THIRTY-FOURTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics II 1990

Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. What is the remainder when $x^{49}+49$ is divided by $x+1$?

   a) 48 b) 50 c) 0 d) -50 e) -48

2. If $F(x) = (3x + 1) / 2$, what is $F^{-1}(x)$?

   a) $2/(3x + 1)$ b) $2x/3 - 1$ c) $2x/3 - 1/2$ d) $(2x - 1)/ 3$

   e) $2(3x - 1)$

3. Given a finite mathematical system's operational table for the binary operation *, as shown, determine the inverse of y.

*	w	x	y	z
w	x	y	z	w
x	y	z	w	x
y	z	w	x	y
z	w	x	y	z

   a) w b) x c) y
   d) z e) 0

4. What is the value of the following?

$$\lim_{x \to 9} \frac{\sqrt{x} - 3}{x - 9}$$

   a) 0 *b)* ∞ *c)* 1 *d)* 1/6 *e)* *doesn't exist*

5. What is the value of the following?

$$\lim_{x \to 4} \frac{x^2 - x - 12}{x^2 - 16}$$

   a) 0 *b)* ∞ *c)* 1 *d)* 7/8 *e)* *doesn't exist*

6. What is the value of the following?

$$\lim_{x\to\infty} \frac{4x^2 + 4}{x^2 - 2}$$

a) 0 *b)* ∞ *c)* 1 *d)* 4 *e)* *doesn't exist*

7. What is the value of k, if

$$\int_1^5 kx\,dx = 60$$

a) -5 b) 5 c) -2.5 d) 2.5 e) 60

8. What is the following indefinite integral?

$$\int e^{2x}(2\,dx)$$

a) $2e + c$ *b)* $2e^x + c$ *c)* $\frac{1}{2}e^{2x+1}$ *d)* $\frac{e^{2x+1}}{2x+1} + c$ *e)* $e^{2x} + c$

9. What is the value of

$$e^{i\pi} = ?$$

a) 0 *b)* $-\sqrt{\pi}$ *c)* $\sqrt{-\pi}$ *d)* 1 *e)* -1

10. If log 2 = 0.3010 and log 3 = 0.4771, then log 36 = ?

a) 1.5562 b) 0.2342 c) 0.0549

d) 0.6054 e) 0.9030

11. Given the curve S with parametric equation x = r cos t, y = r sin t, and $0 \le t \le \pi$, find the arc length of S.

a) 2r b) $2\pi r$ c) 2π d) πr e) $\pi + r$

12. If log e = 0.4343, then ln 10 = ?

a) 0.5657 b) 2.3026 c) -0.4343

d) 0.5658 e) 1.4343

13. What is the value of the following?

$$\int_e^{e^2} \frac{dx}{x \ln x}$$

a) $(\ln 2)^2$ b) -(3/8) c) 7/8 d) 1/2 e) ln2

14. If $y = \cos(e^x)$, find dy/dx.

a) $\sin(e^x)$ b) $-\sin(e^x)$ c) $(-\sin(e^x))(e^x)$

d) $e^x \sin(e^x)$ e) $(\cos(e^x))(e^x)$

15. What is the value of the following?

$$\int_1^e \ln x \, dx$$

a) -e + 1 b) 2e + 1 c) -1 d) 1 e) 0

16. What is the value of the following?

$$\int_{\frac{\pi}{2}}^{\frac{\pi}{4}} \sin x \, dx$$

a) $\frac{\sqrt{2}}{2}$ b) $-\frac{\sqrt{2}}{2}$ c) $\frac{\sqrt{2}}{2} - 1$ d) $-\frac{\sqrt{2}}{2} - 1$ e) $\frac{\sqrt{2}}{2} + 1$

17. Find y' if $2xy = y^2$.

a) 0 b) $\frac{y}{x}$ c) $\frac{y}{y-x}$ d) $\frac{y}{x-y}$ e) $-\frac{y}{x}$

18. What is the value of the following?

$$\lim_{x \to \frac{\pi}{2}} \frac{2x-\pi}{\cos x}$$

a) 0 b) 1 c) -2 d) doesn't exist e) ∞

19. Which of the following is an antiderivative of $f(x) = \sec x \tan x$?

a) $\sec x \tan^2 x + \sec^3 x$ b) $\sec 3x + \pi$

c) $\sec x \tan^2 x - \pi$ d) $\sin x \cos^3 x$

e) $\sec x + \pi/4$

20. Which of the following is an antiderivative of

$$f(x) = \frac{1}{1+4x^2}?$$

a) $\frac{-1}{8x^3}$ b) $\ln(1+4x^2)$ c) $\arcsin(2x)$

d) $\arctan(2x)$ e) $\frac{-1}{8(1+4x^2)^2}$

21. To what does the following series converge?

$$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

a) ln x b) sin x c) e^x d) cos x e) ∞

22. To what does the following series converge?

$$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

a) ln x b) sin x c) e^x d) cos x e) ∞

23. To what does the following series converge?

$$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

a) ln X b) sin X c) e^X d) cos X e) ∞

24. What is the value of the following?

$$\int_1^{\infty} \frac{1}{x^2} dx$$

a) ∞ b) 0 c) 1/3 d) 1 e) -1/3

25. The graph of the polar equation r = 1 + cosθ is which of the following?

a) parabola b) circle c) 4 leaf rose
d) spiral e) cardiod

26. The graph of the polar equation r = θ is which of the following?

a) parabola b) circle c) 4 leaf rose
d) spiral e) cardiod

27. At the point (3,5), what is the slope of the line tangent to the graph

$$y = \sqrt{x^2 + 16}$$

a) 3/5 b) 1/5 c) 6/5 d) 5/2 e) 3/2

28. Determine the unit vector having the same direction as v = 3i - 4j.

a) i - j b) 3i - 4j c) 3/4 i + 4/5 j

d) 3/5 i - 4/5 j e) 5i - 5j

29. Which of the following vectors is orthogonal to v = 3i - 2j + k?

a) -3i + 2j - k b) 3i - 2j c) 2i - 3j + k
d) 2i + 4k e) 2i + j - 4k

30. What is the value of the determinant of the matrix A,

$$where\ A = \begin{bmatrix} 2 & 1 & 3 \\ 3 & -1 & -2 \\ 2 & 3 & 1 \end{bmatrix}$$

a) 11 b) 36 c) 0 d) -2 e) 1

31. What is the inverse under multiplication of matrix B, where

$$B = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

a) $(1/3)\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ b) $(1/3)\begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$ c) $(1/3)\begin{bmatrix} -2 & -1 \\ -1 & -2 \end{bmatrix}$

d) $(1/3)\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$ e) $(1/3)\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$

32. What is the total number of different lists of possible candidates for the election of 4 officers (President, Vice President, Secretary, Treasurer) from a club consisting of 12 members?

a) 11,880 b) 24 c) 495 d) 3,960 e) 12,958,400

33. Which one of the following is equivalent to the expression

$$e^{-\ln x^2}$$

a) $\ln(x^2)$ b) $1/x$ c) x^{-2} d) $1/(2\ln x)$ e) $\ln(2x)$

34. Which is equivalent to the expression

$$|3 - 4i|$$

a) $3 + 4i$ b) -1 c) 1 d) 7 e) 5

35. The expression $[4\cos(240°) + (4i)\sin(240°)]^2$ is the same as

a) $16\cos(240°) + (16i)\sin(240°)$

b) $16\cos(120°) + (16i)\sin(120°)$

c) $8\cos(480°) + (8i)\sin(480°)$

d) $8\cos(120°) + (8i)\sin(120°)$

e) $4\cos(120°) + (4i)\sin(120°)$

36. If Libya is sending in a bomber to bomb New York City and we have 5 radar sets along the coast, each with probability of 60% of detecting an enemy plane, find the probability that no radar set detects the plane and New York City is bombed.

a) $(.6)^5$ b) $5(.4)^4(.6)$ c) $(.4)^5$ d) $5!/(5!0!)$ e) $5(.6)^4(.4)$

37. For all A in the domain, to which one of the following is this expression identical?

$$\frac{1-\cos 2A}{\sin 2A}$$

a) sin 2A b) 1 c) 0 d) sin A e) tan A

38. The sum of the first 15 terms of the geometric progression, where the first term is 3 and the fourth term is 24, is

a) ∞ b) 49,149 c) 98,303

d) 98,301 e) 49,151

39. A local maximum value for the function $y = x^3 - 3x^2 - 3x + 2$ will occur for a value of x in the interval:

a) $0 < x < 1$ b) $1 < x < 2$ c) $-1 < x < 0$

d) $-3 < x < -2$ e) $2 < x < 3$

40. A window has the shape of a rectangle surmounted by a semicircle. If the perimeter of the window is 15 feet, find the length of the radius of the semicircle that will allow the maximum amount of light to enter through the window.

a) 5 b) $30/(2+\pi)$ c) $15/(4+\pi)$ d) $15/(4-\pi)$ e) $30/(4+\pi)$