Algebra II

1. Select from the following the one polynomial which does not have $2x + 3$ as a factor.
   a) 2x+3
   b) $4x^2 - 9$
   c) $2x^2 - x - 6$
   d) $6x^2 - 7x - 9$
   e) $2x^3 + 3x^2 - 2x - 3$

2. Assume that the variables x and y are positive. Simplify the expression $\left(27x^{-\frac{3}{2}}y^{\frac{9}{2}}\right)^{\frac{2}{3}}$.
   a) $\frac{9y^3}{x}$
   b) $\frac{3y^3}{x}$
   c) $\frac{27y^3}{x}$
   d) $\frac{y^3}{27x}$
   e) $\frac{y^3}{9x}$

3. Write the equation of the line that is perpendicular to $y = 3x - 1$ which passes through the point $(-2, -1)$.
   a) $y = -\frac{1}{3}x - \frac{5}{3}$
   b) $y = \frac{1}{3}x - \frac{1}{3}$
   c) $y = -\frac{1}{3}x + \frac{1}{3}$
   d) $y = \frac{1}{3}x + \frac{1}{3}$
   e) $y = -\frac{1}{3}x - \frac{7}{3}$

4. One leg of a right triangle is 2 units longer than the other leg. The hypotenuse of the triangle is 10 units in length. Find the length of the longer of the two legs.
   a) 2
   b) 6
   c) 8
   d) 10
   e) 14

5. Which of the following is a counterexample to the following statement: For every rational number z, $z^2 > z$.
   a) $(-2)^2 > (-2)$
   b) $\left(-\frac{1}{2}\right)^2 > \left(-\frac{1}{2}\right)$
   c) $\left(\sqrt{2}\right)^2 > \left(\sqrt{2}\right)$
   d) $\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)$
   e) $(2)^2 > (2)$

6. Let (a, b) be a point on the graph of $f(x) = \log_c x$. Select from the following points the one that must be a point on the graph of the inverse function $f^{-1}(x) = c^x$.
   a) (a, b)
   b) $(a,\ c^{-b})$
   c) (b, c^a)
   d) (b, a)
   e) $(\ c^{-b},\ a)$

7. Determine the general term a_n for the infinite arithmetic sequence $-4, 1, 6, 11, 16, \ldots$
   a) $a_n = -4n + 6$
   b) $a_n = -4n + 14$
   c) $a_n = 2n + 5$
   d) $a_n = 5n - 4$
   e) $a_n = 5n - 9$

8. Which of the following computations yield a purely real result?
   a) $(5 + 4i) + (5 + 4i)$
   b) $(5 + 4i) + (5 - 4i)$
   c) $(5 + 4i) - (5 - 4i)$
   d) $(5 - 4i) - (5 + 4i)$
   e) $(5 - 4i) + (5 - 4i)$

9. Select from the following the most accurate statement regarding the sets of numbers to which the value of $-3^2 \div 2 \cdot 4$ belongs.
   a) complex numbers $\mathbb{C}$
   b) complex numbers $\mathbb{C}$ and the real numbers $\mathbb{R}$
   c) complex numbers $\mathbb{C}$, the real numbers $\mathbb{R}$, and the rational numbers $\mathbb{Q}$
   d) complex numbers $\mathbb{C}$, the real numbers $\mathbb{R}$, the rational numbers $\mathbb{Q}$, and the integers $\mathbb{Z}$
   e) complex numbers $\mathbb{C}$, the real numbers $\mathbb{R}$, the rational numbers $\mathbb{Q}$, the integers $\mathbb{Z}$, and the natural numbers $\mathbb{N}$

10. Given the function $f(x) = \frac{1}{2}\sqrt{x-3} - 4$, determine the inverse function $f^{-1}(x)$.

a) $f^{-1}(x) = 4(x-4)^2 - 3$
b) $f^{-1}(x) = 2(x+3)^2 + 4$
c) $f^{-1}(x) = 4(x+4)^2 + 3$
d) $f^{-1}(x) = 2(x-3)^2 + 4$
e) $f^{-1}(x) = 4(x+3)^2 - 4$

11. The height of a toy rocket as it rises from and then returns to the ground is modeled by the function $h(t) = -5t^2 + 20t$ where time t is in seconds and height $h(t)$ is in meters. Determine the domain and range of the function **as a model for the rocket flight**.

a) domain is [0, 3] and range is $[0,\ 15]$
b) domain is [0, 20] and range is [0, 4]
c) domain is [0, 4] and range is [0, 20]
d) domain is $[0, \infty)$ and range is $(-\infty, 20]$
e) domain is $(-\infty, \infty)$ and range is $(-\infty, 60]$

12. For $f(x) = 3x + 2$ and $g(x) = \frac{1}{\sqrt{x-2}}$, determine the composition function $f[g(x)]$.

a) $f[g(x)] = \frac{3\sqrt{x}}{x}$
b) $f[g(x)] = \frac{\sqrt{3x}}{3x}$
c) $f[g(x)] = \frac{3}{\sqrt{x}}$
d) $f[g(x)] = \frac{3x+2}{\sqrt{x-2}}$
e) $f[g(x)] = \frac{3}{\sqrt{x-2}} + 2$

13. Simplify and rationalize $\frac{\sqrt[3]{16x^6y^5}}{\sqrt[3]{2y^2z}}$

a) $\frac{x^2y\ \sqrt[3]{8y^2z^2}}{z}$
b) $\frac{x^2y^2\ \sqrt[3]{4yz}}{z}$
c) $\frac{x^2y\ \sqrt[3]{4y^2}}{\sqrt[3]{y^2z}}$
d) $\frac{2\,x^2y\ \sqrt[3]{2}}{\sqrt[3]{z}}$
e) $\frac{2\,x^2y\ \sqrt[3]{z^2}}{z}$

14. Given the matrix $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$, find the inverse matrix $A^{-1} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Determine the sum of the entries of A^{-1}.

a) $a + b + c + d = \frac{1}{2}$
b) $a + b + c + d = \frac{3}{2}$
c) $a + b + c + d = 0$
d) $a + b + c + d = 1$
e) $a + b + c + d = 3$

15. Find the equation of the circle with center $(2, -3)$ and radius 5.

a) $(x-2)^2 + (y+3)^2 = 5$
b) $(x+2)^2 + (y-3)^2 = 5$
c) $(x-2)^2 + (y+3)^2 = 5^2$
d) $(x+2)^2 + (y-3)^2 = 5^2$
e) $(x-2)^2 + (y-3)^2 = 5^2$

16. Determine all intercepts of the graph of the equation $\frac{x^2}{9} - \frac{y^2}{25} = 1$.

a) no intercepts
b) x-intercept $(3, 0)$
c) y-intercepts $(0, -5)$ and $(0, 5)$
d) x-intercepts $(-3, 0)$ and $(3, 0)$
e) y-intercept $(0, 5)$, and x-intercepts $(-3, 0)$ and $(3, 0)$

17. Solve the equation $3^{6x-1} = \frac{1}{81}$. Use the solution to evaluate the expression 2^{-4x+3}.

a) 4
b) 8
c) 16
d) 32
e) 64

18. The product from k = 1 to k = 5 of $9\left(\frac{1}{3}\right)^{k-1}$ is

a) $\frac{1}{9}$
b) $\frac{1}{3}$
c) 1
d) 3
e) 9

19. Given $(x+2)(x-4)(x-1)(x-1) > 0$, write the solution set using interval notation.
    a) $(-\infty, -2) \cup (4, \infty)$
    b) $(-\infty, -2) \cup (1, 4)$
    c) $(-\infty, -2] \cup [4, \infty)$
    d) $(-2, 1) \cup (1, 4)$
    e) $[-2, 1] \cup [4, \infty)$

20. Solve the system of equations $\begin{aligned} x + y + z &= 3 \\ 3x + 3y + 2z &= 7 \\ -x - y + z &= 1 \end{aligned}$. The system has
    a) no solution.
    b) the unique solution $(0, 1, 2)$
    c) the unique solution $(-1, 4, 0)$
    d) the unique solution $(2, 1, 1)$
    e) infinitely many solutions of the form $(t, 1-t, 2)$ where t is any real number.

21. A small e-commerce business sells only t-shirts and boxed greeting cards. The business can manage the sale of no more than 100 items per week, but the business has a contract that guarantees that they will sell at least 20 t-shirts each week. If the profit is \$3 from each t-shirt and is \$2 from each box of greeting cards, what is the maximum profit the business can earn in one week?
    a) \$220
    b) \$240
    c) \$260
    d) \$280
    e) \$300

22. Find the one real number k for which the graph of $y = x^2 + 2x + k$ and the graph of $y = 3x + 1$ intersect in exactly in one point. The value of k is found between which of these pairs of consecutive integers?
    a) -2 and -1
    b) -1 and 0
    c) 0 and 1
    d) 1 and 2
    e) 2 and 3

23. A geometric sequence has first term $a_1 = 64$ and common ratio $r = (-\frac{1}{2})$. Find the sum of the 8th and the 10th terms of the sequence.
    a) $a_8 + a_{10} = \frac{1}{16}$
    b) $a_8 + a_{10} = -\frac{1}{4}$
    c) $a_8 + a_{10} = \frac{3}{8}$
    d) $a_8 + a_{10} = -\frac{5}{8}$
    e) $a_8 + a_{10} = \frac{5}{4}$

24. $|3 + 4i| =$
    a) 5
    b) $\sqrt{7}$
    c) 7
    d) $\sqrt{12}$
    e) 12

25. The coefficient of the 5th term in the binomial expansion of $(x + y)^{100}$is the same as the coefficient of which other term in the expansion?
    a) the 94th term
    b) the 95th term
    c) the 96th term
    d) the 97th term
    e) the 98th term

26. A handyman needs to know the surface area of a roof on a shed whose floor is a 12' by 12' square. The walls of the building are 8' tall. At its central and highest point, the distance between the peak of the roof and the ground is 16'. What is the surface area of the roof? Assume it is a two-sided pitched roof.
    a) 120 square feet.
    b) 144 square feet.
    c) 188 square feet.
    d) 240 square feet.
    e) 288 square feet.

27. The ordered pairs in the accompanying table show the values of variables that are assumed to have a linear relationship. Select the best estimate for the linear regression equation or line of best fit for the data and provide a reasonable estimate for the corresponding correlation coefficient.
    a) $y = 2x + 1$ and $r = -1$
    b) $y = 2x + 1$ and $r = 1$
    c) $y = 2x - 1$ and $r = -1$
    d) $y = 2x - 1$ and $r = 1$
    e) $y = 2x$ and $r = 0$

x	y
0	0.8
1	3.1
2	5.2
3	6.1
5	10.8
6	13.7
7	14.0

28. A sample of data obtained from an approximately normal distribution has values that range from 50 to 170. The best estimate of the standard deviation for the population given only this information would be
    a) 20
    b) 25
    c) 30
    d) 35
    e) 40

29. A coffee machine was designed to dispense 6 ounces of coffee each time a button is pushed, but it has been determined that the amount of coffee dispensed by the machine is actually normally distributed with a variance of 0.25. What percentage of the button pushes on this machine should yield between 5 and 7 ounces of coffee?
    a) 68
    b) 75
    c) 95
    d) 99
    e) 100

30. The region bounded by the graphs of $x = 0$, $y = 0$, and $y = f(x)$ is to be rotated about the y-axis to form a solid. Select from the following the one function that would generate the solid with the greatest volume.
    a) $f(x) = -\frac{1}{3}x^2 + 2x$
    b) $f(x) = -x + 6$
    c) $f(x) = |x - 3|$
    d) $f(x) = x - 5$
    e) $f(x) = \sqrt{x + 4}$

31. $\cos(\sin^{-1}(-\frac{\sqrt{3}}{2}))$=
    a) $-\frac{\sqrt{3}}{2}$
    b) $-\frac{1}{2}$
    c) $\frac{1}{2}$
    d) $\frac{\sqrt{3}}{2}$
    e) undefined.

32. Determine the length of a basic cycle of the function $f(x) = -4\sin(2x + 4) + \sqrt{2}$
    a) $\pi - 2$
    b) $\pi + 2$
    c) $\frac{\pi}{2}$
    d) π
    e) 2π

33. For the acute angle θ for which $\sin(\theta) = x$, find $\tan(\theta)$ in terms of x.
    a) $\tan(\theta) = \frac{x}{\sqrt{1+x^2}}$
    b) $\tan(\theta) = \frac{x}{\sqrt{1-x^2}}$
    c) $\tan(\theta) = \frac{x}{1+x}$
    d) $\tan(\theta) = \frac{x}{1-x}$
    e) $\tan(\theta) = \frac{\sqrt{1-x^2}}{x}$

34. Ten red playing cards and ten black playing cards were shuffled and placed face down in a stack on a table. What is the minimum number of cards that would need to be drawn from the stack in order to guarantee a draw of two pairs if the only requirement for a pair is that they are the same color?
    a) 4
    b) 5
    c) 6
    d) 7
    e) 8

35. Solve the inequality $3^x < 2^x$
    a) $(-\infty, 0)$
    b) $(0, \infty)$
    c) $(-\infty, -1)$
    d) $(-\infty, \infty)$
    e) the solution set is empty.

36. Find the slope of the line passing through the points $(-\frac{1}{2}, x)$ and $(\frac{1}{x}, 3)$.
    a) $m = \frac{2x(3-x)}{2+x}$
    b) $m = \frac{6-x^2}{x}$
    c) $m = \frac{x(6-x)}{1+x}$
    d) $m = \frac{6-x}{2+x}$
    e) $m = 3 - x$

37. Assume that $a, b, c, d,$ and e are natural numbers. Determine the maximum number of positive real zeros which the polynomial function $f(x) = ax^5 - bx^4 - cx^3 + dx - e$ may have.
    a) 2
    b) 3
    c) 4
    d) 5
    e) 6

38. Select from the following numbers, the one which is not a possible rational zero of the polynomial function $f(x) = 4x^4 - 5x^3 + 7x^2 + 3x - 6$.
    a) $\frac{1}{3}$
    b) $\frac{1}{2}$
    c) 1
    d) 2
    e) 3

39. Solve $|2x + 3| - 3 = 5x + 2$. Determine the sum of the solutions.

a) $-\frac{38}{21}$

b) $-\frac{20}{21}$

c) $-\frac{4}{7}$

d) $-\frac{2}{3}$

e) $-\frac{2}{7}$

40. For θ in the interval $[0, \pi]$ and $\theta \neq \frac{\pi}{2}$, $\frac{\sin(\theta)-\cos(\theta)}{\cos(\theta)} - \tan(\theta)$ is equivalent to which of the following.

a) -1

b) $-\frac{\pi}{6}$

c) 0

d) $\frac{\pi}{6}$

e) 1