

FORTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 1997

Prepared by:

Mathematics Faculty
Austin Peay State University

Coordinated by: David Cochener

Reviewed by:

Mathematics Faculty
Roane State CC
Harriman, TN
Oak Ridge, TN

Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State CC, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee

ADVANCED TOPICS I

- If $x^{16} - x^8 + x^2 + 7$ is divided by $3x - 3$, the remainder is
 - $8/3$
 - 3
 - 8
 - $3/8$
 - $-7/3$
- If two cards are drawn simultaneously from a standard deck of 52 cards, the probability that they are both kings or both spades is
 - $1/13$
 - $1/17$
 - $13/221$
 - $14/221$
 - $4/13$
- A strip of paper is 6 inches long and $1/2$ inch wide. It is green on one side and yellow on the other. A dab of blue is placed at each end on the yellow side. The strip is given a half-twist, and the ends are glued together, yellow-to-yellow, with a 1-inch overlap. A starting point is chosen in the middle of the green side of the strip ($1/4$ inch from each edge). From this point, an ant walks forward down the middle of the strip attempting to reach the starting point on the green side of the strip. Which statement is true?
 - The ant will never reach the starting point on the green side of the strip.
 - The ant will travel 6 inches to get back to the starting point on the green side of the strip.
 - The ant will travel 12 inches to get back to the starting point on the green side of the strip.
 - The ant will travel 5 inches to get back to the starting point on the green side of the strip.
 - The ant will travel 10 inches to get back to the starting point on the green side of the strip.
- In order to triple the capacity of a spherical balloon, the surface area must be increased by approximately
 - 70%
 - 133%
 - 108%
 - 200%
 - 233%

5. If x and y are real numbers, then

a. $|x - y| < |x + y|$

b. $x - y < |x + y|$

c. $|x - y| = |x| - |y|$

d. $|x| - |y| \leq |x - y|$

e. $x + y < |x - y|$

6. If n is an odd integer, $i^n + i^{-n}$ equals

a. $(-1)^n$

b. $\frac{1}{n}$

c. $\frac{(-1)^n}{n}$

d. $2i$

e. 0

7. If $A = \{x, y, z, t\}$ and $B = \{1, 2, 3, 4, 5\}$, the total number of possible one-to-one functions from A to B is

a. 20

b. 120

c. 1024

d. 24

e. 625

8. If both " a " and " b " are greater than one and $\log_b a = 2/3$, then $\log_a (ab^2)$ equals

a. $8/27$

b. 4

c. 3

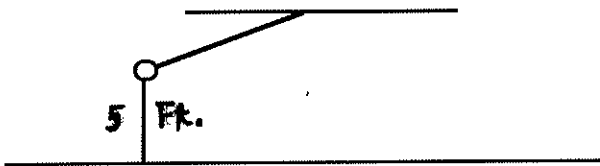
d. $7/3$

e. $3/2$

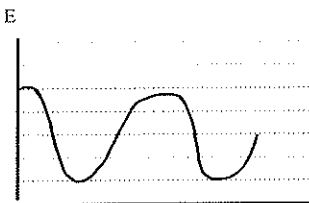
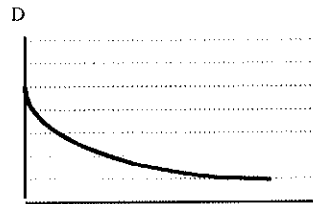
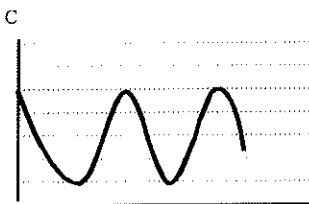
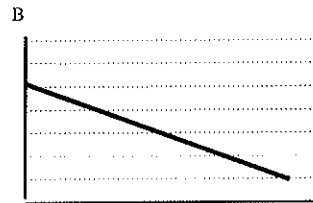
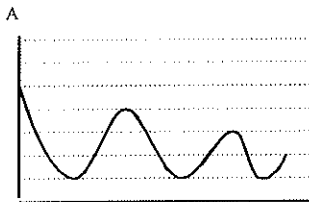
9. If $x - a$, $x - b$ and $2x - 2b$ are the first three terms in a six term geometric progression, then x could be

- a. b/a
- b. a/b
- c. $2a - b$
- d. $a - 2b$
- e. $a - b$

10. A ball is suspended from a 7-foot rope. The rope is attached to a beam 8 feet off the floor. The ball is pulled back until it is in the position shown here.



When the ball is released, it is allowed to swing freely. Which graph shows the distance from the ball to the floor as a function of time?



11. The first three terms of a six term geometric progression are $\sqrt{2}$, $\sqrt[3]{2}$, and $\sqrt[6]{2}$. The fourth term is
- a. $\sqrt[9]{2}$
 - b. $\sqrt[12]{2}$
 - c. $\sqrt[18]{2}$
 - d. 1
 - e. $\sqrt[8]{2}$
12. If 2137^{753} is multiplied out, the units digit in the final product is
- a. 3
 - b. 7
 - c. 9
 - d. 1
 - e. 5
13. A gives B as many cents as B has and C as many cents as C has. Similarly, then B gives A and C as many cents as each of them has. C similarly then gives A and B as many cents as each of them has. If each finally has 16 cents, with how many cents does A start?
- a. 22
 - b. 24
 - c. 26
 - d. 28
 - e. 30
14. If $4a$ is inversely proportional to the square of b , then if b is doubled, $4a$ is
- a. divided by 2
 - b. divided by 4
 - c. multiplied by 2
 - d. multiplied by 4
 - e. divided by 8

15. Let $S = (x - 1)^4 + 4(x - 1)^3 + 6(x - 1)^2 + 4(x - 1) + 1$. Then S equals

- a. $(x - 1)^4$
- b. x^4
- c. $x^4 - 1$
- d. $x^4 + 1$
- e. $(x + 1)^4$

16. The sum of the solutions of $3^{5x - x^2} = 9^{3 - x}$ is

- a. 4
- b. 5
- c. 6
- d. 7
- e. 8

17. If two poles 20 feet and 80 feet high are 100 feet apart, then the height of the intersection of the lines joining the top of each pole to the foot of the opposite pole is

- a. 5 feet
- b. 8 feet
- c. 10 feet
- d. 12 feet
- e. 16 feet

18. If $y = f(x) = \frac{3x + 4}{4x - 3}$, then $f(y)$ in terms of x is

- a. $f(y) = 3x$
- b. $f(y) = -2x$
- c. $f(y) = x$
- d. $f(y) = -x$
- e. $f(y) = 2x$

19. If $\frac{y}{x} = 3$ and $y^x = x^y$, $x \neq 0$, then the value of x is

- a. $\pm\sqrt{3}$
- b. $\pm 3\sqrt{3}$
- c. ± 3
- d. $\pm \frac{\sqrt{3}}{3}$
- e. ± 1

20. If $a^{-1} + 4m^{-1} = (a + m)^{-1}$ ($a \neq 0, m \neq 0, a + m \neq 0$) then a and m are related by

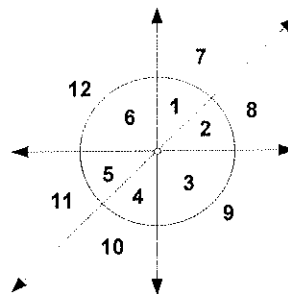
- a. $m = -2a$
- b. $a = 2m$
- c. $a = m$
- d. $m = 2a$
- e. $a = -2m$

21. A fox pursued by a hound has a start of 100 leaps. The fox makes 3 leaps while the hound makes 2, but 3 leaps of the hound is equivalent to 5 of the fox. How many leaps will the fox take before being caught by the hound?

- a. 900
- b. 600
- c. 350
- d. 820
- e. 540

22. The equation of the circle in the figure is $x^2 + y^2 = 16$ and the equation of the line is $y = x$. In which of the numbered regions do the inequalities $x^2 + y^2 > 16$ and $x < y$ both hold?

- a. 1 and 7
- b. 2 and 8
- c. 7, 11 and 12
- d. 8, 9 and 10
- e. 2, 3 and 4



23. The seventh term of an arithmetic progression is 41 and the thirteenth term is 77. The twentieth term is

- a. 119
- b. 113
- c. 125
- d. 117
- e. 123

24. A square and a circle have equal perimeters. The ratio of the area of the circle to the area of the square is

- a. $\frac{\pi}{2}$
- b. $\frac{2}{\pi}$
- c. $\frac{4}{1}$
- d. $\frac{\pi}{4}$
- e. $\frac{4}{\pi}$

25. The radius of a first circle is 1 inch, that of a second circle is $\frac{1}{2}$ inch, that of a third $\frac{1}{4}$ inch, that of a fourth $\frac{1}{8}$ inch, ad infinitum. The sum of the areas of all these circles is

- a. $\frac{3\pi}{4}$
- b. 2π
- c. $\frac{3\pi}{2}$
- d. $\frac{4\pi}{3}$
- e. $\frac{2\pi}{3}$

26. The value of $\cos(\text{Arcsin } a)$ is

- a. $\frac{1}{\sqrt{1 - a^2}}$
- b. $\sqrt{a^2 - 1}$
- c. $\sqrt{1 - a^2}$
- d. $\frac{1}{\sqrt{1 - a^2}}$
- e. $\frac{1}{\sqrt{a^2 - 1}}$

27. Find the number of four digit numbers N , $1000 \leq N \leq 9999$, which read the same both forward and backwards.

- a. 81
- b. 100
- c. 95
- d. 90
- e. 20

28. Let $T = \frac{1}{3 - \sqrt{8}} - \frac{1}{\sqrt{8} - \sqrt{7}} + \frac{1}{\sqrt{7} - \sqrt{6}} - \frac{1}{\sqrt{6} - \sqrt{5}} + \frac{1}{\sqrt{5} - 2}$. Then

- a. $-1 < T < 1$
- b. $1 \leq T < 2$
- c. $2 \leq T < 4$
- d. $4 \leq T < 6$
- e. $T \geq 6$

29. The value of "a" to make the equation $\frac{1}{16} \sin(4x^2) = a \sin(16ax^2) \cos(16ax^2)$ a true statement is

- a. 1
- b. 2
- c. 16
- d. 1/8
- e. 8

30. The smallest period of $f(x) = \sin(3x - \pi) + \sin(4x + \pi)$ is

- a. π
- b. 2π
- c. 3π
- d. 4π
- e. $\pi/2$

31. If (a, b) is a point on the graph of $y = f(x)$, then which of the points below must be on the graph of $y = -f^{-1}(-x) + 1$?

- a. $(1 - a, b)$
- b. $(b, 1 - a)$
- c. $(-b, a + 1)$
- d. $(a, b + 1)$
- e. $(-b, 1 - a)$

32. If it takes 10 men 5 days to lay 1000 bricks, then the number of days it will take 5 men working at the same rate to lay 10 bricks is
- a. 0.1
 - b. 2.4
 - c. $1/12$
 - d. 1.2
 - e. 0.5
33. If θ is in Quadrant III and $\cos \theta = -3/5$, what is $\sin 2\theta + \cos 2\theta$?
- a. $-24/7$
 - b. $24/25$
 - c. $-7/25$
 - d. $42/25$
 - e. $17/25$
34. Assume that each person in Muleshoe has 3 initials, one for each of his first, middle and last name. How many residents must Muleshoe have so that we can be sure that 2 residents have the same initials?
- a. 26^3
 - b. $26^3 + 26^3$
 - c. $3 \cdot 26$
 - d. $26^3 + 1$
 - e. 32
35. A, B, and C are thermometers with different scales. When A reads 12° and 36° , B reads 13° and 29° respectively. When B reads 20° and 32° , C reads 57° and 84° respectively. If the temperature drops 18° using A's scale, how many degrees does it drop using C's scale?
- a. 12°
 - b. 27°
 - c. 15°
 - d. 18°
 - e. 21°

36. An arithmetic progression of 101 natural numbers has a sum of 17,675. The last term is 325. What is the first term?

- a. 175
- b. 15
- c. 25
- d. 54
- e. 150

37. Let D be a 2×2 determinant with integer entries. What is the probability that D is even?

- a. $7/16$
- b. $3/8$
- c. $1/2$
- d. $5/8$
- e. $11/16$

38. If $0 < x < y$, which of the following is largest?

- a. \sqrt{xy}
- b. $\left(\frac{x^{-1} + y^{-1}}{2}\right)^{-1}$
- c. $\left(\frac{x^2 + y^2}{2}\right)^{1/2}$
- d. $\left(\frac{x^{1/2} + y^{1/2}}{2}\right)^2$
- e. $\frac{x + y}{2}$