FIFTY-EIGHTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2014

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the <u>best</u> answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase <u>completely</u>. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Algebra II TMTA Test

1. A point is chosen at random from within a circular board of radius 10 inches. What is the probability that the point will be within two inches of the edge?

   A: 4%
   B: 16%
   C: 25%
   D: 36%
   E: 20%

2. How many numbers are in the following list of numbers: $2, 8, 14, 20, ..., 242$?

   A: 60
   B: 121
   C: 41
   D: 40
   E: 24

3. Find all the solutions of this system: $y = 3x - 2,\ 9x - 3y = 6$.

   A: $x = \frac{2}{3}, y = 0$
   B: $x = 0, y = -2$
   C: no solution
   D: an infinite number of solutions
   E: two solutions: x=2/3, y =0 and x=0, y=-2

4. For what value of a is the determinant of the following matrix equal to 0?

   $$\begin{pmatrix} 2 & 4 \\ 3 & a \end{pmatrix}$$

   A: -6
   B: 0
   C: 5
   D: 6
   E: -5

5. What is the value of the sum $1 + 2 + 3 + \cdots + 2014$?

   A: 1,008,518
   B: 2,016,032
   C: 2,016,036
   D: 2,017,036
   E: 2,029,105

6. $(5x^3 + 13x^2 + 18) \div (x + 3) =$

    A: $5x^2 - 2x + 6$
    B: $5x^2 + 28x + 84$
    C: $5x^2 + 2x + 6$
    D: $5x^2 + 2x - 6$
    E: $5x^2 - 2x - 6$

7. If the volume of a cube is doubled, by what factor does the surface area of the cube increase?

    A: $\sqrt{2}$
    B: $\sqrt[3]{2}$
    C: $\sqrt[3]{4}$
    D: 4
    E: $2\sqrt{2}$

8. Expand: $(1 + 2i)(3 - i)$

    A: $1 - 5i$
    B: $1 + 7i$
    C: $5 - 7i$
    D: $5 + 5i$
    E: $3 + 5i$

9. What values of a and b make $x^3 + ax^2 + bx - 8$ a perfect cube?

    A: $a = -6, b = 12$
    B: $a = 2, b = 4$
    C: $a = 3, b = 3$
    D: $a = 6, b = -12$
    E: $a = 2, b = 3$

10. $A = \sqrt{6 + \sqrt{6 + \sqrt{6 + \ldots}}}$ and $B = \sqrt{9 - \sqrt{9 - \sqrt{9 - \ldots}}}$. Find AB.

    A: $3\sqrt{6}$
    B: $3(\sqrt{26} - 2)/2$
    C: $3(\sqrt{37} - 1)/2$
    D: $4\sqrt{5} - 2\sqrt{2}$
    E: 3

11. If $\sqrt{x} + \sqrt{10} = \sqrt{20}$, then x is equal to:

    A: 10
    B: $(\sqrt{2} - 1)10$
    C: $10\sqrt{3}$
    D: $\sqrt{10}$
    E: $30 - 20\sqrt{2}$

12. The graphs of $y = -3x^2$ and $y = x^2 - 4$ intersect at

    A: $(1, -3)$
    B: $(-1, -3)$
    C: $(1, 3)$ and $(-1, 3)$
    D: no point.
    E: $(1, -3), (-1, -3)$

13. The height (in feet) of a thrown ball at t seconds after release is given by $-16t^2 + 48t + 64$. When does it reach its highest position?

    A: $t = 0$
    B: $t = \frac{3}{2}$
    C: $t = \frac{5}{2}$
    D: $t = 4$
    E: $t = 2$

14. Solve for x: $\log_2(x) + \log_2(x + 6) = 4$

    A: $x = -8$
    B: $x = 2$
    C: $x = -3 + \sqrt{13}$
    D: $x = e^4 - 3$
    E: $x = -3 \pm \sqrt{13}$

15. Determine the equation of a circle with a center at $(-1, 2)$ which goes through the point $(3, 2)$.

    A: $(x - 1)^2 + (y + 2)^2 = 4$
    B: $(x - 1)^2 + (y + 2)^2 = 16$
    C: $(x + 1)^2 + (y - 2)^2 = 4$
    D: $(x + 1)^2 + (y - 2)^2 = 16$
    E: $(x + 1)^2 + (y - 2)^2 = 8$

16. What is the center of the following ellipse? $4x^2 - 16x + 2y^2 + 12y - 12 = 0$

    A: $(2, -3)$
    B: $(2, 3)$
    C: $(-2, -3)$
    D: $(-2, 3)$
    E: $(2.5, 3.5)$

17. Let $f(x) = x^2 + 2x - 1$ and $g(x) = 1 - x^2$. Evaluate $(f \circ g)(2)$.

    A: -48
    B: -21
    C: 14
    D: 2
    E: 21

18. Three pirates, Jack, Liz and Will, discovered a chest of gold coins. Jack took 3/7ths of the coins, Liz took 100 coins, and Will took 1/3rd of the coins, plus an extra 20 coins. How many coins did Jack get?

   A: 188
   B: 210
   C: 216
   D: 504
   E: 630

19. In a standard deck of 52 cards, what is the probability that if you drew one card it is a diamond, a heart, or an ace?

   A: $\dfrac{1}{26}$

   B: $\dfrac{1}{2}$

   C: $\dfrac{15}{26}$

   D: $\dfrac{7}{13}$

   E: $\frac{29}{52}$

20. Simplify $\dfrac{7i}{2+4i}$.

   A: $-\frac{7}{4} + \frac{7}{2}i$
   B: $28 + 14i$
   C: $\frac{7}{5} + \frac{7}{10}i$
   D: $-2 - i$
   E: $\frac{7}{4} + \frac{7}{2}i$

21. Determine the coefficient of the term x^4y^5 in the expansion of $(2x - y)^9$.

   A: -2016
   B: -126
   C: -8
   D: 8
   E: 16

22. If the first term in an arithmetic sequence is 8 and the 10th term is -19, what is the 6th term?

    A: -10
    B: $-\frac{41}{5}$
    C: $-\frac{11}{2}$
    D: -7
    E: -11

23. How many liters of water must we evaporate from 40 liters of a 30% salt solution to obtain a 50% salt solution?

    A: 16
    B: 25
    C: 15
    D: 22
    E: 2420

24. A point located on a chord of a circle is 8 cm from one endpoint of the chord and 7 cm from the center of the circle. If the radius of this circle is 13 cm long, how long is the chord in cm?

    A: 21
    B: 24
    C: 23
    D: 22
    E: 20

25. It takes Chris 1.5 times as long to perform a task as Allison. It takes Allison $\frac{5}{3}$ times as long to perform that task as Barry. If Chris, Allison and Barry work together, they can perform the same task in 3 hours. How long would it take Chris to perform that task if working alone?

    A: 550 minutes
    B: 6 hours
    C: 25 hours
    D: 10 hours
    E: 15 hours

26. Bill sets out on a journey. For the first one third of the distance, he drives at 30 mph. He drives the second third at 40 mph; he drives the last third of the distance at 50 mph. What is his average speed (rounded to one decimal place)?

    A: 35 mph
    B: 38.3 mph
    C: 40 mph
    D: 42.6 mph
    E: 43 mph

27. The third term of an arithmetic sequence is a, and the seventh term of that sequence is b. Find the first term of that sequence.

A: $\frac{b-a}{8}$

B: $\frac{3a-b}{2}$

C: $b - 2a$

D: $\frac{b-2a}{3}$

E: $\frac{5a-b}{4}$

28. Find integer b so that one of the solutions of the equation

$$x^3 + (b^2 - 4b - 3)x^2 - bx + 3b = 0$$

will be $x = -2$.

A: -4

B: -2

C: 2

D: 3

E: 4

29. Let $\log_b(x^3 - 4) + \log_b(2x^2 + 1) = 1$. Find b so that $x = 2$ is a solution.

A: 4

B: 5

C: 9

D: 13

E: 36

30. Find the area of the region which is bounded by these inequalities.
$3x + y \le 15, \quad y \ge 2x, \quad x \ge 0, \quad y \ge 0$

A: 45

B: 15

C: 30

D: 45/2

E: 75/2

31. A ball is thrust up vertically from the ground into the air and hits the ground 2.5 seconds later. What is the maximum height of the ball in feet? Assume that air resistance is negligible. The acceleration due to gravity is -32 ft/sec^2.

A: 50

B: 30

C: 100

D: 75

E: 25

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32. If the geometric series $a + ar + ar^2 + ...$ has a sum of 7, and the terms involving odd powers of r have a sum of 3. What is $a + r$?

    A: 4/3
    B: 12/7
    C: 3/2
    D: 5/2
    E: 1

33. How many different points in 3-dimensional space have 3 positive integral coordinates whose sum is 100?

    A: 98^3
    B: 9702
    C: 4851
    D: 9506
    E: 1000

34. Find all k so that $(1, 2k)$, $(3k, 4)$, and $(5, 6k)$ cannot be 3 points on the same circle.

    A: 2, -2/3
    B: 2/3, 3/5
    C: 1, -4/3
    D: 2/3, 6/5
    E: none of the above

35. What is the base of the system in which 121 represents the same number as the decimal number 324?

    A: 15
    B: 9
    C: 12
    D: 17
    E: 7

36. Given a triangle with two sides measuring 8 and 12 cm and the angle between them measuring 60°, what is the length of the third side?

    A: $6\sqrt{5}$
    B: $9\sqrt{3}$
    C: $2\sqrt{3}$
    D: $4\sqrt{7}$
    E: 10

37. For what value of n does $(2^{2007} - 2^{2006})(2^{1997} - 2^{1996}) = 2^n$?

   A: 4001
   B: 4002
   C: 4000
   D: 4003
   E: 4004

38. Let a and b be distinct real numbers for which: $\dfrac{a}{b} + \dfrac{a + 10b}{b + 10a} = 2$. Find $\dfrac{a}{b}$.

   A: $\frac{5}{6}$

   B: $\frac{5}{4}$

   C: $\frac{4}{5}$

   D: $\frac{2}{3}$

   E: $\frac{3}{4}$

39. For a function f such that $f(1) = 4, f(2) = 1, f(3) = 3, f(4) = 5, f(5) = 2$.
   If $u_0 = 4$ and $u_{n+1} = f(u_n), n = 0, 1, 2, \ldots$, then find u_{2002}.

   A: 4
   B: 2
   C: 3
   D: 5
   E: 1

40. A sphere is inscribed in a cube. The cube has a surface area of 36 square meters. A second cube is then inscribed in the sphere. What is the surface area of the inner cube in square meters?

   A: 12
   B: $9\sqrt{3}$
   C: 9
   D: 18
   E: 24

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