THIRTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 1994

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Scoring formula: 4R - W + 40

Edited by Larry Bouldin, Roane State
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes. The use of calculators is prohibited.

NOTE: 1995 Contest date: April 4

Contributors to TMTA for Annual Mathematics Contest
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1. The product of the slope and the y-intercept of $7x - \frac{4y}{3} - \frac{7}{16} = 0$ is

a. $\frac{441}{256}$ b. $\frac{21}{16}$ c. $\frac{1}{16}$ d. $-\frac{441}{256}$ e. $-\frac{21}{16}$

2. Find the sum of the roots of $\sin 2x = \cos x$, where $0° \le x < 360°$.

a. 120° b. 270° c. 360° d. 540° e. 600°

3. Given $f(2x+1) = 3x^2 + 2$ for all x, find $f(x)$.

a. $3\left(x - \frac{1}{2}\right)^2 + 2$ b. $3(2x+1)^2 + 2$ c. $3\left(x + \frac{1}{2}\right)^2 + 2$ d. $\frac{3(x+1)^2}{4} + 2$

e. $3\left(\frac{x-1}{2}\right)^2 + 2$

4. If the line y = mx passes through the center of the circle $4x^2 + 4y^2 - 4x + 12y - 6 = 0$, then m =

a. $\frac{-1}{3}$ b. -3 c. $\sqrt{2}$ d. $\frac{1}{3}$ e. 3

5. Find the percent of increase (to the nearest whole number) in the volume of a cylinder when the altitude is increased by 10% and the radius is increased by 25%.

a. 35% b. 135% c. 42% d. 72% e. 172%

6. Given $A = P(1 + r)^t$, then t =

a. $\frac{\log A - \log P}{\log r}$ b. $\frac{\log A}{\log[P(1+r)]}$ c. $\frac{\log P - \log A}{\log r}$

d. $\frac{\log A}{\log P - \log(1+r)}$ e. $\frac{\log A - \log P}{\log(1+r)}$

7. An ordinary die and a die whose faces have 2, 3, 4, 6, 7, and 9 dots are tossed and the total number of dots noted. What is the probability that the sum is greater than or equal to 10?

a. $\frac{1}{9}$ b. $\frac{7}{18}$ c. $\frac{13}{36}$ d. $\frac{11}{36}$ e. $\frac{1}{3}$

8. Find the real number, a, for which the solution set of $|ax+2| \le 6$ is $\{x| \ -1 \le x \le 2\}$.

a. -2 b. 4 c. -4 d. 2 e. There is no such a.

9. For which real values of x is $f(x) = \sqrt{\frac{1}{x^2-4x-5}}$ a real number?

a. $\{x| \ x \le -5 \text{ or } x \ge 1\}$ b. $\{x| \ x < -1 \text{ or } x > 5\}$ c. $\{x| \ -1 \le x \le 5\}$
d. $\{x| \ -1 < x < 5\}$ e. $\{x| \ -5 < x < 1\}$

10. The graph of $|x^2+xy| = 0$ is

a. one straight line b. two straight lines c. three straight lines
d. a parabola e. a parabola and a line

11. $\left(-\sqrt{3}+i\right)^9$ =

a. $512-512i$ b. $512+512i$ c. 512 d. $512i$ e. $-512i$

12. Express $\sin^2\theta\cos^2\theta$ in terms of $\cos 4\theta$.

a. $\frac{1}{8}\cos 4\theta$ b. $-\frac{1}{8}\cos 4\theta$ c. $\frac{1}{8}(1+\cos 4\theta)$ d. $\frac{1}{8}(\cos 4\theta - 1)$ e. $\frac{1}{8}(1-\cos 4\theta)$

13. Solve for x. $\log_2 e^2 \cdot \log_e 2 = \log_4 x$

a. e b. 2 c. 16 d. 4 e. 4^e

14. The lengths of the sides of a triangle are 10 inches, 17 inches, and 21 inches. Find the length of the altitude to the 21-inch side.

a. 6 inches b. 7.095 inches c. 8 inches d. 8.095 inches e. 9 inches

15. A group of 40 diplomats gave a symposium for aspiring diplomats. Twenty of the diplomats had majored in history, thirteen has majored in political science, and eighteen had majored in law. Nine had majored in both history and law, six had majored in both history and political science, five had majored in both political science and law, and one diplomat had majored in all three. How many did not major in any of the three areas?

a. 11 b. 10 c. 8 d. 0 e. 9

16. Which of the following third-degree polynomial inequalities has the solution set $\{x \mid x \le -1 \text{ or } 2 \le x \le 4\}$?

a. $(x - 1)(x + 2)(x - 4) \ge 0$ b. $x^3 - 5x^2 + 2x + 8 \le 0$ c. $(x + 1)(x^2 + 2x - 8) \le 0$
d. $x^3 + 3x^2 - 6x - 8 \ge 0$ e. $x^3 - 2x^2 - 6x + 8 \le 0$

17. A man walks from A to B at x miles per hour and returns by the same route at y miles per hour. His average rate for the round trip is

a. $\frac{x+y}{xy}$ b. $\frac{2xy}{x+y}$ c. $\frac{2}{x+y}$ d. $\frac{x+y}{2}$ e. $\frac{x+y}{2xy}$

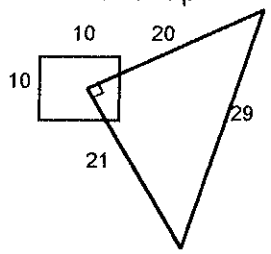
18. In how many ways can a committee of 3 men and 3 women be selected from a group of 4 men and 6 women?

a. 210 b. 9 c. 24 d. 2880 e. 80

19. The common ratio in the geometric series $3, \frac{2}{3}, \frac{4}{27}, \cdots$ is the common difference in an arithmetic series in which the tenth term is 8. The twenty-eighth term of the arithmetic series is

a. $\frac{2}{9}$ b. 20 c. 10 d. 12 e. $\frac{110}{9}$

20. A square whose sides are each 10 units of length and a right triangle with sides of 20, 21, and 29 units overlap as shown. The vertex of the right triangle is at the center of the square. What is the area of overlap?



a. 20 square units b. 25 square units c. 27 square units
d. 30 square units e. 35 square units

21. Given $\sin(A-B)=\frac{7}{25}$ with $(A-B)$ in QI, and $\sin B=\frac{4}{5}$ with B in QI. Find sin A.

a. $-\frac{75}{125}$ b. $\frac{100}{125}$ c. $\frac{117}{125}$ d. $-\frac{44}{125}$ e. $-\frac{13}{25}$

22. The perimeter of an isosceles triangle equals the square of the altitude to the base. What is the smallest possible area of the triangle if both the altitude and the area are positive integers?

a. 6 square units b. 12 square units c. 24 square units
d. 64 square units e. 120 square units

23. If L is a line that intersects the curve with equation $x+2xy-3y=2$ at (1,a) and (2,b), then the slope of L is

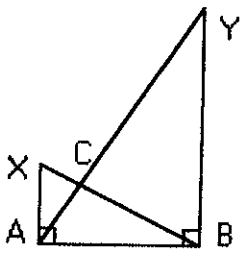
a. 2/3 b. 1 c. 2 d. -1 e. -2/3

24. If $f(x)=x^n-3x^{n-1}+2x-7$ and $g(x)=\frac{x^3+3}{x+1}$, what is $f\left[\frac{6}{g(1/2)-1/12}\right]$?

a. -1 b. -7 c. 1 d. 7 e. n

25. How many solutions are there to the equation $\sin^3 x=\sin x$ if $0\le x\le 2\pi$?

a. 1 b. 2 c. 3 d. 4 e. 5



26. Given: AX ⊥ AB, BY ⊥ AB, m∠ABC = 30°, and m∠BAC = 60°. Find the ratio of the perimeters of ΔACX to ΔYCB.

a. $\frac{1}{3}$ b. $\frac{\sqrt{3}}{3}$ c. $\frac{1+\sqrt{3}}{3}$ d. $\frac{1}{2}$ e. $\frac{\sqrt{3}}{2}$

27. The standard deviation of a set of data is zero if and only if

a. all the data have the same value.
b. all values of the data appear with the same frequency.
c. the mean of the data is also zero.
d. each of the data is zero.
e. the set contains the same number of positive and negative values.

28. The angle between the two lines x - y = 2 and $x-\sqrt{3}y=1$ is

a. 10° b. 15° c. 20° d. 30° e. 45°

29. Find x and y if $3^{x+y} = 9$ and $4^{x-y} = 8$.

a. $\left(\frac{1}{4},\frac{-7}{4}\right)$ b. $\left(\frac{7}{4},\frac{-1}{4}\right)$ c. $\left(\frac{5}{2},\frac{-1}{2}\right)$ d. $\left(\frac{5}{3},\frac{1}{3}\right)$ e. $\left(\frac{7}{4},\frac{1}{4}\right)$

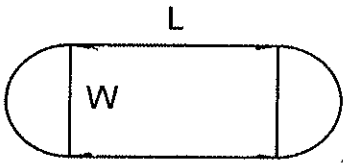
30. Express $\left(\frac{x^{3a+2b-c}}{\left(x^{2a}\right)\left(x^{b}\right)}\right)\left(x^{3c-a-b}\right)$ in simplest form.

a. $\frac{x^{2a+b+2c}}{x^{2a+b}}$ b. x^{2c} c. x^{a+b-c} d. $x^{2a-b+2c}$ e. 2a + b +2c

31. $\dfrac{i^5 - i^6}{i^4 - i^3} =$

a. $1 + i$ b. $1 - i$ c. -1 d. i e. 1

32. An athletic field with perimeter of 440 yards consists of a rectangle with a semicircle at each end as shown. Find the dimensions of the rectangle that yield the greatest possible area for the rectangular region.



a. L = 220, W = 220 b. L = 220, W = 110 c. L = 110, W = 220π
d. L = 110, W = $\dfrac{220}{\pi}$ e. L = 110, W = 110π

33. Box 1 contains 2 gold and 6 silver coins. Box 2 contains 4 gold and 2 silver coins. One of the boxes is randomly selected and then a coin is randomly selected from that box. If the selected coin is gold, what is the probability it came from box 1?

a. $\dfrac{3}{11}$ b. $\dfrac{3}{4}$ c. $\dfrac{1}{2}$ d. $\dfrac{1}{4}$ e. $\dfrac{11}{24}$

34. $4^{e^{\ln\left[\log_4(\ln 16)\right]}} =$

a. $2\ell n4$ b. 16 c. 4^e d. e^4 e. 4^{16}

35. Replace the polar equation $r = a\cos 3\theta$ by an equivalent Cartesian equation.

a. $(x^2+y^2)^2 = a(y^3 - 3x^2y)$ b. $(x^2-y^2)^2 = a(y^3 + 3x^2y)$
c. $(x^2-y^2)^2 = a(y^3 - 3x^2y)$ d. $(x^2-y^2)^2 = a(x^3 + 3xy^2)$
e. $(x^2+y^2)^2 = a(x^3 - 3xy^2)$

36. A box contains four \$10 bills, six \$5 bills, and two \$1 bills. Two bills are taken at random from the box without replacement. What is the probability that the total value of the two bills will be more than \$10?

a. $\frac{19}{33}$ b. $\frac{19}{36}$ c. $\frac{53}{66}$ d. $\frac{5}{9}$ e. $\frac{2}{3}$

37. If F is a function such that $F(0)=2$, $F(1)=3$ and $F(m+2)=2F(m)-F(m+1)$ then $F(5)=$?

a. 13 b. 7 c. -7 d. 3 e. -3

38. From a starting point A on a circle, a chord is drawn so as to make a 35° angle with the radius to point A. From the other endpoint of the chord another chord is drawn so as to make a 35° angle with the radius to this endpoint. This process is repeated until one arrives back at the starting point A. How many chords will there be?

a. 10 b. 11 c. 36 d. 72 e. 396

39. Two circles are concentric. A tangent to the inner circle forms a chord 12 inches long in the larger circle. Find the area of the ring between the circles rounded to the nearest whole unit.

a. 19 square inches b. 36 square inches c. 63 square inches
d. 113 square inches e. 144 square inches

40. What is the solution set of $\left|\frac{5+2x}{2-x}\right| \geq 1$?

a. $\{x \mid x \leq -7 \text{ or } x \geq -1\}$ b. $\{x \mid x \leq -7 \text{ or } x > 2\}$ c. $\{x \mid -7 \leq x \leq -1\}$
d. $\{x \mid x \leq -7 \text{ or } -1 \leq x < 2 \text{ or } x > 2\}$ e. $\{x \mid -1 \leq x < 2\}$