THIRTY-SECOND ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

ADVANCED TOPICS I 1988

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem there are listed 5 possible answers. You are to work each problem, determine the best answer, and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all questions. Do your very best on the questions you feel you know how to work. You will be penalized for incorrect answers, so it is advisable not to do wild guessing.

If you should change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on your answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will be able to keep this booklet after the test is completed.

When told to do so, open your test booklet to page 2 and begin. When you have finished one page, go on to the next. The working time for the entire test is 80 minutes.

Contributors to TMTA for Annual Mathematics Contest:

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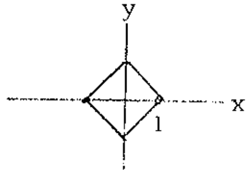
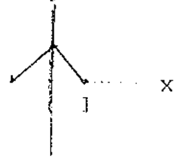
1. Which of the following best represents the graph of $|x| + |y| = 1$?

(a)



(b)



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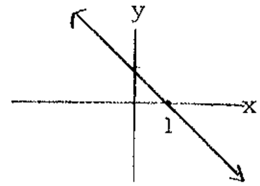
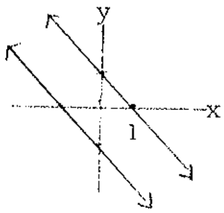
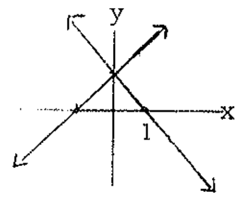


(d)



(e)



2. If $P(x) = x^4 + Ax^3 + Ax + 4$ and $P(2) = 6$, then $P(-2) =$

(a) 34 (b) 6 (c) -6 (d) -7/5 (e) 0

3. The point P divides the line segment $\overline{AB}$ where $A = (-3,1)$ and $B = (5,9)$ so that $|\overline{AP}|:|\overline{PB}| = 3:5$. Then P is the point

(a) (-1,3) (b) (0,4) (c) (1,5) (d) (2,6) (e) (3,7)

4. The distance between the lines $x + y = 1$ and $x + y = -1$ is

(a) 2 (b) $\sqrt{2}$ (c) 1 (d) $1/\sqrt{2}$ (E) 1/2

5. In one grading system, G, 60 is passing and 100 is perfect. In a second grading system, H, 70 is passing and 100 is perfect. A linear relation that converts passing in G to passing in H is

(a) $H = \frac{3}{4} G + 25$ (b) $H = \frac{4}{3} G + 25$ (c) $H = G$

(d) $H = -\frac{3}{4} G + 25$ (e) $H = -\frac{4}{3} G + 25$

6. $[(a^{-1} + b^{-1})^{-1} + c^{-1}]^{-1}$ expressed as a single fraction without negative exponents is

(a) $\dfrac{ac + bc}{abc + a + b}$ (b) $\dfrac{a + b + c}{1}$ (c) $\dfrac{abc}{bc + ac + ab}$

(d) $\dfrac{1}{a + b + c}$ (e) $\dfrac{1 + ac + bc}{a + b}$

7. If the radius of a circle is increased by 2 cm., its area increases by 16π cm^2. The radius of the original circle is

(a) 2 cm. (b) 3 cm. (c) 4 cm. (d) 5 cm. (e) 6 cm.

8. The S and T that satisfy $\frac{S}{x-2} + \frac{T}{x+3} = \frac{8x-1}{(x-2)(x+3)}$ are

(a) $S = 3,\ T = 5$ (b) $S = 5,\ T = 3$ (c) $S = 1,\ T = 1$ (d) $S = 2,\ T = 2$ (e) $S = 8,\ T = -1$

9. If $f(x) = 3x - 1$ and $g(x) = 2x + k$, then the value of k for which $f(g(x)) = g(f(x))$ is

(a) -2 (b) $-1/2$ (c) 0 (d) $1/2$ (e) 2

10. If $x - \frac{1}{x} = 2$, then $x^3 - \frac{1}{x^3}$ is equal to

(a) $1 + \sqrt{2}$ (b) 7 (c) 8 (d) $11 + \sqrt{2}$ (e) 14

11. $\mathrm{Tan}^{-1}(\sec(\sin 0)) =$

(a) 0 (b) $\pi/6$ (c) $\pi/4$ (d) $\pi/3$ (e) $\pi/2$

12. The asymptotes of the graph of $y = \frac{1+x}{1-x}$ are the lines

(a) $x = -1$, only (b) $y = 1$, only (c) $x = 1$ and $y = 1$ (d) $x = -1$ and $y = 1$ (e) $x = 1$ and $y = -1$

13. $\mathrm{Sin}\ 15^\circ =$

(a) $\sqrt{1 + (1/2)}/\sqrt{2}$ (b) $\sqrt{1 - (1/2)}/\sqrt{2}$ (c) $1/4$ (d) $\sqrt{2 + \sqrt{3}}/2$ (e) $\sqrt{2 - \sqrt{3}}/2$

14. The difference $2^{10} - 2^9$ is

(a) 2 (b) 2^3 (c) 2^4 (d) 2^5 (e) 2^9

15. The repeating decimal $0.3333\cdots$ in base 10 arithmetic represents the fraction $1/3$. What fraction does it represent in base 5 arithmetic?

(a) $1/9$ (b) $1/6$ (c) $1/3$ (d) $2/3$ (e) $3/4$

16. The largest set of real numbers, x, for which

$f(x) = \ln\sqrt{\pi - 4\,\mathrm{Tan}^{-1}x}$ is defined is

(a) $(-\infty,1)$ (b) $(-\infty,1]$ (c) $(0,1)$ (d) $(1,\infty)$ (e) $[1,\infty)$

17. A basketball tournament has eight teams and a team is out of the tournament if it loses two games. The minimum number of games needed to determine a winner is

(a) 11 (b) 12 (c) 13 (d) 14 (e) 15

18. The third term in the binomial expansion of $\left(a + (2/a)\right)^{12}$ is

(a) $66a^8$ (b) $264a^8$ (c) $1760a^6$ (d) $220a^6$ (e) $66a^6$

19. The graph of the equation $4x^2 - 4xy + 4y^2 - 2x + 8y - 73 = 0$ is

(a) a circle. (b) an ellipse. (c) a parabola. (d) a hyperbola. (e) a line.

20. The multiplicative inverse of the matrix $\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ is

(a) $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$ (e) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

21. What is the largest positive value of k for which the graph of $y = 4x^2 + 2kx + 9$ lies on or above the x-axis?

(a) 2 (b) 4 (c) 6 (d) 10 (e) 12

22. If $x = \log_b a$ and $y = \log_a b$ where $b \neq 1$ and $a \neq 1$, then $x/y =$

(a) 1 (b) a/b (c) $\log_b a^2$ (d) $(\log_b a)^2$ (e) b/a

23. The number of 9 letter "words" that can be constructed from the letters in the word TENNESSEE is

(a) 24 (b) 126 (c) 630 (d) 3780 (e) 362,880

24. The complete solution set of $16^{|3x-2|} = 8^{x-1}$ is

(a) $\{1/3\}$ (b) $\{5/9\}$ (c) $\{5/9, 11/15\}$ (d) $\{-5/9, -11/9\}$ (e) $\{5/9, 1/3\}$

25. The ordered pair form of the complex number $z = x + yi$ is (x,y). If $z = 1 + i$, then the ordered pair form of $(3z-1)/(z-1)$ is

(a) $(-2,3)$ (b) $(-3,2)$ (c) $(1,1)$ (d) $(3,-2)$ (e) (∞,∞)

26. The smallest period of the function $f(x) = \sin 2x - \frac{1}{3}\sin 6x$ is

(a) $\pi/3$ (b) $2\pi/3$ (c) π (d) 2π (e) 3π

27. The graph of the polar equation $r = 13/(4 + 4\cos\theta)$ is

(a) an ellipse. (b) a parabola. (c) a hyperbola. (d) a cardiod. (e) a lemniscate.

28. If $1/x = \sqrt{2 + \sqrt{2}}$, then which of the following is true?

(a) $2 + x^2 = 1/2x^2$ (b) $(2x^2 - 1)/x^2 = \sqrt{2}$ (c) $x = 1/(\sqrt{2} + \sqrt[4]{2})$ (d) $2x^4 - 4x^2 + 1 = 0$ (e) $2x^4 + 4x^2 + 1 = 0$

29. Which of the following is NOT logically equivalent to the logical implication "p implies q" ?

(a) q is necessary for p. (b) p only if q. (c) q provided p. (d) q is sufficient for p. (e) If p then q.

30. The number of diagonals of a convex polygon of ten sides is

(a) 8 (b) 28 (c) 35 (d) 40 (e) 45

31. Given the function $f(x) = x^2 - 4$, $x \geq 2$, then the inverse function $f^{-1}(x)$ exists and satisfies the formula

(a) $-\sqrt{4-x}$, $x \geq 0$ (b) $-\sqrt{x+4}$ (c) $\sqrt{x+4}$, $x \geq 0$ (d) $\sqrt{4-x}$ (e) $2 + \sqrt{x}$

32. If $\log_{10}(x^2-1) - \log_{10}(x-1) = 0$, then the complete solution set is

(a) $\emptyset$ (b) $\{0\}$ (c) $\{-1\}$ (d) $\{0,1\}$ (e) $\{-1,0\}$

33. For the system of linear equations
$$\begin{aligned} 3x + 4y - bz &= 17 \\ ax - ay + az &= -4 \\ bx - 3y + az &= -7 \end{aligned}$$
to have the solution $(1,2,-3)$, the values of a and b should be, respectively,

(a) 0 and 0 (b) -2 and -1 (c) 1 and 3 (d) 1 and 2 (e) 1 and -5

34. If S_n is a convergent infinite sequence such that $S_{n+1} = \frac{1}{2} S_n + 1$ for all positive integers n, then $\lim\limits_{n\to\infty} S_n$

(a) 0 (b) 1/2 (c) 1 (d) 3/2 (e) 2

35. The complete solution set of the equation $\sin(x-1) = \sin(1-x)$ is

(a) $\{x \mid x = k\pi$, k is an integer.$\}$
(b) $\{x \mid x = k\pi/2$, k is an integer.$\}$
(c) $\{x \mid x = 1 + k\pi$, k is an integer.$\}$
(d) $\{x \mid x = 1 + 2k\pi$, k is an integer.$\}$
(e) $\{x \mid x = 2k\pi$, k is an integer.$\}$

36. If P and Q are points in the plane with polar coordinates $(-1,\pi/4)$ and $(3,-\pi/4)$, respectively, then the distance between P and Q is

(a) $\sqrt{10}$ (b) $\sqrt{64+\pi}/4$ (c) $\sqrt{64+\pi^2}/4$ (d) $\sqrt{256+\pi^2}/4$ (e) 4

37. The probability of Harvey, James, Thomas and William each solving this problem correctly is 1/6, 1/2, 2/3 and 3/4, respectively. What is the probability that all four will solve it?

(a) 1/24 (b) 1/12 (c) 3/8 (d) 1/2 (e) 5/8

38. If $\det\begin{pmatrix} x-1 & -2 \\ 1 & x-4 \end{pmatrix} = 0$ then the complete solution set is

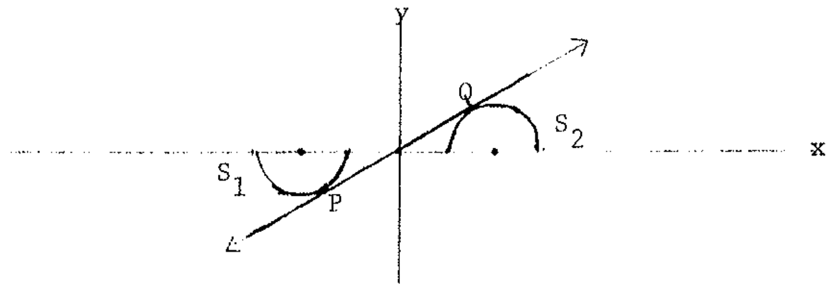
(a) $\{2,3\}$ (b) $\{-2,-3\}$ (c) $\{1,4\}$ (d) $\{-1,-4\}$ (e) $\{1,6\}$

39. If $y = (e^{2x} - e^{-2x})/2$ then $x =$

(a) $\frac{1}{2}\ell n(y + \sqrt{y^2 - 1})$ (b) $\frac{1}{2}\ell n(y - \sqrt{y^2 + 1})$ (c) $\frac{1}{2}\ell n(y + \sqrt{y^2 + 1})$

(d) $\ell n(y + \sqrt{y^2 - 1})$ (e) $\ell n(y - \sqrt{y^2 + 1})$

40. The semicircles S_1 and S_2 each have radius 1 and centers at the points $(-2,0)$ and $(2,0)$, respectively. (See Figure below.) There is a unique line $\overleftrightarrow{PQ}$ that is tangent to S_1 at P and is tangent to S_2 at Q.



The coordinates of the point Q are

(a) $(\sqrt{3}/2, 3/2)$ (b) $(1, \sqrt{3})$ (c) $(\sqrt{3}, 1)$ (d) $(3/2, \sqrt{3}/2)$ (e) $(3/2, \sqrt{3})$