THIRTY-SEVENTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I 1993

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Coordinated by Jeff Knisley

Scoring formula: 4R - W + 40

Edited by: Larry Bouldin, Roane State
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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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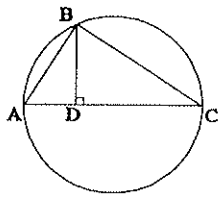
NOTE: 1994 CONTEST DATE --APRIL 12

1. θ measured in degrees is equal to θ measured in radians if and only if
   a. $\theta=\pi$ b. $\theta=\pi/2$ c. $\theta=2\pi$ d. $\sin\theta=0$ e. $\theta=0$

2. How many different values of x will satisfy $\sqrt{x}+x^2=x+x$?
   a. 0 b. 1 c. 3 d. 4 e. infinitely many

3. The solution set of $|x| + |x+9| = 9$ contains
   a. no numbers
   b. one number
   c. two numbers
   d. more than three numbers
   e. all real numbers

4. If ABC is a right triangle and BD $\perp$ AC, and if the lengths of AD and AB are 18 and 30, respectively, then the area of a circle circumscribing triangle ABC is
   a. 81π
   b. 225π
   c. 324π
   d. 625π
   e. 900π



5. One solution of $x^3+x-2=0$ is
   a. $\frac{-1+\sqrt{9}}{2}$
   b. $\frac{-1+\sqrt{-9}}{2}$
   c. $\frac{-1+\sqrt{7}}{2}$
   d. $\frac{-1+\sqrt{-7}}{2}$
   e. -1

6. If $x\geq 0$ and $\frac{e^{4x}}{16}=e^{2x}-2e^{x}+1$, then x is
   a. 0 b. 1 c. ln 2 d. ln 4 e. 3i

7. One factor of x^4+1 is
   a. $x^2+\sqrt{2}x+1$
   b. $x^2+x+\sqrt{2}$
   c. $x^2+\sqrt{2}x+\sqrt{2}$
   d. $x^2+2x+\sqrt{2}$
   e. $x^2-x+\sqrt{2}$

8. $\sqrt{2}^{\sqrt{2}^{\sqrt{2}^{\cdots}}}$ =

a. 0 b. $\sqrt{2}$ c. 2 d. ∞ e. π

9. When graphed together, which 4 of the following 8 equations form a happy face?

1. $x^2+y^2=9$
2. $x^2+y^2=25$
3. $y=x^2-4,\ -2\le x\le 2$
4. $y=4-x^2,\ -2\le x\le 2$
5. $(x+2)^2+(y+2)^2=1$
6. $(x+2)^2+(y-2)^2=1$
7. $(x-2)^2+(y+2)^2=1$
8. $(x-2)^2+(y-2)^2=1$

a. 1,3,5,6
b. 2,3,6,8
c. 2,4,5,6
d. 2,3,7,8
e. 1,3,6,8

10. Let $A=\begin{bmatrix}1 & 2 & 0\\ 1 & -1 & 2\\ 0 & 1 & 3\end{bmatrix}$ and $B=\begin{bmatrix}3 & 0 & 4\\ 0 & 5 & 4\\ 1 & 2 & 11\end{bmatrix}$. Which of the following statements is true?

a. A = B
b. AB = A
c. A - B = B - A
d. 3A = B
e. AB = BA

11. If the third term of an arithmetic sequence is 11, and the 30th term is 119, then the first term is

a. 1 b. 3 c. 4 d. 6 e. 7

12. A survey of 60 graduating seniors at East High School produced the following results: 38 of the students had taken Spanish, 15 of the students had taken French, 5 had taken Spanish and French, 4 had taken French and German, 2 had taken Spanish and German, 1 had taken all three and 5 had taken none of the three. How many students had taken German?

a. 7
b. 10
c. 12
d. 5
e. 9

13. $\cos^4(x) - \sin^4(x) =$
    a. cos(2x)
    b. sin(2x)
    c. cos(4x)
    d. sin(4x)
    e. 1

14. If $f(x) = \frac{3x+4}{2x-3}$, then $f(f(f(x)))=$
    a. x
    b. $f(x)$
    c. $(f(x))^2$
    d. $f(f(x))$
    e. $5f(x)$

15. If $\sin(x+A) = \cos(x-2A)$ and if $0 \le A \le \pi/2$, then $A =$
    a. 0
    b. $\pi/6$
    c. $\pi/4$
    d. $\pi/3$
    e. $\pi/2$

16. If $2\cos(3x) = \cos(x)$ and $\cos(x) \neq 0$, then $\cos(2x) =$
    a. 0 b. 1/2 c. 2/3 d. 3/4 e. 1

17. If $(1+i)^n = -4$, then $n=$
    a. 2 b. 3 c. 4 d. 5 e. 6

18. A rocket fired into the air has a height of $s(t) = -16t^2 + vt$ feet after t seconds. If the projectile returns to the launch site after 5 seconds, what was the greatest height it reached during its flight?
    a. 50 feet
    b. 75 feet
    c. 100 feet
    d. 125 feet
    e. 200 feet

19. Let $f(x) = \begin{cases} 3x & \text{if } x \le .5 \\ 3-3x & \text{if } x > .5 \end{cases}$. A solution to $f(f(x))=x$ is
    a. 1/3
    b. 1/2
    c. 2/7
    d. 3/10
    e. 7/9

20. The center of a circle connecting (0,0), (1,5) and (5,5) is
    a. (2,2)
    b. (3,2)
    c. (2,3)
    d. (3,3)
    e. (4,3)

21. The sum of the geometric series $5+\frac{5i}{2}-\frac{5}{4}-\frac{5i}{8}+\ldots$ is
    a. -2
    b. 2
    c. $2 - i$
    d. $2 + i$
    e. $4 + 2i$

22. A street light is at the top of a 15 foot pole. A man who is 6 feet tall is standing x feet from the bottom of the pole and is casting a shadow whose tip is y feet from the bottom of the pole. Assuming the ground is level and the pole forms a right angle with the ground, which of the following is true?
    a. $x + y = 21$
    b. $15x = 6y$
    c. $90 = y(y-x)$
    d. $9y = 15x$
    e. $15x + 6y = 9$

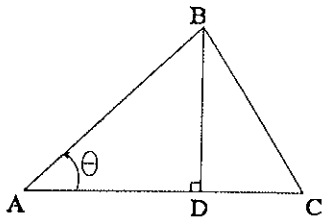
23. If $\sin(A+30°)=\frac{\sqrt{2}}{2}$ and $\sin(A+45°)=\frac{\sqrt{3}}{2}$, then $\sin(A)$ is
    a. $\frac{1}{4}$
    b. $\frac{1}{2}$
    c. $\frac{2-\sqrt{3}}{2}$
    d. $\frac{\sqrt{6}-\sqrt{2}}{4}$
    e. 1

24. If $\sigma_1=\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}$, $\sigma_2=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$ and $\sigma_3=\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$, then $(\sigma_2+\sigma_1)(\sigma_2-\sigma_1)=$
    a. $i\sigma_1$
    b. $2i\sigma_3$
    c. $\sigma_2-\sigma_3$
    d. $\sigma_2+\sigma_1$
    e. $3i\sigma_1$

25. If the lengths of AB and BC are 8 and 5, respectively, and $\theta=30°$, then the length of DC is

a. 1
b. 2
c. 3
d. 4
e. 5

(Assume AC ⊥ BD)

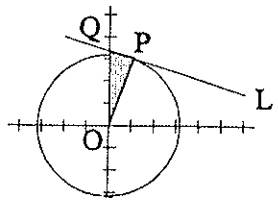


26. If $f(x)=x^4+2x^2-k$ and if $k > 0$, how many times does the graph of $f(x)$ cross the x-axis?

a. 0
b. 1
c. 2
d. 3
e. 4

27. If the line, L, is tangent to the graph of $x^2+y^2=10$ at the point P(1,3) and if Q is the y-intercept of L, then the area of triangle OPQ is

a. 5/3
b. 1
c. 2/3
d. 10/3
e. π



28. Three black balls and 1 red ball are placed into a box, and three balls are withdrawn at random. What is the probability that one of the balls is red?

a. 0
b. 1/4
c. 1/2
d. 3/4
e. 1

29. The product of the solutions to $[\ln(x)]^3=\ln(x^3)$ is

a. 0
b. 1
c. e
d. $e^{2\sqrt{3}}$
e. e^3

30. The graphs of $x^2+4y=4$ and $x^2+4y^2=4$ intersect in exactly how many points?

a. 0 b. 1 c. 2 d. 3 e. 4

31. If $\sin\theta=\sqrt{x}$ and $\cos\theta=\sqrt{2}\,x$ and if $0\le\theta\le 2\pi$, then x is

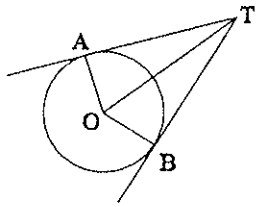
a. -1
b. 0
c. $\sqrt{2}/2$
d. $\sqrt{3}/2$
e. 1/2

32. For all $x>0$, the series $\sum_{n=0}^{\infty} e^{-nx}$ converges to

a. $\dfrac{e^x}{e^x-1}$
b. $\dfrac{1}{1-e^x}$
c. e^x
d. e^x+e^{-x}
e. $\dfrac{1}{e^x-e^{-x}}$

33. If TA and TB are tangents to a circle with center O, then which of the following is not necessarily true?

a. OA and OB have the same length
b. TA and TB have the same length
c. $\angle AOB=120°$
d. $\Delta AOT \cong \Delta BOT$
e. ΔAOT is a right triangle



34. How many ways can 8 loaves of bread be divided among 4 hungry people if each is to receive at least one loaf and if each receives a whole number of loaves?

a. 32
b. 35
c. 4
d. 8
e. 70

35. If 2 is a root of the polynomial $2x^3-3x^2+kx-4$, then k =

a. 0
b. -4
c. 2
d. 5
e. 3

36. Let $f(x)=\begin{cases} x+e & \text{if } x\le -1 \\ x^2 & \text{if } -1\le x<2 \\ 3x & \text{if } x\ge 2 \end{cases}$ and let $g(x)=e^x$. Then $f(g(-1))$ is

a. 0
b. $3e$
c. $-3e$
d. $(e-1)^2$
e. e^{-2}

37. If $\tan\theta=\frac{1}{3}$, then $\sin 2\theta$ =

a. 3/5 b. 3/10 c. $6/\sqrt{10}$ d. 2/9 e. $3/\sqrt{10}$

38. A number is divided by its sum with 2, resulting in x. The original number is

a. $\frac{x}{x+2}$
b. $\frac{x}{x-2}$
c. $\frac{x}{2-x}$
d. $\frac{2x}{1+x}$
e. $\frac{2x}{1-x}$

39. $\frac{5}{2-i}-\frac{2}{1-i}$ =

a. -1
b. 1
c. 2
d. i
e. $2i$

40. Which of the following functions satisfies $\frac{1}{f(x)}=f\left(\frac{1}{x}\right)$?

a. $f(x)=\frac{x}{x+1}$
b. $f(x)=\frac{x+1}{x+2}$
c. $f(x)=\frac{x+2}{2x+1}$
d. $f(x)=\frac{1-x}{1+x}$
e. $f(x)=\frac{2x+3}{3x+4}$