

THIRTY-THIRD ANNUAL MATHEMATICS CONTEST  
sponsored by  
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS II 1989

Prepared by:

Mathematics Department  
Belmont College  
Nashville, TN

Co-ordinated by: Raymond Medley

Scoring formula:  $4R - W + 40$

Edited by: Larry Bouldin, Roane State  
Community College

---

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

---

Contributors to TMTA for Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin,  
Tennessee

Donnelley Printing Company, Gallatin, Tennessee

Sears, Madison, Tennessee

TRW, Ross Gear Division, Lebanon, Tennessee

IBM, Nashville, Tennessee



## Advanced Topics II

1. If  $x = 3 \sin t$  and  $y = 4 \cos t$ , then  $\frac{dy}{dx} = ?$
- a.  $-\frac{4}{3} \tan t$
- b.  $-\frac{3}{4} \cot t$
- c.  $-12 \sin t \cos t$
- d.  $\frac{3}{4} \tan t$
- e.  $\frac{4}{3} \cot t$
2.  $\lim_{x \rightarrow \infty} x^2 e^{-x} = ?$
- a.  $-\infty$
- b.  $-2$
- c.  $0$
- d.  $2$
- e.  $\infty$
3. Suppose that a basketball player has a probability  $p$  of hitting a free-throw. In a given series of free-throws, what is the probability that the first free-throw missed is the fourth one tossed? (Assume that the free-throws are independent.)
- a.  $p^4$
- b.  $0$
- c.  $p^3(1-p)$
- d.  $4p^3(1-p)$
- e.  $3p^3(1-p)$
4. If the substitution  $x = \sin \theta$  is made in the integral  $\int \frac{dx}{x \sqrt{1-x^2}}$  the integral becomes
- a.  $\int \csc \theta \sec^2 \theta \, d\theta$
- b.  $\int \csc \theta \sec \theta \, d\theta$
- c.  $\int \sec \theta \, d\theta$
- d.  $\int \csc \theta \, d\theta$
- e.  $\int d\theta$
5.  $\lim_{x \rightarrow 0} \frac{3x \cos x}{\sin 3x} = ?$
- a.  $0$
- b.  $1/3$
- c.  $1$
- d.  $3$
- e.  $\infty$

### Advanced Topics II

6. The exact value of  $2 \cos^2 \frac{\pi}{8} - 1$  is
- a.  $1/2$
  - b.  $-1/2$
  - c.  $\sqrt{2} - 1$
  - d.  $\sqrt{2}/2$
  - e.  $-\sqrt{2}/2$
7. In a certain population, 1 out of 9 individuals suffer from anemia. If a person suffers from anemia, the probability that he/she has a certain bacterial infection is 0.4. If a person does not suffer from anemia, the probability that he/she has the bacterial infection is 0.05. Suppose a person is selected at random from this population and is found to have the bacterial infection. What is the probability that he/she suffers from anemia?
- a. 0.1
  - b. 0.5
  - c.  $1/9$
  - d. 0.36
  - e. 0.044
8.  $\sin^{-1}(-\sqrt{3}/2)$  is equal to
- a.  $2\pi/3$
  - b.  $-5\pi/3$
  - c.  $\pi/3$
  - d.  $-\pi/3$
  - e.  $5\pi/6$
9. The sponsor of a contest offers to give the winner of the contest \$1000 dollars the first second after midnight, April 1, 1989, and half the amount given the previous second for every second after that for the rest of the winner's life. The sponsor is required by law to place enough money in trust to cover the entire amount to be paid out. From among the numbers given below, choose the smallest amount the sponsor can put into trust.
- a. \$500
  - b. \$10,000
  - c. \$8244.44
  - d. \$2,000,000
  - e. \$2,000
10. The polar coordinate equation of  $y = x^2$  is
- a.  $r = \cos \theta \tan \theta$
  - b.  $r = \cos^2 \theta$
  - c.  $r = \tan \theta \sec \theta$
  - d.  $r = \sin^2 \theta$
  - e.  $r = \tan \theta \csc \theta$

## Advanced Topics II

11. Let  $f(x) = \int_1^x (t^2 - 1) dt$ . At  $x = 1$  the curve  $y = f(x)$  has
- a. a relative maximum
  - b. an asymptote
  - c. a relative minimum
  - d. a discontinuity
  - e. an inflection point
12. The curve  $y = 1 + 3x^2 + x^3$  has a minimum slope at  $x = ?$
- a. 0
  - b. 1
  - c. -2
  - d. -1
  - e. 2
13. Let  $x = 2t$  and  $y = 2(1 - \sin t)$  where  $0 \leq t \leq \pi$ . The area between the curve and x-axis is
- a.  $4\pi$
  - b.  $4(\pi - 2)$
  - c.  $2\pi$
  - d.  $2\pi - 4$
  - e.  $8\pi$
14. The region bounded by the x-axis, the line  $x = 2$ , and the curve  $y = \sqrt{2x}$  is rotated about the x-axis. The volume for this solid is
- a.  $4\pi$
  - b.  $8\pi$
  - c.  $\frac{8}{3}\pi$
  - d.  $\frac{16}{3}\pi$
  - e.  $2\pi$
15. A particle moves on the x-axis so that its distance from the origin at any time  $t$  is given by  $x = t^2 - 6t + 5$ . For what value of  $t$  is the velocity zero?
- a. 1
  - b. 5
  - c. 3
  - d. 4
  - e. 2

## Advanced Topics II

16. If the line  $y = 3x + b$  is tangent to the curve  $y = x^2 - x + 9$ , then  $b = ?$
- a. -1  
b. 1  
c. 3  
d. 5  
e. 9
17. What is the slope of the line normal to the curve  $y = \cos 3x$  at the point  $(\pi/6, 0)$ ?
- a. -3  
b. 3  
c. -1/3  
d. 1/3  
e. 1
18. What is the equation of a curve that passes through the point (2,3) and also satisfies the equation  $x dy + y dx = 0$  ?
- a.  $x^2 + y^2 = 13$   
b.  $y^2 - x^2 = 5$   
c.  $x + y = 5$   
d.  $xy = 6$   
e.  $2y = 3x$
19. Which of the following lines are asymptotes of the graph of  $y = \frac{3x(x-2)}{(x+1)(x-3)}$  ?
- I.  $x = -1$ , II.  $x = 3$ , III.  $x = 0$ , IV.  $x = 2$ , V.  $y = 3$
- a. V only  
b. I and II only  
c. III and IV only  
d. I, II, and V only  
e. I, II, III, IV, and V
20. For what values of  $x$  is the curve  $y = x^2 + \frac{1}{x}$  concave downward?
- a.  $x < 0$   
b.  $0 < x < 1$   
c.  $-1 < x < 0$   
d.  $x > 0$   
e.  $x < 0$  or  $x > 0$

## Advanced Topics II

21. If  $\sin \theta = 3/5$  and  $\tan \theta < 0$ , then
- a.  $\tan \theta = 3/4$
  - b.  $\cot \theta = -4/3$
  - c.  $\sec \theta = -4/5$
  - d.  $\cos \theta = 5/4$
  - e.  $\csc \theta = -5/3$
22. The straight line defined by  $x = -3t + 1$ ,  $y = t - 4$ ,  $z = 2t + 3$  intersects the  $xz$ -plane at the point
- a.  $(13, 0, -5)$
  - b.  $(13, -4, -5)$
  - c.  $(-11, -4, 11)$
  - d.  $(-11, 0, 11)$
  - e.  $(-11, 4, 11)$
23. Suppose the system of equations  $2ax - by = 4$  has a solution:  
 $ax + by = 20$   
 $x = 8$  and  $y = -4$ . Which of the following must be true?
- a.  $b = -3$
  - b.  $a = 2$
  - c.  $a + b = 4$
  - d.  $b = 1$
  - e.  $a = -1$
24. The value of  $x$ ,  $1 < x < 5$ , where the tangent to the graph of  $y = \frac{1}{x}$  will be parallel to the secant through the points where  $x = 1$  and  $x = 5$  is
- a. 4
  - b.  $\sqrt{5}$
  - c. 2
  - d. 3
  - e.  $\sqrt{6}$
25. The graph of the equation  $4x^2 + 3y^2 - 16x + 18y + 43 = 0$  is
- a. a parabola
  - b. an ellipse
  - c. a hyperbola
  - d. a point
  - e. a pair of intersecting lines

26. Given the matrix:

$$A = \begin{bmatrix} 2 & 5 & 4 \\ 1 & 4 & 3 \\ 1 & -3 & -2 \end{bmatrix}$$

Which of the following is the second row of  $A^{-1}$  ( $A$  inverse)?

- a. (1, 3, 5)
- b. (-1, -5, -2)
- c. (1/2, 3/8, 4/3)
- d. (-5, 8, 2)
- e. (0, 0, 4)

27. The rectangular equation of the vector function

$$\mathbf{r}(t) = (t + 2)\mathbf{i} + (t^2 + 7t + 12)\mathbf{j} \quad \text{is}$$

- a.  $y = x^2 + 6x + 10$
- b.  $y = x^2 + 3x + 2$
- c.  $y = x^2 + x - 7$
- d.  $y = 2x^2 + 10x + 3$
- e.  $y = x^2 + 8x + 14$

28. Evaluate  $\ln(e^{2x+1})$  when  $x = -2$ .

- a.  $e^{-3}$
- b. -3
- c.  $-3 \ln(5)$
- d. e
- e. undefined

29. The point (5, -2) is at a distance  $\sqrt{13}$  from the midpoint of the segment joining (5,  $y$ ) and (-1, 1). The value of  $y$  is

- a. 1 or 9
- b. 1 or -9
- c. -1 or 9
- d. -1 or -9
- e. -1 only

30. Given  $y = x^2 - 4x + 3$ , find the equation of the axis of symmetry.

- a.  $x = 2$
- b.  $y = 3$
- c.  $y = 2x - 4$
- d.  $x = 3$
- e.  $y = x(x - 4) + 3$



## Advanced Topics II

31. The area of the region bounded by  $y = x^2$  and  $y = 2 - x$  is
- a.  $3/2$
  - b.  $17/6$
  - c.  $16/3$
  - d.  $6$
  - e.  $9/2$
32. A unit square has vertices at  $(0,0)$ ,  $(0,1)$ ,  $(1,1)$ , and  $(1,0)$ . The curve  $y = x^2$  divides the unit square into two (2) regions. The ratio of the smaller region to the larger region is
- a.  $1/4$
  - b.  $1/3$
  - c.  $2/3$
  - d.  $1/2$
  - e.  $3/4$
33. The slope of the tangent line to the graph of  $y = \sin^{-1} \frac{x}{4}$  at the point  $(0,0)$  is
- a.  $-1$
  - b.  $-1/4$
  - c.  $1/4$
  - d.  $1$
  - e.  $4$
34. A particle is moving along the curve  $y = \ln x$ . The length of the arc it has traveled along the curve is changing at the rate of 4 units per second. How many units per second is its x-coordinate changing at the instant  $x = 1$ ?
- a.  $4$
  - b.  $2\sqrt{2}$
  - c.  $2$
  - d.  $8$
  - e.  $2$
35. The mean (average) value of the function  $y = x^2 - 2x$  over the interval from  $x = 1$  to  $x = 3$  is
- a.  $1$
  - b.  $1/3$
  - c.  $-2/3$
  - d.  $2/3$
  - e.  $-1$

### Advanced Topics II

36. Which of the following equations is the equation of a line tangent to the ellipse  $(x^2/9) + (y^2/5) = 1$  and having slope equal to  $-1$  ?
- a.  $y = 2x + 14$
- b.  $y = -x - 90$
- c.  $y = -x + \sqrt{14}$
- d.  $3x + 5y = 45$
- e.  $\frac{2}{9}x + \frac{2}{5}y = -1$
37. Find an equation of the line through the centers of the two circles,  $x^2 + y^2 + 2x - 19 = 0$  and  $x^2 + y^2 - 10x - 12y + 41 = 0$ .
- a.  $3x - 2y - 3 = 0$
- b.  $3x + 2y + 3 = 0$
- c.  $x - y + 1 = 0$
- d.  $x + y - 1 = 0$
- e.  $3x + 2y - 27 = 0$
38. How many different subcommittees consisting of 2 representatives and 1 senator can be selected from a committee consisting of 10 representatives and 8 senators?
- a. 720
- b. 40
- c.  $(10! 8!)/(2! 1!)$
- d. 16
- e. 360
39. The expression  $\frac{\tan \theta + \cot \theta}{\sec \theta \csc \theta}$  is equal to
- a.  $\sin \theta + \cos \theta$
- b. 1
- c.  $\sec^2 \theta + \csc^2 \theta$
- d.  $2 \sin \theta \cos \theta$
- e.  $2 \cos^2 \theta$
40. Your math teacher asks you to select 3 trig books from the 5 trig books available and to select 2 calculus books from the 4 which are available. Then you are to arrange these 5 books in order on your desk with the restriction that the 3 trig books must be together and the 2 calculus books must be together. How many different arrangements are possible?
- a. 5!
- b.  $12(5!)$
- c.  $5!/2!$
- d. 60
- e. 480



