THIRTY-NINTH ANNUAL MATHEMATICS CONTEST

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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 1995

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Advanced Topics I

1. Find the domain of $h(t) = \sqrt{\frac{t-2}{t-1}}$.

   (a) $t = 1,\ t = 2$
   (b) $t \neq 1,\ t \neq 2$
   (c) all real numbers except $t = -1$ and $t = -2$
   (d) all real numbers except $t = 1$
   (e) all real numbers except $1 \leq t < 2$

2. A point on the graph of the function in problem one could have which of the following as a y- coordinate?

   (a) $-\frac{3}{2}$ (b) $3\frac{1}{2}$ (c) -1 (d) 1 (e) $-3\frac{1}{2}$

3. Find functions r(x) and s(x) such that $r(s(x)) = s(r(x))$.

   (a) $r(x) = |x|,\ s(x) = 3x - 7x^2$ (b) $r(x) = |x|,\ s(x) = 3|x|$

   (c) $r(x) = |3x|,\ s(x) = |3x| - |7x^2|$ (d) $r(x) = 3x - 7x^2,\ s(x) = |x|$

   (e) $r(x) = -x,\ s(x) = -|3x + 7x^2|$

4. Which of the following is the inverse of $f(a) = a^5 + 4$? (Assume f(a) is one to one.)

   (a) $\sqrt[5]{a} - 4$ (b) $\sqrt[5]{a - 4}$ (c) $(4 - a)\frac{1}{5}$

   (d) $(a + 4)\frac{1}{5}$ (e) $\sqrt[5]{a} + 4$

5. Which of the following is symmetrical with respect to the origin?

   (a) $y = x^3 - 27$ (b) $x^2 + y - 9 = 0$ (c) $4x^2 + 9y^2 = 36$

   (d) $x^2 = y$ (e) $(x - 2)^4 + 1 - y = 3$

6. Find k, given that (3,k) is equidistant from (2,9) and (8,3).

   (a) $k = 5$ (b) $k = 4$ (c) $k = 6$ (d) $k = 2$ (e) $k = -4$

7. Find (x,y) if the line through (0,0) and (x,y) has a slope of $\frac{2}{3}$, and the line through (x,y) and (-4,1) has a slope of -3.

   (a) (-3,-2) (b) (3,2) (c) (-3,2) (d) (-2,-2) (e) (-2,-3)

8. Where does the circle of radius 7 centered at the origin intersect the line of slope -1 through the origin?

(a) (-4.9,4,9) (b) (4.8,-4.8) and (-4.8,4.8)
(c) (4.8,-4.8) (d) (4.9,-4.9) and (-4.9,4.9) (e) (4.7,-4.9)

9. A rocket is fired vertically upward and is h feet above the ground t seconds after being fired, where $h = 96t - 16t^2$. Find the maximum height to which the rocket ascends.

(a) 144 (b) 96 (c) 16 (d) 6 (e) 80

10. Given that $f(x) = x^4 - 4x^3 + 5x^2 - 4x + 4$ has i as one of its zeros, the total number of zeros (real and complex) is:

(a) 1 (b) 2 (c) 4 (d) 5 (e) 7

11. The function $f(x) = (x+\pi)(x-2)(x+1)$ has how many rational roots?

(a) 3 (b) 2 (c) 1 (d) π (e) 0

12. The function $f(x) = x^3 - 6x^2 + 10x - 3$ has how many irrational roots?

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

13. Which is a factor of $f(x) = 3x^5 - 38x^3 + 5x^2 - 720$?

(a) x (b) x-1 (c) x-2 (d) x-3 (e) x-4

14. Identify one corner in the region of feasibility determined by the following constraints:

$x + y \le 8$ $y \le 4$ $x \le 6$ $x \ge 0, y \ge 0$

(a) (1,1) (b) (2,6) (c) (6,2) (d) (4,5) (e) (5,4)

15. Maximize: $P = 143x + 60y$ subject to: $\begin{cases} x \ge 0 \\ y \ge 0 \\ x+y \le 100 \\ 120x+210y \le 15000 \\ 110x+30y \ge 4000 \end{cases}$

Which constraint is superfluous?

(a) $110x + 30y \ge 4000$ (b) $x + y \le 100$ (c) $x \ge 0$

(d) $120x + 210y \le 15000$ (e) $y \ge 0$

16. Find: $(e^{\ln 2x})^2$

(a) $4x^2$ (b) $2x^2$ (c) $2x$ (d) e^{4x} (e) e^{4x^2}

17. Find: $(\ln e^{3x^2})^3$

(a) e^{27x^2} (b) $\ln 27x^6$ (c) $9x^6$ (d) $27x^6$ (e) e^{27x^6}

18. A woman plans to spend \$120 on an ornamental fence around a rectangular vegetable garden. The fencing for three sides of the garden costs \$2 per foot and for the fourth side \$3 per foot. If x represents the length of the side which costs \$3 per foot, what is the area of the garden?

(a) $-\frac{5}{4}x^2 + 30x$ (b) $-30x^2 + \frac{5}{4}$ (c) $3x + 6y$

(d) $\frac{120 - 5x}{4}$ (e) $\frac{4}{120 - x}$

19. Each widget requires 2 aluminum pieces, 3 steel pieces and 5 fasteners. Each wodget requires 1 aluminum piece, 2 steel pieces and 3 fasteners. Each gadget requires 3 aluminum pieces, 3 steel pieces and 6 fasteners. Let a, s and f denote respectively the number of aluminum pieces, steel pieces and fasteners required for a production run. If we produce 17 widgets, 37 wodgets and 57 gadgets, then $[a\ s\ f]$ = ?

(a) $\begin{bmatrix}242\\296\\538\end{bmatrix}$ (b) [102 296 798] (c) [170 222 684]

(d) [430 262 504] (e) [242 296 538]

20. In an ecology experiment, each carnivore consumes 1 herbivore, 2 units of food supplement and 7 units of water per week. In the same time each herbivore consumes 7 units of pasturage, 1 unit of food supplement and 14 units of water. Let N = $\begin{bmatrix}c\\h\end{bmatrix}$ denote the respective number of carnivores and herbivores to be maintained. Also let R = $\begin{bmatrix}h\\p\\s\\w\end{bmatrix}$ denote the weekly nutritional requirements of the system of herbivores, pasturage, food supplement and water respectively.
Then N = AR where A = ?

(a) $\begin{bmatrix}1&0&2&7\\0&7&1&14\end{bmatrix}$ (b) $\begin{bmatrix}1&0\\1&7\\2&1\\7&14\end{bmatrix}$ (c) $\begin{bmatrix}1&1&2&7\\0&7&1&14\end{bmatrix}$

(d) $\begin{bmatrix}1&1\\0&7\\2&1\\7&14\end{bmatrix}$ (e) $\begin{bmatrix}1&1&2&7\\0&7&1&14\end{bmatrix}$

21. Which of the following measures of central tendency is most affected by extreme values?

(a) mean (b) median (c) mode (d) range (e) midrange

22. Fifty elementary school students were asked the question, "Which is your favorite dessert, ice cream or cake or neither?" Of the 24 that preferred cake, 18 were girls. Twelve of the twenty-six boys preferred ice cream. If a child is chosen at random, what is the probability that the child is a boy or prefers cake?

(a) 12/25 (b) 3/25 (c) 19/25 (d) 22/25 (e) 1/2

23. A pair of octahedral dice is tossed. The faces on each die are numbered 1 through 8. If the total number of dots is counted, what is the probability that the number is at most 5?

(a) 5/12 (b) 1/4 (c) 5/18 (d) 7/64 (e) 5/32

24. You are offered the chance to choose one gem from either of two bags. Bag A has two diamonds and 4 sapphires while Bag B has 3 diamonds and 1 sapphire. The bags are not labeled. You choose one at random and pick a gem from it. What is the probability that you get a diamond?

(a) 1/2 (b) 1/3 (c) 3/4 (d) 1/6 (e) 13/24

25. A cookie jar has 6 chocolate chip cookies, 4 peanut butter cookies and 2 oatmeal cookies. Two cookies are taken from the jar without replacement. What is the probability that there is one chocolate chip cookie and one peanut butter cookie taken?

(a) 2/11 (b) 1/6 (c) 5/22 (d) 4/11 (e) 5/6

26. Find: $\tan\left(csc^{-1}\frac{5}{3}\right)$

(a) 3/4 (b) 4/3 (c) 5/3 (d) 3/5 (e) 5/6

27. Find: $(1 + i\sqrt{3})^6$

(a) 12 (b) 64 (c) $64i$ (d) -64 (e) -1

28. Solve for x between 0° and 360°: $\frac{\sin x}{1 + \cos x} = 1$

(a) 90° (b) 90°, 180°, 270° (c) 180°

(d) 90°, 270° (e) 180°, 270°

29. Find the sum of the roots of $4\cos^2 x = 1$

(a) 60 (b) 180 (c) 420 (d) 720 (e) 540

30. In triangle ABC, the measure of angle A is 48°, the measure of angle C is 22° and the length of side a is 4 inches. Find the length of side b to the nearest tenth.

(a) 110 (b) 5.1 (c) 12.5 (d) 2.5 (e) 9.9

31. The weight of a body is inversely proportional to the square of its distance from the center of the earth. If the radius of the earth is 4000 miles, and a body weighs 200 pounds on the earth's surface, what will be this body's weight, to the nearest pound, at a distance of 400 miles above the earth's surface?

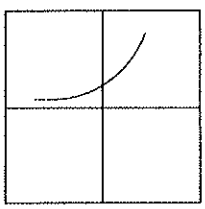
(a) 150 (b) 165 (c) 140 (d) 170 (e) 100

32. Let A = $[a_{ij}]$, B = $[b_{ij}]$ and C = $[c_{ij}]$ be n x n matrices with AB = C. Then c_{ij} = ?

(a) $\sum_{i=1}^{n} a_{ij}b_{ij}$ (b) $\sum_{j=1}^{n} a_{ij}b_{ij}$ (c) $\sum_{i=1}^{n} a_{ik}b_{ik}$

(d) $\sum_{k=1}^{n} a_{ik}b_{kj}$ (e) $\sum_{j=1}^{n} a_{ik}b_{ik}$

33. Which is the best approximation to the graph of $y = e^{x^2}$?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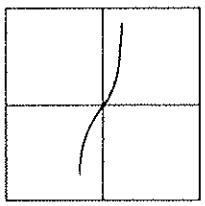
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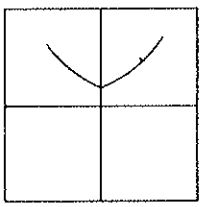
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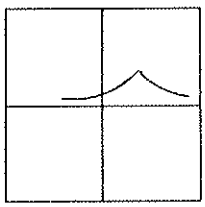
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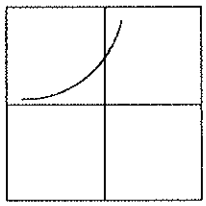
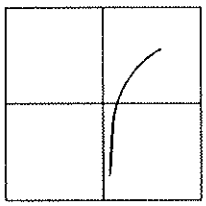

(e)

34. Which is the best approximation to the graph of $y = \ln\left(\frac{1}{x}\right)$?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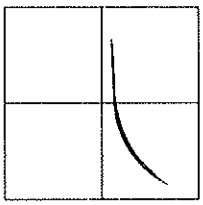
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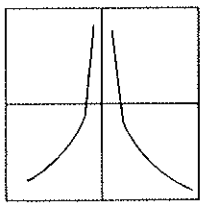
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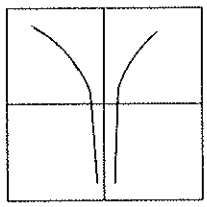
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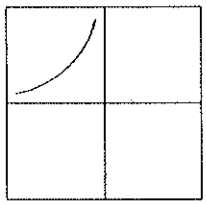
(e)

35. The sum of the series $\sum_{n=10}^{100} \frac{1}{n(n+1)}$ is:

(a) -91/100 (b) 91/1010 (c) 101/10

(d) 1010/91 (e) 10/101

36. A solution of the equation $4^x = 3(2^x)$ is:

(a) $\log_2 3$ (b) $\ln 3$ (c) $\log_3 4$ (d) $\log_4 3$ (e) $\log_3 2$

37. For all values of θ in its domain, $\frac{\tan\theta + \sin\theta}{\cot\theta + \csc\theta}$ is equal to:

(a) $\cos\theta \sin\theta$ (b) $\csc\theta \cos\theta$ (c) $\tan^2\theta$

(d) $\sin\theta \tan\theta$ (e) $\tan\theta \csc\theta$

38. If the fourth term of an arithmetic progression is 9 and the eighth term is 17, then the 431st term is:

(a) 863 (b) $3(2^{430})$ (c) $3(430^2)$

(d) 1292 (e) $2(3^{430})$

39. $\frac{80!}{78! + 79!}$ is equal to:

(a) 78 (b) 79 (c) 80 (d) $\frac{2!}{79!}$ (e) $\frac{80!}{157!}$

40. From a container filled with 1 gallon of grape juice, 1 pint is withdrawn and then the container is filled with water and thoroughly mixed. If this procedure is carried out exactly three times, which of the following is closest to the fractional part of the original contents left in the container?

(a) 2/3 (b) 1/3 (c) 1/2 (d) 3/4 (e) 1/4