FIFTIETH ANNUAL MATHEMATICS CONTEST
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THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2006

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Scoring formula: 4R − W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Algebra II Test

1. A lot is in the shape of a rectangle with a perimeter of 36 meters. The length is 4 meters more than twice the width. Find the length of the lot.

   a. 7 m   b. 40/3 m   c. 3 m   d. 13 m   e. 14/3 m

2. Which of the following equations has two different imaginary solutions?

   a. $(3x-2)^2 = 5$ b. $(4r+1)^2 = 0$ c. $(5r+2)^2 = 16$

   d. $(3r-2)^2 = -4$ e. $-(2x+1)^2 = -1$

3. For the graph of $y = (x+4)^2 - 2$, give the axis, vertex, domain, range.

   a. axis $x = 4$, vertex $(4,-2)$, domain $[4,\infty)$, range $(-\infty,\infty)$

   b. axis $x = -2$, vertex $(-4,2)$, domain $[-4,2]$, range $(\infty,-2]$

   c. axis $x = 2$, vertex $(-4,-2)$, domain $[-4,\infty)$, range $(-\infty,\infty)$

   d. axis $x = -4$, vertex $(-4,2)$, domain $(-\infty,\infty)$, range $[-2,\infty)$

   e. axis $x = -4$, vertex $(-4,-2)$, domain $(-\infty,\infty)$, range $[-2,\infty)$

4. Simplify:

   $$\dfrac{2-\dfrac{3m}{m^2-9}}{\dfrac{5}{m+3}}$$

   a. $\dfrac{-m+6}{5}$ b. $\dfrac{2m^2-3m-18}{5(m-3)}$ c. $\dfrac{2m^2-3m-18}{5}$ d. $\dfrac{2-3m}{5m-15}$ e. $\dfrac{2(m+3)}{15}$

5. Solve the inequality. Write solutions in interval notation: $\left|\dfrac{1}{2}x - \dfrac{1}{3}\right| \le 4$

   a. $\left[\dfrac{-22}{3}, \dfrac{26}{3}\right]$ b. $\left(-\infty, \dfrac{-22}{3}\right]$ c. $(-\infty,\infty)$ d. $\left[\dfrac{26}{3},\infty\right)$ e. $\left(-\infty, \dfrac{-22}{3}\right] \cup \left[\dfrac{26}{3},\infty\right)$

6. Solve: $\sqrt{2x+7} - 1 = \sqrt{x+3}$

   a. $\{-3,1\}$ b. $\{3,-1\}$ c. $\{-3,-1\}$ d. $\{3,1\}$ e. $\{\ \}$

7. Write the equation in standard form through $(4,1)$ which is parallel to the line $2x - y = -9$.

   a. $2x - y = -2$ b. $2x - y = 7$ c. $x - 2y = 2$ d. $x + 2y = 6$ e. $2x - y = 9$

8. Which of the following is a factor of $P(x) = 5x^3 - 9x^2 + x + 219$?

   a. $x+3$ b. $x+73$ c. $x-73$ d. $x-3$ e. $5x+7$

9. Which one of the functions defined below has a graph with no vertical asymptotes?

   a. $f(x) = \dfrac{3x-2}{2x-3}$

   b. $f(x) = \dfrac{x^2}{x^2+4}$

   c. $f(x) = \dfrac{x^2+4}{x^2}$

   d. $f(x) = \dfrac{2}{x}$

   e. $f(x) = \dfrac{x}{x^2-4}$

10. In which of the following equations is y a function of x?

    a. $2x + 3y - 1 = 0$

    b. $x^2 + 3y^2 = 7$

    c. $2x^2 y = 9$

    d. both a and b

    e. both a and c

11. Find the equation of the line through (4,2) that is perpendicular to the line through (9,7) and (11,4).

    a. $y = \frac{2}{3}x + 1$

    b. $y = -\frac{3}{2}x + \frac{41}{2}$

    c. $y = -\frac{3}{2}x + 8$

    d. $y = \frac{2}{3}x - \frac{2}{3}$

    e. $y = -\frac{3}{4}x + 7$

12. Decide whether the equation $y = 10 - 2x^2$ has a graph that is symmetric to the x-axis, y-axis, and/or the origin.

    a. x-axis   b. y-axis   c. origin   d. x-axis, y-axis, origin   e. x-axis, y-axis

13. Let $f(x) = 2x^2 - 13x + 15$ and $g(x) = 3x + 4$. Find $(f \circ g)(3/2)$.

    a. 13/2   b. 49   c. 2/13   d. –5   e. 22

14. A company logo is composed of a triangle superimposed on a rectangle so that the base of the triangle coincides with the length of the rectangle, with the areas intersecting. If the area of the rectangle must be 1/3 of the area of the triangle, what is the ratio of the height of the triangle to the width of the rectangle?

    a. 6:1   b. 1:6   c. 3:1   d. 1:3   e. 6:1/6

15. Suppose y varies directly as t and inversely as z. If y is 2 when t is 24 and z is 3, find y when t is 40 and z is 2.

    a. 32   b. 1/80   c. 5   d. 1/5   e. 80

16. A fifth-degree polynomial function having real coefficients has known zeros of 2, i, and $4 + 3i$. The other zeros are:

    a. -2 and $2i$ b. ± 2 c. $-i$ and $4 - 3i$ d. $-i$ and $-4 + 3i$ e. $-i$ and $-4 - 3i$

17. The given functions intersect at what points?
    $f(x) = 2x^2 + 8x + 9$ and $g(x) = x + 3$

    a. $(2,5), \left(\dfrac{-2}{3}, \dfrac{7}{3}\right)$

    b. $(-2,1), \left(\dfrac{-2}{3}, \dfrac{7}{3}\right)$

    c. $\left(\dfrac{-3}{2}, \dfrac{3}{2}\right), (-2,1)$

    d. $(-3,0), (-4,-1)$

    e. $(-2,1), (-4,-1)$

18. Solve over the set of real numbers and write solutions in interval notation: $15 - x^2 \geq -1$

    a. $(-4,4)$ b. $[-4,4]$ c. $(-\infty,4) \cup (4,\infty)$ d. $(-\infty,4] \cup [4,\infty)$ e. $(-\infty,-4] \cup [4,\infty)$

19. The equation $x^2 + y^2 + 12x - 8y + 5 = 0$ represents a circle with what center and radius?
   a. $C(-6,4), r = 9$ b. $C(6,-4), r = \sqrt{47}$ c. $C(-6,4), r = \sqrt{47}$
   d. $C(-6,4), r = 47^2$ e. $C(6,-4), r = 47$

20. A factor of $a^{3n} - 125$ is:
   a. $(a^n - 5)^3$ b. $a^{2n} - 5a^n + 25$ c. $a^{2n} + 10a^n + 25$ d. $a^{2n} + 5a^n + 25$ e. $a^3 - 5$

21. A radioactive substance is decaying so that the number of grams present after t days is given by the function, $A(t) = 4000e^{-0.03t}$. Find the half-life of the substance to the nearest day.
   a. 10 days   b. 21 days   c. 4 days   d. 23 days   e. 17 days

22. Use properties of logarithms to write $\log_7\left(\dfrac{x^3 z}{\sqrt{y}}\right)$ as a sum, difference, quotient or product of logarithms.
   a. $\dfrac{(3\log_7 x)(\log_7 z)}{\frac{1}{2}\log_7 y}$ b. $(3\log_7 x)(\log_7 z) - \frac{1}{2}\log_7 y$ c. $3\log_7 x + \log_7 z - \frac{1}{2}\log_7 y$
   d. $(3\log_7 x)(\log_7 z)(\frac{1}{2}\log_7 y)$ e. $3(\log_7 x + \log_7 z) - \frac{1}{2}\log_7 y$

23. Solve the following system of equations for x.
$$\begin{cases} 2x - y + 2z = -7 \\ x + 2y + z = -1 \\ 3x - y - z = -6 \end{cases}$$
   a. -2 b. -1 c. 0 d. 1 e. 2

24. Give the determinant of the following matrix.
$$\begin{bmatrix} -3 & 1 & 2 \\ -5 & 6 & 0 \\ -2 & 3 & -1 \end{bmatrix}$$
   a. -23 b. -27 c. -31 d. 18 e. 7

25. Write the first five terms for the following sequence: $a_n = \dfrac{n-1}{n}$.
   a. 0, 1/2, 2/3, 3/4, 4/5   b. 1, 1/2, 3/4, 5/6, 7/8   c. undefined, 1/2, 2/3, 3/4, 4/5
   d. 1, 1/2, 1/3, 1/4, 1/5   e. undefined, 1, 1/2, 2/3, 3/4

26. In how many ways can the president, the first lady, the vice-president, the vice-president's wife, the secretary of state and the chief of staff be lined up on stage?
   a. 240   b. 720   c. 360   d. 36   e. 120

27. A blue 64 cm^3 cube is perfectly circumscribed by a red sphere. What is the exact volume of the sphere?

   a. $64\pi\sqrt{3}$ b. $32\pi\sqrt{3}$ c. $64\pi\sqrt[3]{3}$ d. $32\pi\sqrt[3]{3}$ e. $\dfrac{64\pi\sqrt{2}}{3}$

28. Solve the equation over the given interval: $\sec(2x)\tan(2x) = \tan(2x)$, $[0,2\pi)$.

   a. $\left\{0,\dfrac{\pi}{2},\pi,\dfrac{3\pi}{2}\right\}$ b. $\{\ \}$ c. $\{0,\pi\}$ d. $\left\{\dfrac{\pi}{2},\dfrac{3\pi}{2}\right\}$ e. $\left\{0,\dfrac{\pi}{2},\pi,\dfrac{3\pi}{2},2\pi\right\}$

29. The following results were reported by a Medical Center. Of the 100 people in a control group who were suffering from arthritis, 75 were given a new drug and the remaining were given a placebo. Of the control group 60 showed improvement but only 54 of those were actually given the drug. How many patients not receiving the drug showed no improvement?

   a. 21   b. 19   c. 40   d. 18   e. 22

30. Given $f(x) = x^3 + 6x^2 + 11x + 6$, the set that best describes the possible rational zeros is:

   a. $\{\pm 6,\pm 3,\pm 2,\pm 1\}$ b. $\{-6,-3,-2,-1\}$ c. $\left\{\pm 1,\pm\dfrac{1}{3},\pm\dfrac{1}{2},\pm\dfrac{1}{6}\right\}$

   d. $\{6,3,2,1\}$ e. $\left\{-1,-\dfrac{1}{3},-\dfrac{1}{2},-\dfrac{1}{6}\right\}$

31. Find the number of solutions to the following system of linear equations:
$$\begin{cases} 2x - y = 6 \\ 6x - 3y = 12 \end{cases}$$

   a. none   b. 1   c. 2   d. 3   e. infinitely many

32. For the given matrices, find the product AB:
$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 5 & 6 & 7 \\ 8 & 9 & 0 \end{bmatrix}$$

   a. $\begin{bmatrix} 21 & 24 & 7 \\ 47 & 54 & 21 \end{bmatrix}$ b. $\begin{bmatrix} 5 & 12 \\ 24 & 36 \end{bmatrix}$ c. $\begin{bmatrix} 5 & 12 & 0 \\ 24 & 36 & 0 \end{bmatrix}$ d. $\begin{bmatrix} 21 & 24 \\ 47 & 54 \end{bmatrix}$

   e. the operation cannot be performed

33. If a quadratic equation has imaginary solutions, then the discriminant is:

   a. positive   b. negative   c. zero   d. undefined   e. none of these

34. Solve the logarithmic equation: $3\log_2 x = -\log_2 27$

   a. $-1/3$ b. 3   c. 9   d. -3 e. 1/3

35. Solve the exponential equation: $4^{2x-1} = 8$

   a. 5/4   b. 2   c. 1   d. 1/4   e. no real solutions

36. Find the domain of the function, $f(x) = \dfrac{-1}{\sqrt{x-2}}$.

    a. $x \geq 2$ b. $x \leq 2$ c. $x \neq 2$ d. $x > 2$ e. $x < 2$

37. Find the inverse of the given function, $g(x) = 3^x$.

    a. 3^{-x} b. x^3 c. $\dfrac{1}{3^x}$ d. $\log_3 x$ e. does not exist

38. Solve the rational equation:

$$\frac{z}{z+3} - \frac{4}{z-3} = \frac{-24}{z^2-9}$$

    a. $\{4\}$ b. $\{4,3\}$ c. $\{1/2\}$ d. $\{4,1/2\}$ e. $\{3\}$

39. A store manager wants to offer a blended coffee by mixing an unflavored coffee that sells for $3 per pound with a flavored coffee that sells for $5 per pound. If she mixes 100 pounds of the new blend and plans to sell it for $4.50 per pound, how much of the unflavored coffee should she use?

    a. 75 pounds   b. $33\frac{1}{3}$ pounds   c. 25 pounds   d. $66\frac{2}{3}$ pounds   e. 50 pounds

40. If Donald can perform a job working alone in 4 hours, and his brother Mark can do the same job in 6 hours, how long will it take for them to do the job working together?

    a. 4.5 hours   b. 5 hours   c. 3 hours   d. 3.2 hours   e. 2.4 hours