model 600s. If two computers are sold at random, find the probability that one of each model is sold.

a. $\frac{35}{66}$ b. $\frac{15}{66}$ c. $\frac{9}{33}$ d. $\frac{9}{22}$ e. $\frac{5}{66}$

40. There is a 0.39 probability that John will buy a new car, a 0.73 probability that Jane will buy a new car, and a 0.36 probability that both will buy a new car. Find the probability that neither will buy a new car.

a. .76 b. 1.12 c. .02

d. .72 e. .24