TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

FIFTY-NINTH ANNUAL MATHEMATICS CONTEST

2015

Algebra II

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Scoring formula: 4 x (Number Right) – (Number Wrong) + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft lead (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you; if you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly eighty minutes to work.

1. Find the solution (x, y) to the following system of equations. Then find xy.

$$4x - 3y = 6$$
$$-5x + 7y = -1$$

A. -5 B. 5 C. $\frac{3}{2}$ D. -8 E. 6

2. Approximate the distance between the origin and the line $5x + 3y = 4$.

A. 0.53 B. 0.69 C. 1.07 D. 0.89 E. 0.78

3. A rhombus with sides of length 417 inches has a diagonal of 385 inches. Find the approximate length of the other diagonal.

A. 417" B. 590" C. 740" D. 802" E. 834"

4. A committee of 3 cats and 2 dogs is to be chosen from a group of 12 cats and 8 dogs. Determine the number of different ways of selecting the committee.

A. 73,920 B. 6160 C. 1320 D. 248 E. 770

5. Find the equation of the perpendicular bisector of the line segment between the points (4,2) and $(-2, -6)$. Express the equation in the form $px + qy = 1$. What is the value of $p + q$?

A. 7 B. -7 C. $-\frac{7}{5}$ D. $\frac{7}{4}$ E. $\frac{1}{5}$

6. Find the sum of the roots of the equation $x^3 - 8x^2 + 46x - 68 = 0$

A. 2 B. $5 + 5i$ C. 15 D. 6 E. 8

7. $\log_2 x + \log_8 x + \log_{16} x =$

A. $\frac{\log x^{19}}{\log 24}$ B. $\frac{\log x^{19}}{\log 4096}$ C. $\log_{26} x$ D. $\log_{256} x$ E. $\frac{\log x^7}{\log 2048}$

8. For this particular question, one fourth of you will omit the answer. Of those of you answering this question, 10 will select A, 12 will select B, 5 will select C, 8 will select D, and 1 will select E. How many of you will take this test?

A. 48 B. 38 C. 40 D. 36 E. 46

9. The line $2x + y = -1$ and the circle $(x - 3)^2 + (y + 4)^2 = 5$ intersect at points $P(a, b)$ and $Q(c, d)$. Find $a + b + c + d$.

A. $\frac{-21}{5}$ B. $\frac{-28}{5}$ C. $\frac{-13}{5}$ D. $\frac{31}{5}$ E. $\frac{-18}{5}$

10. Simplify the expression $(\tan\alpha + \cot\alpha)(\cos\alpha + \sin\alpha)$

A. $\sin\alpha + \cos\alpha$

B. $\sin\alpha \cos\alpha$

C. 1

D. $\csc\alpha + \sec\alpha$

E. $\csc\alpha \sec\alpha$

11. Let $f(x) = \frac{3x+2}{2x-5}$. If $(f \circ g)(x) = x$, then $g(x) =$

A. $\frac{2x-5}{3x+2}$ B. $\frac{2+5x}{2x-3}$ C. $\frac{3x-2}{2x+5}$ D. $\frac{2+3x}{-5+2x}$ E. $\frac{2x+5}{3x-2}$

12. Find an equation for the ellipse with an eccentricity of $\frac{2}{3}$ and endpoints of minor axis $(\pm 5,0)$.

A. $\frac{x^2}{9} + \frac{y^2}{25} = 1$

B. $\frac{x^2}{25} + \frac{y^2}{29} = 1$

C. $\frac{x^2}{25} + \frac{y^2}{9} = 1$

D. $\frac{x^2}{25} + \frac{y^2}{45} = 1$

E. $\frac{x^2}{9} + \frac{y^2}{5} = 1$

13. Find the sum of the solutions of the equation

$$\frac{9^{x^2+x}}{81} = 3^{24} \cdot 27^{x^2-3x}$$

A. 3 B. 11 C. 4 D. -3 E. 28

14. Find the number of integral values of x that satisfy the inequality

$$7 < |3x + 5| < 34$$

A. 9 B. 8 C. 16 D. 17 E. 18

15. If $a + bi = \frac{4+i}{1-i}$, then $a + b =$

A. 5 B. 3 C. 8 D. -1 E. 4

16. Find the maximum value of $C = 3x + 4y$ on the region determined by the constraints $x \geq 0, \quad y \geq 0, \quad y - x \geq -2, \quad 2x + y \leq 6, \quad x + 2y \leq 8.$

A. $10\frac{2}{3}$ B. $17\frac{1}{3}$ C. $18\frac{1}{3}$ D. $25\frac{1}{3}$ E. $12\frac{2}{3}$

17. The distance from Bristol to Memphis is approximately 500 miles. Tennesslow Bus Lines advertises that it can make the trip in 8.5 hours. But, as usual, traffic is bad and the driver can only drive 45 mph for the first 100 miles. Approximately how fast must he drive for the remainder of the trip to stay on schedule?

A. 63.7 mph

B. 65.0 mph

C. 64.5 mph

D. 66.7 mph

E. 72.6 mph

18. The difference between two positive numbers is $3\sqrt{2}$. The product of the two numbers is 9. What is the absolute value of the difference of their reciprocals?

A. $3\sqrt{2}$ B. $\frac{\sqrt{2}}{9}$ C. $9\sqrt{2}$ D. $\frac{\sqrt{2}}{6}$ E. $\frac{\sqrt{2}}{3}$

19. If $x^* = x^2 + 2$, find $(x^*)^*$

A. $x^4 + 4x^2 + 6$

B. $x^4 + 4x^2 + 4$

C. $x^2 + 6$

D. $x^2 + 4$

E. $x^2 + 4x + 6$

20. For what values of k does the equation $kx^2 + 3x\sqrt{5} - 4 = 0$ have two imaginary roots?

A. $k \leq -\frac{45}{16}$

B. $k \geq -\frac{45}{16}$

C. $k > \frac{45}{16}$

D. $k < -\frac{45}{16}$

E. $k < \frac{45}{16}$

21. For all real x, $f(x) = x - 3$ and $g(x)$ is a polynomial of degree 2 such that $g(f(x)) = 2x^2 + 3x - 30$. Find $g(3)$.

A. 60

B. -3

C. -30

D. -9

E. 0

22. Find m if $\sqrt{\frac{a}{b}\left(\frac{b}{a}\sqrt{\frac{a}{b}}\right)^{1/3}} = \left(\frac{a}{b}\right)^m$

A. $\frac{1}{12}$ B. $\frac{1}{6}$ C. $-\frac{1}{6}$ D. $\frac{5}{6}$ E. $\frac{5}{12}$

23. If $f(x) = \frac{a}{x+3}$, $g(x) = \frac{b}{x}$, $(f \circ g)(1) = \frac{1}{2}$ and $(f \circ g)(-1) = -2$, find $3a + b$.

A. 17 B. 7 C. 9 D. 1 E. 19

24. Find the sum of the roots of $|x-5|^2 + 4|x-5| = 21$

A. 4 B. 8 C. 12 D. 10 E. 20

25. The product of the values of a, b, and c that require the graph of $y = ax^2 + bx + c$ to pass through the points $(0,-4)$, $(1,5)$, and $(2,18)$ is

A. -56 B. -28 C. 16 D. 28 E. -48

26. If the length and width of a rectangle were each increased by 1, the area would be 96. The area would be 44 if the length and width were each decreased by one. Find the perimeter P of the original rectangle.

A. $P = 26$

B. $P = 35$

C. $P = 52$

D. $P = 50$

E. $P = 70$

27. An aquarium of height 3 feet is to have a volume of 15 ft^3. Let x denote the length of the base and y the width of the base. Express the total number S of square feet of glass needed as a function of x (assume the aquarium has no lid!).

A. $10 + \frac{15}{x} + 3x$

B. $9x^2y^2$

C. $\frac{5}{x} + 6x^2$

D. $\frac{5}{x} + 12x^2$

E. $5 + \frac{30}{x} + 6x$

28. An airplane, flying with a tail wind, travels 1400 miles in 5 hours. The return trip, against the wind, takes 7 hours. Assuming both speeds are constant, find the cruising speed of the plane and the speed of the wind, respectively.

A. 200 mph, 80 mph

B. 220 mph, 40 mph

C. 240 mph, 20 mph

D. 240 mph, 40 mph

E. 250 mph, 30 mph

29. For a second-quadrant angle α, let $\sin\alpha = \frac{4}{5}$. For a first-quadrant angle β, let $\cos\beta = \frac{5}{13}$. Find $\tan(\alpha - \beta)$.

A. $\frac{56}{15}$ B. $-\frac{16}{63}$ C. $\frac{56}{33}$ D. $\frac{16}{63}$ E. $-\frac{56}{33}$

30. Find the product of the amplitude, period, and phase shift of the function

$$f(x) = 4\sin\left(\frac{\pi}{3}x - \frac{\pi}{4}\right)$$

A. $-\frac{\pi^2}{3}$ B. 18 C. $\frac{\pi^2}{12}$ D. $\frac{3}{4}$ E. -18

31. From a point P on level ground at a distance x from a tower, the angle of elevation of the top of a tower is 23°. From a point 30 meters closer to the tower and on the same line with P and the base of the tower, the angle of elevation of the top of the tower is 37°. Determine the approximate height y of the tower in meters. Then find $x + y$.

A. 59.2 m B. 97.9 m C. 67.8 m D. 98.7 m E. 68.7 m

32. Jake and Elwood play a match of Ping-Pong every Friday night. They play until one player has won two sets, and then they retire for the evening. The probability that Jake will win a given set is 0.73, and the probability that Elwood will win a given set is 0.27. What is the probability that Elwood will win only one set?

A. 0.143883

B. 0.0729

C. 0.287766

D. 0.1558

E. 0.4671

33. Find the product of the solution(s) of

$$\log_3(x-2)+\log_3(x-4)=2$$

A. -1

B. 24

C. $\sqrt{10}-3$

D. 1

E. $3+\sqrt{10}$

34. A student (but not you!) has finished 20 classes with a grade point average (GPA) of 2.75. How many additional classes y at 4.0 will raise the student's GPA to some desired value x?

A. $y=\frac{2.75+x}{55+x}$

B. $y=\frac{55+20x}{x}$

C. $y=\frac{80}{20+x}$

D. $y=\frac{55-20x}{x-4}$

E. $y=\frac{55+20x}{4+x}$

35. Find $a+b+c+d$.

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix}=\begin{bmatrix} -20 & 26 \\ -11 & 14 \end{bmatrix}$$

A. 9 B. 13 C. -14 D. 5 E. -1

36. A street sign 18 feet from a lamp post casts a shadow 6 feet long. If the lamp post were 1 foot taller the shadow would only be 5 feet 6 inches long. How high is the lamp on the post?

A. 12 ft. B. $14\frac{2}{3}$ ft. C. $17\frac{1}{2}$ ft. D. $15\frac{2}{3}$ ft. E. $14\frac{1}{6}$ ft.

37. Let $P(a, b)$ and $Q(c, d)$ denote two distinct points on the graph of $f(x) = x^2$. Suppose that the slope of line PQ is 3 and the x coordinates of P and Q differ by two. Find $b + d$.

A. 3 B. 6 C. 6.5 D. 2.75 E. 6.75

38. Find the sum of the coefficients of the first two terms in the expansion of $(2x^4 + 3x^{-2})^6$, when the terms are listed in descending order by exponent.

A. 576 B. 2160 C. 640 D. 793 E. 1440

39. Find an exponential function of the form $f(x) = ba^{-x} + c$ that has horizontal asymptote $y = 32$, y-intercept 212, and passes through the point $(2, 112)$. Then find $a + b + c$.

A. 245.5 B. 145.5 C. 213.5 D. 293.5 E. 356.5

40. Suppose (in theory!) you just give up and throw this test booklet vertically into the air with an initial velocity of 48 ft/sec. The height h of your test booklet in feet above the ground after t seconds is given by

$$h(t) = -16t^2 + 48t + 3$$

For approximately how many seconds will your test booklet be at least ten feet above the ground?

A. 3.00 B. 2.69 C. 2.85 D. 1.50 E. 0.3074