



TENNESSEE MATHEMATICS TEACHERS ASSOCIATION

SIXTY-FIFTH ANNUAL MATHEMATICS CONTEST

2023

Algebra II/Integrated Math III

Prepared by:

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Scoring Formula: $4 \times (\text{Number Right}) - (\text{Number Wrong}) + 40$

Directions:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

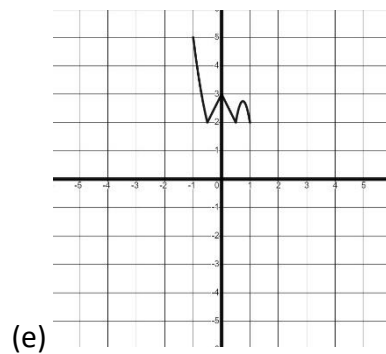
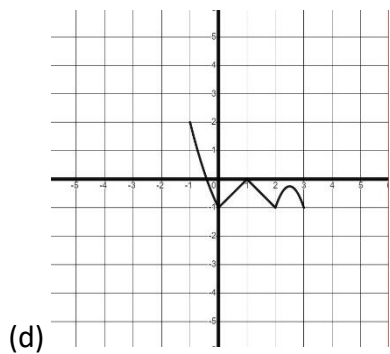
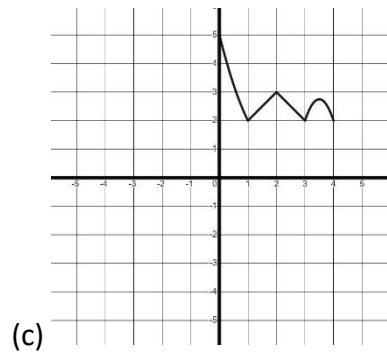
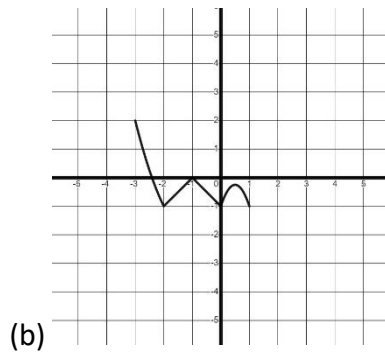
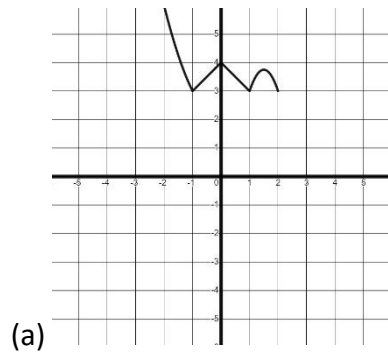
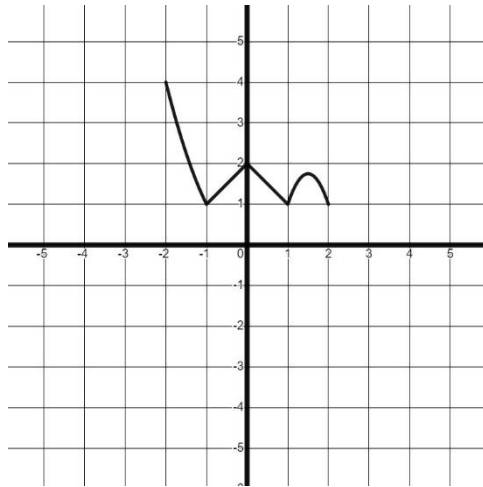
This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. What are the x -values of the points where the line $y = 5 - x$ and the parabola $y = x^2 - 2x + 3$ intersect?
- (a) -1 and 2
(b) -2 and 1
(c) $1 + \sqrt{3}$ and $1 - \sqrt{3}$
(d) $\frac{3+\sqrt{17}}{2}$ and $\frac{3-\sqrt{17}}{2}$
(e) These two graphs do not intersect.
2. For which of the following values of θ is $\sin \theta \neq \frac{1}{2}$?
- (a) 30° (b) $\frac{\pi}{6}$ (c) 570° (d) $\frac{29\pi}{6}$ (e) -330°
3. If the number of bacteria in a culture increases 6% every hour, approximately how long will it take the original sample to double?
- (a) 1.9 hours (b) 11.9 hours (c) 16.7 hours (d) 33.3 hours (e) 94.3 hours
4. Which of the following is the equation of a circle with radius $r = 2$ centered at the point $(-1, 3)$?
- (a) $x^2 + 2x + y^2 - 6y + 8 = 0$
(b) $x^2 + 2x + y^2 - 6y + 6 = 0$
(c) $x^2 - 2x + y^2 + 6y + 8 = 0$
(d) $x^2 - 2x + y^2 + 6y + 6 = 0$
(e) $x^2 + 1 + y^2 - 9 - 4 = 0$
5. What is the equation of the line passing through the point $(1, -2)$ that is perpendicular to the graph of the line $2x - 3y = 7$?
- (a) $3x + 2y = -1$
(b) $-2x + 3y = -7$
(c) $2x - 3y = -\frac{1}{7}$
(d) $\frac{1}{3}x + \frac{1}{2}y = -1$
(e) $-\frac{1}{2}x + \frac{1}{3}y = 7$

6. If the pictured graph is the graph of $y = f(x)$, which of the choices shows the graph of $y = f(x + 1) - 2$?



7. If θ is a third-quadrant angle with $\cos \theta = -\frac{7}{12}$, what is the value of $\tan \theta$?

(a) $\frac{7}{\sqrt{95}}$

(b) $-\frac{7}{\sqrt{95}}$

(c) $-\frac{\sqrt{95}}{12}$

(d) $\frac{\sqrt{95}}{7}$

(e) $\frac{\sqrt{95}}{12}$

8. The parabolas $y = 2x^2$ and $y = 4x - 4x^2$ intersect at two points, one of which is the origin. What is the x -coordinate of the non-origin intersection point?

- (a) 0 (b) $\frac{4}{3}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{2}{3}$ (e) $\frac{\sqrt{3}}{2}$

9. What is the value of the sum $\frac{4}{x+1} + \frac{-1}{x-1}$?

- (a) $\frac{3}{2x}$ (b) $\frac{3x}{x^2-1}$ (c) $\frac{3x-1}{x^2-1}$ (d) $\frac{3x-2}{x^2-1}$ (e) $\frac{3x-5}{x^2-1}$

10. Molly drove the entire length of Interstate 40 in Tennessee, beginning at the Arkansas border in Shelby County and travelling 732.7 km to the North Carolina border in Cocke County. If it took her 9 hours and 40 minutes, which of the following must be true?

- (a) Molly drove approximately 75.8 kilometers every hour for her entire trip.
(b) Molly's average speed was approximately 75.8 kilometers per hour.
(c) Molly never drove slower than 37.9 kilometers per hour.
(d) Molly never drove faster than 113.7 kilometers per hour.
(e) Molly spent the night in Nashville.

11. Max's "special" blend of tea is just chamomile (which costs \$24/pound) and decaffeinated black tea (which costs \$45/pound). If Max wants the special blend to cost \$30/pound, what is the ratio of pounds of chamomile to pounds of decaffeinated black tea Max must mix to make the blend?

- (a) 2:5 (b) 2:7 (c) 5:2 (d) 5:7 (e) 7:2

12. Given a polynomial $p(x)$ and a real number c , which of the following statements is not equivalent to the others?

- (a) c is a root of $p(x) = 0$.
(b) $(x - c)$ is a factor of $p(x)$.
(c) $(x + c)$ is a factor of $p(x)$.
(d) $p(c) = 0$.
(e) The remainder in the division $p(x) \div (x - c)$ is zero.

13. A quadratic function of the form $f(x) = ax^2 + bx + c$ has one root equal to $3 + 2\sqrt{2}i$. What is the value of $\frac{b}{a}$?

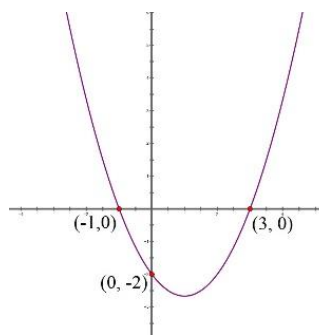
- (a) -3 (b) 3 (c) $\frac{17}{3}$ (d) 6 (e) -6

14. What are the real solutions to the equation $x^{2/3} - 1 = x^{1/3}$?

- (a) $\left\{\frac{1+\sqrt{5}}{2}, \frac{1-\sqrt{5}}{2}\right\}$
- (b) $\{2 + \sqrt{5}, 2 - \sqrt{5}\}$
- (c) $\left\{\sqrt[3]{\frac{1+\sqrt{5}}{2}}, \sqrt[3]{\frac{1-\sqrt{5}}{2}}\right\}$
- (d) $\left\{\frac{1+\sqrt{5}}{2}\right\}$
- (e) There are no real solutions

15. The graph displayed shows the graph of a parabola with intercepts as labeled. What is the equation of the parabola?

- (a) $y = x^2 - 3x - 2$
- (b) $y = x^2 - 2x - 3$
- (c) $y = \frac{2}{3}x^2 - \frac{4}{3}x - 2$
- (d) $y = \frac{1}{3}x^2 - \frac{2}{3}x - 2$
- (e) $y = x^2 - 2x - 2$



16. Which of the following expressions is equivalent to $\frac{x^2+5x+6}{x^2+2x-3} \div \frac{x^2+3x+2}{x^2-1}$ (on the intervals on which they are both defined)?

- (a) 1
- (b) $\frac{(x+2)^2}{(x-1)^2}$
- (c) $\frac{1}{x^2+2x-3}$
- (d) $\frac{x^2+2x-3}{(x+1)^2}$
- (e) $\frac{x^2+2x-3}{x^2-2x-3}$

17. Jo has three cats named Amy, Beth, and Meg. She needs to know how much each of them weighs, but they are skittish and will only get on the scale with one of the other cats. She knows, then, that Amy and Meg together weigh 17 pounds, Amy and Beth together weigh 19 pounds, and Meg and Beth together weigh 22 pounds. How much does Beth weigh?

- (a) 7 pounds
- (b) 8 pounds
- (c) 10 pounds
- (d) 11 pounds
- (e) 12 pounds

18. What is the equation of the line tangent to the circle $(x - 3)^2 + (y + 4)^2 = 25$ that passes through the origin?

- (a) $y = 24 - x$
- (b) $y = -\frac{4}{3}x$
- (c) $y = \frac{3}{4}x$
- (d) $y = -\frac{3}{4}x - \frac{7}{4}$
- (e) $y = \frac{3}{4}x + \frac{7}{4}$

19. Two real numbers have both a sum and a product of -6 . What is the value of the larger of the two numbers?

- (a) -6
- (b) -3
- (c) $3 + \sqrt{3}$
- (d) $3 - \sqrt{3}$
- (e) $-3 + \sqrt{15}$

20. Consider the equation $(2023^x)^y = 2023^x 2023^y$. For which values of x are there no values of y that make the equation true?

- (a) $\ln(2023)$
- (b) 1
- (c) 0
- (d) $\frac{1}{2023}$
- (e) The equation has solutions for every real value of x .

21. Consider a triangle whose vertices are the origin and the intercepts of $y = \frac{5-2x}{3+2x}$. What is the area of the triangle if the point $(1, 0)$ is 1 cm from the origin?

- (a) $\frac{1}{3} \text{ cm}^2$
- (b) $\frac{2}{3} \text{ cm}^2$
- (c) $\frac{3}{2} \text{ cm}^2$
- (d) $\frac{25}{6} \text{ cm}^2$
- (e) $\frac{25}{12} \text{ cm}^2$

22. Which of the following is equivalent to $\sqrt[3]{2}\sqrt{3}$?

- (a) $\sqrt[5]{6}$
- (b) $\sqrt[6]{6}$
- (c) $\sqrt[6]{72}$
- (d) $\sqrt[6]{108}$
- (e) $\sqrt[6]{6^5}$

23. For which value of x does $3e^{2x} = 2e^{3x}$?

- (a) 0
- (b) 1
- (c) $\ln 5$
- (d) $\ln \frac{2}{3}$
- (e) $\ln \frac{3}{2}$

24. Laurie is taking a dual-enrollment statistics class, and “forgot” to go to class last Friday. Unfortunately, he missed a quiz and received a zero on that quiz. Luckily, his kindly old statistics professor will drop his lowest quiz grade. He’s had ten quizzes, and the average (mean) of his quiz grades with the zero is 15.3. What will his average be when the professor drops the zero?

- (a) 16.8
- (b) 17
- (c) 17.3
- (d) 18
- (e) There is insufficient information to solve this problem.

25. If the sum of three consecutive integers is s , what is their product?

- (a) $\frac{s^3}{27}$
- (b) $\frac{s^3-s}{27}$
- (c) $\frac{s^3-9s}{27}$
- (d) $\frac{s^3+9s^2+18s}{27}$
- (e) $\frac{s^3-9s^2+18s}{27}$

26. The Mossy Creek Bottling Company produces cans of Fresh Cola Drink, the volumes of which vary normally with mean 12 ounces and standard deviation 0.5 ounce. Approximately what proportion of cans of Fresh Cola drink contain between 11 ounces and 12.5 ounces?

- (a) 47.5%
- (b) 68%
- (c) 81.5%
- (d) 95%
- (e) 97.35%

27. Which of the following is equivalent to $\frac{1}{\sqrt{3}-\sqrt{2}}$?

- (a) $\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{2}}$ (b) $\sqrt{3} + \sqrt{2}$ (c) 1 (d) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{6}}$ (e) $\frac{-1}{\sqrt{6}}$

28. If one of the zeroes of $f(x) = x^4 - 8x^3 + 25x^2 - 36x + 20$ is $2 + i$, what is the complete set of solutions to the equation $x^4 - 8x^3 + 25x^2 - 36x + 20 = 0$?

- (a) $\{2 + i, 2\}$
- (b) $\{2 + i, 2 - i\}$
- (c) $\{2 + i, 2 - i, -2\}$
- (d) $\{2 + i, 2 - i, 2\}$
- (e) $\{2 + i, 2 - i, 2, -2\}$

29. What is the solution set of the inequality $\frac{x+1}{x-2} \leq 1$?

- (a) $[0, 3]$
- (b) $(-\infty, 2]$
- (c) $(-\infty, 2)$
- (d) $(-\infty, 0] \cup [2, \infty)$
- (e) $(-\infty, 0] \cup (2, \infty)$

30. If $\log_2 a = m$ and $\log_4 b = n$, what is the value of $\log_8 \frac{a}{2b}$?

- (a) $\frac{m}{2n}$
- (b) $m - 2n$
- (c) $\frac{m+2n-1}{3}$
- (d) $\frac{m-2n-1}{3}$
- (e) $3m + 6n - \frac{1}{3}$

31. A geometric sequence has first term $a_1 = 1$ and fourth term $a_4 = 4$. What is the sum of the first six terms of the sequence?

- (a) 21
- (b) 15
- (c) $5 + 5\sqrt[3]{4} + 5\sqrt[3]{16}$
- (d) $21 + 5\sqrt[3]{4} + 10\sqrt[3]{2}$
- (e) $21 + 10\sqrt[3]{4} + 5\sqrt[3]{2}$

32. The points $A = \left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ and $B = \left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ both lie on the circle whose equation is $x^2 + y^2 = 1$. What is the length of the longer of the two arcs of the circle having A and B as endpoints?

- (a) $\frac{5\pi}{12}$
- (b) $\frac{7\pi}{12}$
- (c) $\frac{15\pi}{12}$
- (d) $\frac{17\pi}{12}$
- (e) $\frac{19\pi}{12}$

33. The function $f(x) = \frac{3x+2}{4x+3}$ has domain $\{x|x \neq -\frac{3}{4}\}$. What is the value of $f^{-1}(x)$?

- (a) $f^{-1}(x) = \frac{4x+3}{3x+1}$
- (b) $f^{-1}(x) = \frac{-3x+2}{4x-3}$
- (c) $f^{-1}(x) = -\frac{3x+2}{4x+3}$
- (d) $f^{-1}(x) = \frac{\frac{1}{3}x + \frac{1}{2}}{\frac{1}{4}x + \frac{1}{3}}$
- (e) $f^{-1}(x) = \frac{3+2x}{4+3x}$

34. If $\log_2 x = \log_x 4$, what is the set of possible values of x ?

- (a) $\{2\}$
- (b) $\{\sqrt{2}, \frac{1}{\sqrt{2}}\}$
- (c) $\{2^{\sqrt{2}}\}$
- (d) $\{2^{\sqrt{2}}, \frac{1}{2^{\sqrt{2}}}\}$
- (e) $\frac{1}{\ln \sqrt{2}}$

35. Each spring Jessica, Sam, and Michael work together to score the TMTA High School Math Contest exams. If Jessica and Sam work together, they can finish scoring in 2 hours. If Jessica and Michael work together, they can finish in 90 minutes. Sam and Michael are the slowest: if they work together, it takes 3 hours to finish. How long would it take if all three of them work together?

- (a) 30 minutes
- (b) 48 minutes
- (c) 1 hour & 20 minutes
- (d) 1 hour & 33 minutes
- (e) 2 hours & 10 minutes

36. A function f is said to be *self-inverse* if $f^{-1}(x) = f(x)$. How many linear functions are self-inverse?

- (a) There are no self-inverse linear functions.
- (b) There is exactly one self-inverse linear function.
- (c) There are exactly two self-inverse linear functions.
- (d) There are exactly four self-inverse linear functions.
- (e) There are infinitely many self-inverse linear functions.

37. A circle of radius 3 centered in the first quadrant passes through both $(1, 0)$ and $(0, 1)$. What are the coordinates of the center of the circle?

- (a) $(2, \sqrt{8})$
- (b) $(3, 3)$
- (c) $\left(\frac{1+\sqrt{5}}{2}, \frac{1+\sqrt{5}}{2}\right)$
- (d) $\left(\frac{1+\sqrt{17}}{2}, \frac{1+\sqrt{17}}{2}\right)$
- (e) $(4, 0)$

38. The virus *mathematicus extremus* infects about 1 in every 250 people. The only test for *mathematicus extremus* is very reliable, returning accurate results (positive or negative) 99.5% of the time. Suppose that Cameron tests positive for *mathematicus extremus*. Which of the following is closest to the probability that Cameron is actually infected?

- (a) 0.4% (b) 44% (c) 50% (d) 99.5% (e) 99.6%

39. Suppose that $f(x) = 3x + 2$. If $g(x)$ is another linear function satisfying the properties $f \circ g = g \circ f$ and $g(1) = 3$, what is the slope of the line $y = g(x)$?

- (a) 3 (b) -3 (c) $-\frac{1}{3}$ (d) 2 (e) $-\frac{1}{2}$

40. Suppose that Rand leaves home travelling at a constant rate of 20 kilometers per hour (kph), and that Mat leaves three hours later, travelling at a constant rate 15 kilometers per hour for the first hour, then at a constant rate of 18 kph the second hour, 21 kph the third hour, and so on with his speeds forming an arithmetic progression. To the nearest minute, how long has Mat been traveling when he catches Rand?

- (a) 8 hours, 21 minutes
- (b) 8 hours, 51 minutes
- (c) 9 hours, 25 minutes
- (d) 11 hours, 51 minutes
- (e) Mat cannot catch Rand traveling at these speeds.