THIRTY-FOURTH ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Advanced Topics I   1990

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all the questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish to have a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. The working time for the entire test is 80 minutes.

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1. One point of intersection of the parabola $y - 5 = (x - 3)^2$ and the line $y = 3x$ is

   A. $(7,-6)$
   B. $(3,9)$
   C. $(2,7)$
   D. $(2,3)$
   E. $(7,21)$

2. The remainder when $3x^4 - 5x^2 + 2x - 7$ is divided by $(x + 2)$ is

   A. -55
   B. 17
   C. 25
   D. 1
   E. 0

3. If $F(x) = x^4 + Ax^3 + 3x^2 + A$ and $F(2) = -8$ then A is equal to

   A. -4
   B. $-22/5$
   C. 3
   D. 0
   E. $20/9$

4. The solution set for the inequality $\dfrac{1 - x}{x} \geq 1$ is

   A. $\{x \mid x \leq 1/2\}$
   B. $\{x \mid 0 \leq x \leq 1/2\}$
   C. $\{x \mid 0 < x\}$
   D. $\{x \mid x < 0\}$
   E. $\{x \mid 0 < x \leq 1/2\}$

5. If $\theta = 120°$, then $\sin^2\theta + \cos^2\theta + \tan^2\theta + \sec^2\theta + \cot^2\theta + \csc^2\theta$ is equal to

   A. 3
   B. 0
   C. $29/3$
   D. -3
   E. $23/2$

6. Let n_1 be the number of positive roots and n_2 be the number of negative roots of the polynomial equation $x^4 + 2x^3 - 3x^2 - 1 = 0$. Then $n_1 + n_2$ must be equal to

   A. 0
   B. 1
   C. 2
   D. 3
   E. 4

7. The smallest integer that is an upper bound for the real zeros of $f(x) = x^3 + x^2 + x - 9$ is

   A. 3
   B. 1
   C. 0
   D. 2
   E. 9
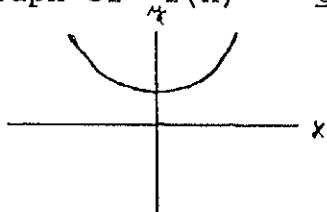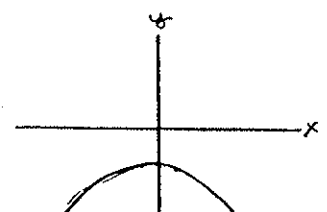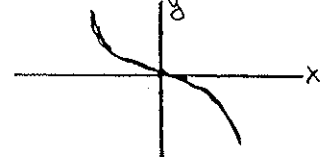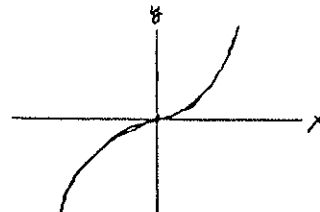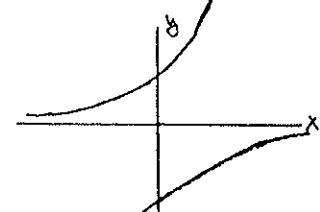
8. $\dfrac{80!}{78! + 79!}$ is equal to

 A. 78 D. 2!/79!
 B. 79 E. 80!/157!
 C. 80

9. If $f(x) = \sqrt{(x - 5)^3}$ and $g(x) = \dfrac{2 \ln(x)}{3}$, then the formula for the composition $g \circ f$, where defined, is

 A. $\sqrt{\left(\dfrac{2}{3} \ln(x) - 5\right)^3}$ D. $\ln(x - 5)^{2/3}$

 B. $\ln(x - 5)$ E. $\left(\dfrac{2}{3} \ln(x) - 5\right)^{2/3}$

 C. $\ln(x - 5)^{9/4}$

10. The graph of $f(x) = \dfrac{e^x - e^{-x}}{2}$ is

 A.

 B.

 C.

 D.

 E.

11. Given $g(x) = \sqrt{\dfrac{x^2 + x - 2}{x + 3}}$. The largest possible domain of g is

 A. $\{ x \mid x \neq -3 \text{ and } -2 \leq x \leq 1 \}$
 B. $\{ x \mid x \neq +3 \text{ and } -2 \leq x \leq 1 \}$
 C. $\{ x \mid x \neq -3 \}$
 D. $\{ x \mid -3 < x \leq -2 \text{ or } x \geq 1 \}$
 E. $\{ x \mid x \leq -2 \text{ or } x \geq 1 \}$

12. The period of $f(x) = -3\tan(x/3 - \pi/3)$ is

    A. 3π D. 6π
    B. π E. $2\pi/3$
    C. $\pi/3$

13. For all values of θ in its domain $\dfrac{\tan\theta + \sin\theta}{\cot\theta + \csc\theta}$ is equal to

    A. $\cos\theta\sin\theta$ D. $\sin\theta\tan\theta$
    B. $\csc\theta\cos\theta$ E. $\tan\theta\csc\theta$
    C. $\tan^2\theta$

14. The sum, if it exists, of the infinite series $\displaystyle\sum_{n=1}^{\infty} \dfrac{(-1)^{n-1}\,7}{5^{n-1}}$ is

    A. series has no sum   D. $35/4$
    B. $35/6$ E. $35/8$
    C. $5/4$

15. The roots of the equation $2x^3 - 3x^2 - 17x + 30 = 0$ are

    A. 2 , 6 , and 5 D. $3/2$, -3 , and -2
    B. 2 , -3 , and $5/2$ E. -1 , 3 , and $5/2$
    C. -2 , 3 , and 6

16. The rectangular equation that is equivalent to the polar equation $r = 4\cos\theta$ is

    A. $x^2 + y^2 = 4$ D. $x^2 + (y - 2)^2 = 4$
    B. $x^2 + y^2 = 4x$ E. $x^2 + y^2 - 4y = 0$
    C. $y^2 = 4x$

17. The expression i^{793} simplifies to

    A. 1 D. $-i$
    B. -1 E. 0
    C. i

18. The complex number $\sqrt{3} - i$ can be written in trigonometric form as

    A. $2(\cos(\pi/6) + i\sin(\pi/6))$
    B. $2(\cos(5\pi/6) + i\sin(5\pi/6))$
    C. $2(\cos(5\pi/3) + i\sin(5\pi/3))$
    D. $2(\cos(-4\pi/3) + i\sin(-4\pi/3))$
    E. $2(\cos(-\pi/6) + i\sin(-\pi/6))$

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19. If $f(x) = |x| + |x - 1|$ and $g(x)$ is the piecewise function defined by:

$$g(x) = \begin{cases} 2x - 1 & x > 1 \\ 1 & 0 \leq x \leq 1 \\ 1 - 2x & x < 0 \end{cases}$$
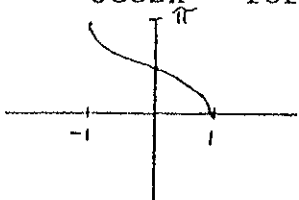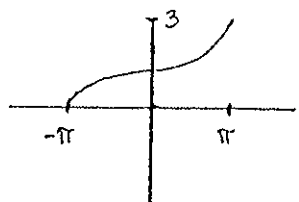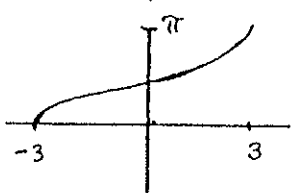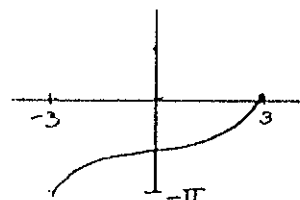
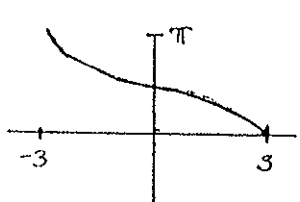Which of the following is true?

   A. $f(x) = g(x)$ only if $x \geq 1$
   B. $f(x) = g(x)$ only if $x \leq 0$
   C. $f(x) = g(x)$ only if $x \geq 1$ or $x \leq 0$
   D. $f(x) = g(x)$ only if $0 \leq x \leq 1$
   E. $f(x) = g(x)$ for all real numbers x

20. The exact value of $4\cos 285°$ is

   A. $\sqrt{2} - \sqrt{6}$ D. $\sqrt{6} - \sqrt{2}$
   B. $\sqrt{3} - 1$ E. $\sqrt{6} + \sqrt{2}$
   C. $\sqrt{3} - \sqrt{2}$

21. If $\pi/2 < x < y < \pi$, which of the following are true?

   1. $\sin y < \sin x$ 2. $\sin y > \sin x$ 3. $\cos y < \cos x$

   A.   1 only   D.   2 and 3 only
   B.   1 and 2 only   E.   1 , 2 , and 3
   C.   1 and 3 only

22. The function $f(x) = 2 - \sqrt[3]{(x - 2)^5}$ is one-to-one. A formula for $f^{-1}(x)$ is

   A. $\sqrt[3]{2} + (2 - x)^3$ D. $\dfrac{1}{2 - \sqrt[3]{(x - 2)^5}}$
   B. $2 + (2 - x)^{5/3}$
   C. $2 + (2 - x)^{3/5}$ E. $\dfrac{1}{2 - \sqrt[5]{(x - 2)^3}}$

23. A parabola has focus at F(3,0) and directrix the line x = 5. An equation for this parabola can be written as

   A. $y^2 + 4x = 16$ D. $x^2 - 4y + 16 = 0$
   B. $x^2 + 4y = 16$ E. $y^2 + x = 4$
   C. $y^2 - 4x + 16 = 0$

24. The middle term of the binomial expansion of $(2x - 1/(4x))^6$ is

   A. $15x^2$ D. $-5/2$
   B. $5/2$ E. $-15x^2$
   C. $15/(64x^2)$

25. If the fourth term of an arithmetic progression is 9 and the eighth term is 17, then the 431st term is

   A.   863
   B. $3(2^{430})$
   C. $3(430^2)$

   D.   1292
   E. $2(3^{430})$

26. Which graph represents the graph of the inverse of the function $f(x) = -3\cos x$ for $0 \le x \le \pi$?

   A.

   D.

   B.

   E.

   C.

27. Which of the following are equivalent to $\cos(3t)$?
      1. $3\cos t$ 2. $\cos(-3t)$ 3. $-\cos(3t)$

   A.   2 only
   B.   3 only
   C.   1 and 2 only

   D.   1 and 3 only
   E.   2 and 3 only

28. For the graph of the function $f(x) = \dfrac{1 - 2x}{x + 3}$ equations for the vertical asymptote and horizontal asymptote, respectively, are

   A. $x = 3$ and $y = 1/2$
   B. $x = -3$ and $y = -2$
   C. $x = -3$ and no horizontal

   D.   No vertical, $x = 1/2$
   E. $y = -3$ and $x = 1/2$

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29. The weight of a body is inversely proportional to the square of its distance from the center of the earth. If the radius of the earth is 4000 miles, and a body weighs 200 pounds on the earth's surface, what will be this body's weight, to the nearest pound, at a distance of 400 miles above the earth's surface?

    A. 150   D. 400
    B. 165   E. 100
    C. 200

30. What is the probability of getting at least one head in six tosses of a fair coin?

    A. 1/64   D. 5/6
    B. 3/32   E. 63/64
    C. 1/6

31. Given the vectors U and V such that $U = 2i + j$ and $V = -3i + 2j$. The magnitude of $2U + V$ is

    A. $\sqrt{10}$ D. $\sqrt{17}$
    B. 4   E. 3
    C. $\sqrt{15}$

32. The angle between the vector $V = 2i + j$ and the line $2x + y = 5$ is

    A. $\pi/3$ D. $\pi/4$
    B. 0   E. $\pi/2$
    C. $\pi/6$

33. A solution of the equation

    $$\ln \sqrt{x^2 - 1} - \ln \sqrt{x + 1} = 4$$ is

    A. $1 + e^2$ D. $1 + e^4$
    B. $1 - e^2$ E. has no real solution
    C. $e^8 + 1$

34. The sum of the series $\displaystyle\sum_{n=10}^{100} \frac{1}{n(n + 1)}$ is

    A. -91/100   D. 1010/91
    B. 91/1010   E. 10/101
    C. 101/10

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35. A solution of the equation $4^x = 3(2^x)$ is

    A. $\log_2 3$ D. $\log_4 3$
    B. $\ln 3$ E. $\log_3 2$
    C. $\log_3 4$

36. All acute values of x that are solutions of $\sin(4x - \pi/6) = 0$ are

    A. $\pi/6$ D. $\pi/24$ and $7\pi/24$
    B. $\pi/6$, $\pi/4$, and $\pi/3$ E. $7\pi/24$
    C. $\pi/24$, $7\pi/24$, and $13\pi/24$

37. In the given figure, $\tan\beta$ is equal to

    A. $3/7$ D. $21/59$
    B. $5/7$ E. $\sqrt{3}/3$
    C. $49/59$

38. From a container filled with 1 gallon of grape juice, 1 pint is withdrawn and then the container is filled with water and thoroughly mixed. If this procedure is followed exactly three times, which of the following is closest to the fractional part of the original contents left in the container?

    A. $2/3$ D. $3/4$
    B. $1/3$ E. $1/4$
    C. $1/2$

39. A group of students planned a trip and agreed to contribute equal amounts toward a fixed transportation cost of $270.00. Later 5 more students decided to go on the trip and the transportation cost for each student was reduced by $4.50. What amount did each pay for transportation?

    A. $9.00   D. $15.00
    B. $10.00   E. $18.00
    C. $13.50

40. The series $\sum_{n=0}^{\infty} \pi x^n$ converges for

    A. all real numbers x   D. only for $x = 0$
    B. only positive values of x   E. $\{x \mid \;|x| < 1\}$
    C. only negative values of x

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