THIRTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS I 1989

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Scoring formula: 4R - W + 40

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DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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1. Between what pair of consecutive integers do we find $\log_2 25$?

   A. 1 and 2
   B. 2 and 3
   C. 4 and 5
   D. 5 and 6
   E. 12 and 13

2. Let $f(x) = 2x - 3$. Then $g = f^{-1}$ will map (send)

   A. 11 back to 11
   B. every real r back to r
   C. 7 back to 11
   D. 3 back to 3
   E. -3 back to -9

3. If a and b are positive integers and if 9 is a factor of their product, which of the following must be true?

   A. 9 is a factor of both a and b
   B. 9 is a factor of one number but not the other
   C. 9 is a factor of at least one of the numbers a and b
   D. 3 is a factor of both a and b
   E. 3 is a factor of at least one of the numbers a and b

4. The probability of obtaining more heads than tails in five tosses of a fair coin is

   A. 5/16
   B. 2/5
   C. 1/2
   D. 3/5
   E. 5/8

5. For what value of k do the equations

   $$2x - 3y = 7$$
   $$6x - ky = 21$$

   represent the same line?

   A. -9
   B. -1
   C. 1
   D. 9
   E. -3

6. A car traveling at 90 miles per hour travels at

   A. 132 feet per second
   B. 88 feet per second
   C. 110 feet per second
   D. 77 feet per second
   E. 100 feet per second

7. If $|x - 2| = 7$, then x equals

   A. 9 or -5
   B. -9 or 5
   C. 9 only
   D. 9 or -9
   E. 5 or -5

8. If $\log_b x = -1/2$ and $b > 1$, then x equals

   A. $1/b$
   B. b
   C. b^{-2}
   D. $b^{-1/2}$
   E. $b^{1/2}$

9. Select the statement that negates the statement, "Sam plans both to wash his car and to mow his lawn."

   A. Sam does not plan to wash his car and he does not plan to mow his lawn.
   B. Either Sam does not plan to wash his car or he does plan to mow his lawn.
   C. If Sam plans to mow his lawn, then he plans to wash his car.
   D. If Sam plans to wash his car, then he does not plan to mow his lawn.
   E. If Sam does not plan to wash his car, then he plans to mow his lawn.

10. If $0 < y < x < \pi/2$, which of the following are true?

    1. $\sin y < \sin x$ 2. $\cos y < \cos x$ 3. $\tan y < \tan x$

    A. 1 only
    B. 1 and 2 only
    C. 1 and 3 only
    D. 2 and 3 only
    E. 1, 2, and 3

11. The roots of $x^2 - 3x + 1 = 0$ are

    A. both rational
    B. both complex
    C. one rational and one complex
    D. one irrational and one complex
    E. both irrational

12. Sam, Mike, and John invest \$5,000, \$7,000, and \$12,000 respectively in a business. If the profits are distributed proportionally, what share of a \$1,111 profit will John receive?

    A. \$231.40
    B. \$555.50
    C. \$780.00
    D. \$370.33
    E. \$101.11

13. If $\sqrt{144 - 169} = a + bi$, then (a,b) is the ordered pair

    A. (12,13)
    B. (12,-13)
    C. (0,1)
    D. (0,5)
    E. (0,-1)

14. What is the fifth Z-value that this BASIC computer program will print?

```
10  Let X = 6
20  Let Y = 4
30  Let Z = X  + Y - 2
40  PRINT Z
50  Let X = X + 1
60  Let Y = Y + 2
70  IF Y < 100, THEN GO TO 30
80  END
```

    A. 15
    B. 17
    C. 19
    D. 21
    E. 20

15. If $x > 1$, which of the following decreases as x decreases?

1. $x + x^2$ 2. $2x^2 - x$ 3. $\frac{1}{x+1}$

A. 1
B. 2
C. 3
D. 1 and 2
E. 2 and 3

16. The proposition $(p \Longrightarrow q) \wedge (\sim q)$ is true if

A. p is false and q is false
B. p is false and q is true
C. p is true and q is false
D. p is true and q is true
E. $p \Longrightarrow q$ is true

17. An operation * is defined on the integers such that when a,b,c are integers then $a*b = ab + a$. Such an operation * is

A. associative
B. commutative
C. self-distributive
D. both associative and commutative
E. neither associative nor commutative

18. The geometric series

$$4 + 4/3 + 4/9 + 4/27 + \ldots$$

has the sum

A. 8/3
B. 3/8
C. 6
D. 4
E. 5

19. The multiplication

$$\begin{array}{r} 1\ 2\ 3 \\ \times\quad 4\ 2 \\ \hline 2\ 5\ 0 \\ 5\ 4\ 0 \\ \hline 1\ 0\ 0\ 5\ 0 \end{array}$$

is carried out in base

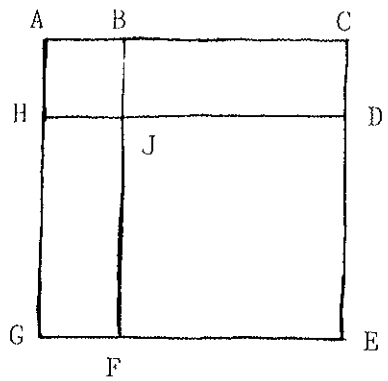
A. ten
B. nine
C. eight
D. seven
E. six

20. In the accompanying figure ABJH, JDEF, and ACEG are squares. If the ratio of BC to AB is 3, then the ratio of areas

area BCDJ to area JDEF

must be

A. 1/9
B. 1/3
C. 2/3
D. 3
E. 3/2



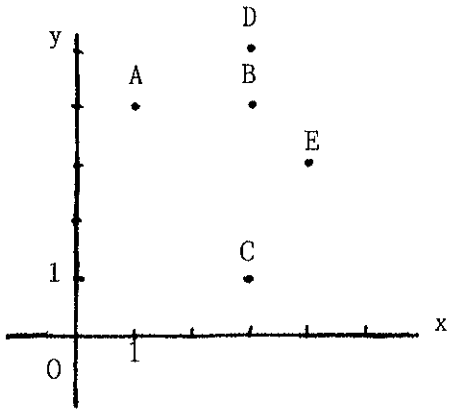
21. A day on Planet X consists of 16 "hours." This means that "noon" is at 0800 hours, "1:00 P.M." is 0900 hours, etc. A clock is set to strike on every "4" (as ours might on 6) and on each hour the number of times the hour indicates. After "midnight" on X we begin counting the strikes. Through noon the next day the clock will strike how many times?

A. 44
B. 43
C. 42
D. 45
E. 46

22. The polar equation $r = \frac{1}{2 \sin \theta + \cos \theta}$ can be expressed in rectangular coordinates as

A. $2y + x = 1$
B. $x^2 + (y - 2)^2 = 1$
C. $2x + y = 1$
D. $(x - 2)^2 + y^2 = 1$
E. $x + y = 2$

23. For the accompanying figure, let p denote that the x-coordinate is 3 and q denote that the y-coordinate is 4. Which of the lettered points on the figure does not satisfy p v q?

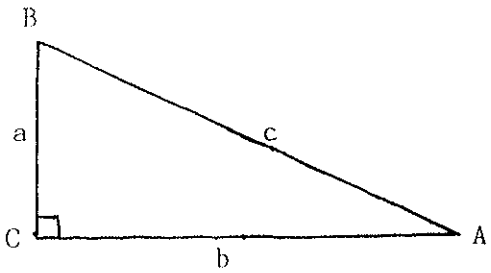


A. A
B. B
C. C
D. D
E. E

24. The expression $(x^{-1} + y^{-1})^{-1}$ simplifies to

A. $x + y$
B. $\frac{1}{x + y}$
C. $\frac{xy}{x + y}$
D. $\frac{x + y}{xy}$
E. $1/x + 1/y$

25. Which of the following is true about the figure?



A. $\sin^2 A + \cos^2 A = 1$
B. $\tan A + \tan B = c^2/ab$
C. $\frac{\sin A}{\cos A} = \tan A$
D. $b \sin A = a \sin B$
E. All the above are true

26. The value of x when $2^{3x-1} = 4^x$ is

A. 0
B. 2
C. -1
D. 1
E. -2

27. All of the following arguments have true conclusions, but not all of the arguments are valid. Select the argument that is not valid.

A. All integers are rational. All rational numbers are real numbers. Therefore, all integers are real numbers.
B. All squares have four sides. All rhombuses have four sides. Therefore, all squares are rhombuses.
C. All multiples of eight are divisible by two. All numbers divisible by two are even. Therefore, all multiples of eight are even.
D. All trapezoids have four sides. All four-sided figures are quadrilaterals. Therefore, all trapezoids are quadrilaterals.
E. All integers are numbers. Therefore, all integers are numbers or Superboy can jump tall buildings in a single bounce.

28. If x is the measure of an acute angle such that $\tan x = k/3$, then $\sin x$ equals

A. $\frac{k}{3 + k}$
B. $\frac{3}{\sqrt{9 + k^2}}$
C. $\frac{k}{\sqrt{9 - k^2}}$
D. $\frac{3}{9 + k^2}$
E. $\frac{k}{\sqrt{9 + k^2}}$

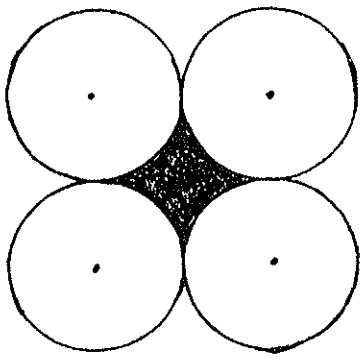
29. The natural domain of the function $h(x) = \log_{10} \frac{\sqrt{x}}{x - 2}$ is

A. all real x
B. $x > 2$
C. $x \geq 2$
D. $x < 0$
E. $0 < x < 2$

30. Let $f(x) = \sqrt{x - 4}$ for $x \geq 4$. The range of f is the set of all numbers y such that

A. $y \geq 4$
B. y is any rational number
C. $y \geq -4$
D. y is any real number
E. $y \geq 0$

31. Four equal circles of diameter 1 foot touch at four points as shown. What is the area of the shaded portion of the figure?



A. 4π
B. 2π
C. $2\pi^2$
D. $4 - \pi$
E. $\pi - 4$

32. If the average (arithmetic mean) of $x + 1$, $x + 4$, and $x + 7$ is 20, what is the x value?

A. 16
B. 17
C. 20
D. 24
E. 19

33. Which equation below has roots that are each 4 greater than the roots of the equation $3x^2 - 2x - 4 = 0$?

A. $3x^2 + 14x - 12 = 0$
B. $3x^2 - 26x + 52 = 0$
C. $6x^2 + 3x - 28 = 0$
D. $6x^2 + 16x + 9 = 0$
E. $3x^2 - 15x - 18 = 0$

34. If M(3,8) is the midpoint of the segment from A(-2,4) to B(x,y), then B is the point

A. (1,12)
B. (-5,-4)
C. (-6,32)
D. (8,4)
E. (8,12)

35. If $(2x - 3y)^{10}$ is expanded according to the binomial theorem, then the coefficient of the term containing xy^9 is

A. 1,029
B. 10,390
C. -15,360
D. 15,360
E. -10,390

36. The inequality $-4 < 4 - 2x < 10$ is satisfied by all x such that

A. $4 < x < -3$
B. $-3 < x < 4$
C. x is real
D. $x > 4$ or $x < -3$
E. $3 < x$ or $x < -4$

37. If the expression

$$\begin{vmatrix} a & c \\ d & b \end{vmatrix}$$

is defined to have the value ab - cd for all values of a, b, c, and d, then the equation

$$\begin{vmatrix} 2x & 1 \\ x & x \end{vmatrix} = 3$$

is satisfied for

A. only 1 value of x
B. 2 values of x
C. no values of x
D. an infinite number of values of x
E. x values of x

38. Insert two geometric means between 3 and 1/72:

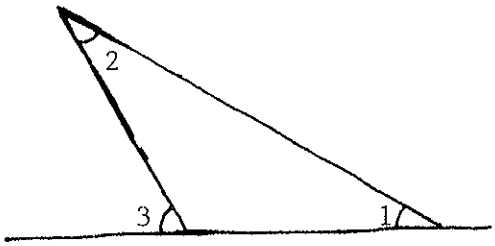
A. 1/6, 1/12
B. 1/3, 1/48
C. 1/2, 1/12
D. 1/70, 1/2
E. 2, 1/6

39. Which of the following is (are) always true about the figure regardless of the measures of 1, 2, and 3 ?

1. $\angle 1 + \angle 2 = \angle 3$

2. $\angle 3 > \angle 2$

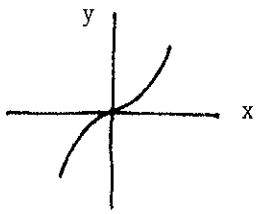
3. $180° > \angle 2 + \angle 3$



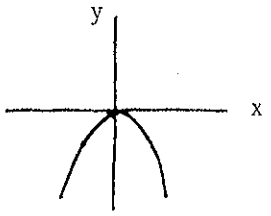
A. 1 and 2
B. 2 and 3
C. 1 and 3
D. 1 only
E. 2 only

40. Which of the following is the graph of $y = |x| \cdot x$?

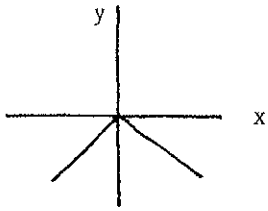
A.



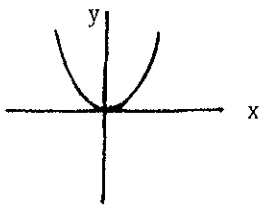
B.



C.



D.



E.