FIFTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2007

Prepared by:

Department of Mathematics
Jackson State Community College
Jackson, Tennessee

Reviewed by:

Mathematics Faculty
Austin Peay State University
Clarksville, Tennessee

Coordinated by: Tim Britt

Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee

TMTA
ALGEBRA II Test
2007

1. Coordinates of the vertex for the parabola $y = -3x^2 + 12x - 3$ are:

   A. (3, 12) B. (2, -9) C. (-3, 12) D. (2, 9) E. (-2, -39)

2. Find the solution to: $\begin{cases} 3x - 2y = 4 \\ 5x + 3y = 13 \\ x + y = 4 \end{cases}$.

   A. $(2,1)$ B. $(1,2)$ C. $\left(12/5, 8/5\right)$ D. $\left(1/2, 7/2\right)$ E. $\varnothing$

3. If the expression $(x^2+4)^3(x^4-2)(5x-3)$ is expanded, the highest power of x would be:

   A. 10 B. 11 C. 13 D. 7 E. 8

4. The solution of $\frac{3}{x-2} + \frac{3}{x+2} = \frac{5x-4}{x^2-2x}$ is:

   A. $\{4,2\}$ B. $\{4\}$ C. $\{0,1\}$ D. $\{-4\}$ E. $\{-4,0\}$

5. If $(x+3)$ is a factor of $x^3 - 2kx + k^2$, then k is:

   A. 3 B. -3 or 9 C. 3 or -9 D. -3 E. -9

6. A polynomial with real coefficients and zeroes 3, -1, and 1+2i is:

   A. $x^4 - 2x^2 + 16x - 15$ B. $x^4 - 4x^3 + 6x^2 - 4x - 15$ C. $x^4 - 6x^2 - 8x - 3$
   D. $x^4 - 2x^3 + x^2 - 8x - 12$ E. $x^3 - 7x - 6$

7. A car leaves a town traveling at 40 mph. Two hours later, a second car leaves the same town, on the same road, traveling at 60 mph. The second car drives how many hours to overtake the first car?

   A. 2 hours B. 1 hour C. 1.5 hours D. 4 hours E. 3 hours

8. If $f(y)=2y^2+3$ and $g(y)=\sqrt{y+2}$, find $f\circ g$.

A. $2\sqrt{y+2}+3$ B. $2y+7, y\geq -2$ C. $2y+5$

D. $\sqrt{2y^2+5}$ E. $(2y^3+3)(\sqrt{y+2})$

9. Suppose the function $f(x)=2x^3+4x^2-3x-5$ is divided by $x+2$. The remainder is:

A. 31 B. -11 C. $f(2)$ D. -31 E. $f(-2)$

10. The equation of the circle with endpoints of the diameter at $(-3,-2)$ and $(5,4)$ is:

A. $x^2+y^2-2x-2y+27=0$ B. $x^2+y^2-2x-2y-23=0$

C. $x^2+y^2-2x-2y-27=0$ D. $x^2+y^2+2x+2y+23=0$

E. $x^2+y^2+2x+2y-23=0$

11. What is the coefficient of the term that contains x^4y^5 in the expansion of $(2x-y)^9$?

A. -16 B. 126 C. -126(16) D. -126 E. 16

12. If $3^{x+2}=2^{2x-1}$ then x is:

A. $\frac{\log 18}{\log \frac{4}{3}}$ B. $\frac{\log 12}{\log \frac{4}{3}}$ C. $\frac{\log 27}{\log 2}$ D. $\frac{\log 3}{\log 2}$ E. $\frac{\log \frac{4}{3}}{\log 18}$

13. If Jack can do a job in 104 hours and it takes Jack and Jill working together 40 hours to do the same job, how many hours will Jill take to do the job alone?

A. 64 B. 80 C. 28 D. 72 E. 65

14. The height h in feet above the ground at time t in seconds of an object launched vertically is given by: $h(t)=-16t^2+16t+32$. In how many seconds will it hit the ground?

A. 3 B. 1 C. 4 D. 2 E. 1/2

15. Given $\frac{a^2}{r+t} = m$ and if $m \neq 0$ then r = :

A. $\frac{m}{a^2} - t$ B. $\frac{a^2 - mt}{m}$ C. $\frac{a^2 - t}{m}$ D. $\frac{a^2 + mt}{m}$ E. $a^2 m - t$

16. Simplify by rationalizing the denominator. Assume all radicands represent positive real numbers.

$$\frac{\sqrt{x} + \sqrt{x+1}}{\sqrt{x} - \sqrt{x+1}}$$

A. $-2x - 2\sqrt{x(x+1)} - 1$ B. $2x - 2\sqrt{x(x+1)} - 1$ C. $-2x + 2\sqrt{x(x+1)} - 1$

D. $2x + 2\sqrt{x(x+1)} + 1$ E. $-2x - 2\sqrt{x(x+1)} + 1$

17. Simplify using positive exponents. Assume that y>0.

$$\frac{8y^{\frac{2}{3}}y^{-1}}{2^{-1}y^{\frac{3}{4}}y^{-\frac{1}{6}}}$$

A. $\frac{61}{y^{\frac{11}{12}}}$ B. $\frac{16}{y^{\frac{7}{12}}}$ C. $\frac{16}{y^{\frac{11}{12}}}$ D. $\frac{12}{y^{\frac{7}{15}}}$ E. $\frac{4}{y^{\frac{11}{12}}}$

18. The inverse function of $f(x) = 3 - \sqrt{3-x}$ is:

A. $\sqrt{3-x} - 3$ B. $x, x \geq 3$ C. $-x^2 + 6x - 6, x \geq 3$

D. $x^2 - 6x + 9, x \leq 3$ E. $-x^2 + 6x - 6, x \leq 3$

19. The graph of the following quadratic equations intersect each other in how many points?

$x^2 + y^2 = 4$

$2x^2 - y^2 = 8$

A. 0 B. 1 C. 2 D. 3 E. 4

20. The simplified form of $2i^{34} - 3i^{24}$ is:

A. 5 B. -5 C. -5i D. 5i E. -1

21. Which of the following pairs of lines are perpendicular?

(1) $2x+3y=5$

(2) $2y-4x=6$

(3) $2y+12=x$

(4) $-3x+2y=4$

A. 1 *and* 2 B. 2 *and* 3 C. 1 *and* 3 D. 3 *and* 4 E. 1 *and* 4

22. The roots of a quadratic equation whose discriminant is 72 may best be described as:

A. unequal and rational
B. unequal and irrational
C. imaginary
D. equal
E. additive inverses of each other

23. Simplify $\dfrac{1-\dfrac{1}{m+1}}{1+\dfrac{1}{m-1}}$ $(m \neq 0,1,-1)$

A. -1 B. 1 C. $\dfrac{m-1}{m+1}$ D. $\dfrac{1-m}{1+m}$ E. 2

24. All the values of x that satisfy the inequality $-8 \leq 1-3(x-2) < 13$ are:

A. $-2 < x \leq 5$ B. $-6 < x \leq 1$ C. $5 \leq x < -2$ D. $1 \leq x < -6$
E. $-4\frac{1}{2} < x \leq 6$

25. Multiply: $\dfrac{x^3-27}{5x-15} \cdot \dfrac{2x^2-18}{3x^2+9x+27}$.

A. $\dfrac{2x+6}{15}$ B. $\dfrac{x^2+6x+9}{x-3}$ C. $\dfrac{2x^2-18}{15}$ D. $\dfrac{2}{15}$ E. $\dfrac{2x-6}{5x+15}$

26. The complex number $\dfrac{2+3i}{4-2i}$ written in $a+bi$ form is:

A. $\dfrac{7}{6}+\dfrac{2}{3}i$ B. $\dfrac{7}{6}-\dfrac{2}{3}i$ C. $\dfrac{7}{6}+\dfrac{4}{3}i$ D. $\dfrac{1}{10}-\dfrac{4}{5}i$ E. $\dfrac{1}{10}+\dfrac{4}{5}i$

27. Consider the graph of $\frac{x^2-2x-3}{2x^2-8}$. Which of the following is NOT true?

A. $x=2$ is a vertical asymptote
B. $x=-2$ is a vertical asymptote
C. $y=1$ is a horizontal asymptote
D. $x=3$ is zero of the function
E. The graph intercepts the y axis only once.

28. In an arithmetic sequence, if the eighth term is 10 and the fifteenth term is 23, find the third term.

A. -3 B. 13/7 C. 5/7 D. 75/14 E. -34/7

29. The equation for the line through $\left(\frac{1}{2},-\frac{1}{3}\right)$ with slope -2 is:

A. $3y+6x-2=0$ B. $y+2x+1=0$ C. $3y-6x+2=0$
D. $3y+6x+2=0$ E. $3y+6x-4=0$

30. If $f(x)=x-x^2$ then $\frac{f(x+h)-f(x)}{h}$ for $h\neq 0$ is:

A. $1+2x+h$ B. $\frac{2x^2+2x(1+h)-h^2}{h}$ C. $1-2x+h$
D. $1+2x-h$ E. $1-2x-h$

31. Given $\log_a 2=.301, \log_a 3=.477$ and $\log_a 5=.699$ then $\log_a\left(\frac{25}{8}\right)=$:

A. -.495 B. .386 C. .541 D. -.541 E. .495

32. Alice has \$2.60 in nickels and dimes. The number of dimes is two less than three times the number of nickels. How many of each coin does Alice have?

A. 8 nickels, 22 dimes B. 7 nickels, 19 dimes C. 12 nickels, 20 dimes
D. 2 nickels, 25 dimes E. 32 nickels, 10 dimes

33. When the polynomial $x^4-3x^3-7x^2+7x+2$ is divided by $x+2$ the quotient is x^3+Bx^2+Cx+1. The value of $B+C$ is:

A. -8 B. -2 C. 0 D. 2 E. 10

34. Solve the equation $2x^2 + x - 1 = 0$.

A. $\{1/2, -1\}$ B. $\{-1/2, 1\}$ C. $\frac{-1 \pm i\sqrt{7}}{4}$ D. $\frac{1 \pm i\sqrt{7}}{4}$ E. $\frac{-1 \pm i\sqrt{7}}{2}$

35. The solution of the inequality $\frac{2}{x-3} < \frac{3}{x+4}$ is:

A. $(-\infty, -4) \cup (3, \infty)$ B. $(-4, \infty)$ C. $(-4, 3) \cup (17, \infty)$ D. $(-\infty, \infty)$ E. $\varnothing$

36. The domain of $f(x) = \sqrt{3x - 2x^2}$ is:

A. all real numbers B. $\left[0, 3/2\right]$ C. $[0,1]$ D. $(0, \infty)$ E. $(-\infty, 0] \cup [2, \infty)$

37. A pizza restaurant offers 8 different toppings and 4 different cheeses. A deluxe pizza contains your choice of 5 different toppings and your choice of 2 different cheeses. How many different deluxe pizzas are possible?

A. 10 B. 48 C. 336 D. 80,640 E. $8^5 \cdot 4^2$

38. If $x = \log_4 \frac{1}{16}$, then x equals:

A. -1/4 B. -4 C. ½ D. 2 E. -2

39. The solution to the equation $\sqrt{3x + 4} = 1 + \sqrt{x + 5}$ is:

A. 1 B. -1 C. -1 and 4 D. 4 E. 1 or -4

40. The number 2.53535.... can be written as a fraction. When this fraction is reduced to lowest terms, the sum of the numerator and denominator is:

A. 7 B. 350 C. 141 D. 257 E. 349