FORTY-NINTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2005

Prepared by:

Department of Mathematics and Statistics
UT-Martin
Martin, Tennessee

Reviewed by:

Mathematics Faculty
Austin Peay State University
Clarksville, Tennessee

Coordinated by: Louis Kolitsch

Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

Dr. Hal Ramer, President, Volunteer State Community College, Gallatin, Tennessee
Donnelley Printing Company, Gallatin, Tennessee
TRW Commercial Steering Division, Lebanon, Tennessee
Wright Industries, Inc., Nashville, Tennessee

1. The vertical asymptote(s) to the graph of the function $f(x) = \dfrac{x^2 + x - 6}{x^2 - 6x + 8}$ is/are

   a) $x = 2$ and $x = 4$

   b) $x = -3$, $x = 2$, and $x = 4$

   c) $x = -3$ and $x = 2$

   d) only $x = 4$

   e) only $x = -3$

2. Find all values of x satisfying $|2x - 3| = |5x + 4|$.

   a) $x = \dfrac{3}{2}$ or $x = \dfrac{-4}{5}$

   b) $x = \dfrac{-7}{3}$ or $x = \dfrac{-1}{7}$

   c) $x = \dfrac{7}{3}$ or $x = \dfrac{1}{7}$

   d) $x = \dfrac{1}{3}$ or $x = \dfrac{-1}{7}$

   e) No solution exists.

3. If the point (2, 1) is the midpoint of the line segment $\overline{AB}$ and A has coordinates $\left(\dfrac{-1}{2}, 6\right)$, then the coordinates of B are

   a) $\left(\dfrac{9}{2}, -4\right)$ b) $(-3, 11)$ c) $\left(\dfrac{3}{4}, \dfrac{7}{2}\right)$ d) $\left(\dfrac{5}{4}, \dfrac{5}{2}\right)$ e) $\left(\dfrac{-7}{2}, -4\right)$

4. The equation of the line that passes through the point (2, 3) and is perpendicular to the line $4x + 12y - 3 = 0$ is

   a) $y = 3x - \frac{3}{2}$

   b) $y = 3x - 3$

   c) $y = 2x - 3$

   d) $y = \frac{1}{3}x - 2$

   e) $y = \frac{-1}{3}x + \frac{1}{4}$

5. Find the quadratic function that fits the data points (0, 0), (2, 2), and (–4, 2).

   a) $f(x) = \frac{1}{2}x^2 - \frac{1}{2}x$

   b) $f(x) = \frac{1}{2}x^2 + \frac{1}{2}x$

   c) $f(x) = \frac{3}{8}x^2 + \frac{1}{2}x$

   d) $f(x) = \frac{1}{4}x^2 - \frac{1}{2}x$

   e) $f(x) = \frac{1}{4}x^2 + \frac{1}{2}x$

6. Determine the solution of the inequality $\frac{x+7}{x-3} \geq 0$.

a) $-7 \leq x \leq 3$

b) $x \leq 7$ or $x \leq 3$

c) $-7 \leq x < 3$

d) $x \leq -7$ or $x > 3$

e) $-3 \leq x \leq 7$

7. What is the coefficient of x^5 in the expansion of $(2x + 0.5)^{10}$?

a) 1 b) 16 c) 32 d) 252 e) 1024

8. If $f(x) = x^4 - 2x^3 + 3x + 4$ and $g(x) = 5x$, then $g(f(x)) =$

a) $x^5 - 10x^4 + 3x^2 + 4x$

b) $5x^4 - 10x^3 + 15x + 20$

c) $5x$

d) $625x^4 - 250x^3 + 15x + 4$

e) $5x^5 - 10x^4 + 15x^2 + 20x$

9. If $\log_b a = c$ where a, b, and c are positive real numbers, find $\log_a b$.

a) $\frac{1}{c}$

b) $-c$

c) $\log_c a$

d) $\log_c b$

e) cannot be determined with the given information

10. If $f(x) = x^7 + x + 2$ and $f(a) = 9$, then $f(-a) =$

a) 4 b) –4 c) 5 d) –5 e) 6

11. Simplify the expression $(2 + 3i)(1 - i) + (2 + i)$:

a) $7 + 2i$ b) $6 + 9i$ c) $4 + 4i$ d) $9 - i$ e) $5 + 3i$

12. Solve for x. $\log_2(x - 5) + \log_2(x + 2) = 3$

a) $x = -3$ or $x = 6$

b) $x = 6$

c) $x = 5$ or $x = -2$

d) $x = -3$

e) $x = 4.5$

13. Find $f^{-1}(x)$ if $f(x) = \dfrac{10x + 3}{2x - 9}$.

a) $f^{-1}(x) = \dfrac{2x - 9}{10x + 3}$

b) $f^{-1}(x) = \dfrac{10x - 3}{2x + 9}$

c) $f^{-1}(x) = \dfrac{9x + 3}{2x - 10}$

d) $f^{-1}(x) = \dfrac{2x - 10}{9x + 3}$

e) $f^{-1}(x) = \dfrac{\frac{1}{10}x + \frac{1}{3}}{\frac{1}{2}x - \frac{1}{9}}$

14. Solve for x: $2x = 5^{1+\log_5 4}$

a) 3 b) 4 c) 5 d) 8 e) 10

15. In the arithmetic sequence 4, 11, 18, 25, . . . , the 150^{th} term is

a) 1043 b) 1050 c) 1047 d) 1054 e) 1061

16. What is the maximum product of two numbers having a sum of 24?

a) 128 b) 140 c) 144 d) 156 e) No maximum exists

17. An investment with interest compounded continuously, tripled itself in 12 years. What is the interest rate?

a) 3.98% b) 5.78% c) 17.31% d) 9.16% e) 3.00%

18. How much of a 40% solution of an acid should be mixed with pure water to obtain 80ml of a 30% acid solution?

a) 24 ml b) 28 ml c) 32 ml d) 60 ml e) 80 ml

19. How many minutes is it until six o'clock if fifty minutes ago, it was four times as many minutes past three o'clock?

a) 15 b) 17 c) 20 d) 26 e) 30

20. Find k so that the line containing (–3, k) and (2, 7) is perpendicular to the line containing (3, –2) and (–5, 7).

a) $\frac{23}{9}$ b) $\frac{101}{8}$ c) $\frac{103}{9}$ d) $\frac{83}{8}$ e) $\frac{11}{8}$

21. A six-sided die has 2 blue faces, 1 red face and 3 yellow faces. If the die is rolled twice, what is the probability that both rolls will result in a blue face?

a) $\frac{1}{3}$ b) $\frac{1}{9}$ c) $\frac{2}{3}$ d) $\frac{1}{6}$ e) $\frac{1}{4}$

22. If the curve $y = \log_a(2x + b)$ has a vertical asymptote where $x = 1$, then the value of b is

a) -2 b) -1 c) 0 d) 1 e) 2

23. A ball is dropped straight down from a height of 9 feet. With each bounce the ball returns to $\frac{2}{3}$ of its previous height. What is the total distance the ball will have traveled at the instant it hits the ground for the third time?

a) 29 feet b) 19 feet c) 38 feet d) $\frac{65}{3}$ feet e) $\frac{95}{3}$ feet

24. What is the radius of a circle with area 10 square inches? Give your answer accurate to two decimal places.

a) 1.78 inches b) 3.18 inches c) 10.13 inches d) 5.60 inches e) 1.01 inches

25. Find h, where $3x^2 - hx + 9 = 0$ and the sum of the solutions is -4 and the product of the solutions is 3.

a) 12 b) 4 c) 3 d) 0 e) -12

26. For what value(s) of B does the equation $x^2 + Bx + 1 = 0$ have no real solutions?

a) $-2 < B < 2$ b) $B < 2$ c) $B > 0$ d) $0 < B < 2$ e) $B = 2$

27. What is the domain of $f(x)=(x^2 + 4x + 3)^{1/2}$?

a) $x \leq -3$ or $x \geq -1$ b) $-4 \leq x \leq 3$ c) $x \geq -3$ d) $1 \leq x \leq 3$ e) $x \geq 0$

28. If opening valve A will fill a swimming pool in 8 hours and opening valve B will fill the same pool in 5 hours, how much time is required to fill the pool if both valves are opened simultaneously?

a) $\frac{13}{2}$ hours b) $\frac{40}{13}$ hours c) $\frac{13}{40}$ hours d) $5\frac{3}{8}$ hours e) 3 hours

29. If $15x^5 + bx^3 - cx - 6 = 0$, where b and c are integers, which of the following could not be a root of the polynomial equation?

a) $\frac{2}{3}$ b) 6 c) $\frac{4}{3}$ d) $\frac{3}{5}$ e) $\frac{-1}{15}$

30. The cost for one print run of a book is jointly proportional to the number of pages in the book and the number of books in the print run. If it costs \$20000 to print 400 copies of a 100-page book what is the cost to print 400 copies of a 293-page book?

a) \$68260 b) \$117200 c) \$23440 d) \$58600 e) \$146500

31. Given the functions $f(x) = \sqrt{x^2+1}$ and $g(x) = \sqrt{25-x^2}$, what is the domain of $f+g$ within the set of real numbers?

a) All real numbers b) $-1 \le x \le 1$ c) $0 \le x \le 5$ d) $1 \le x \le 25$ e) $-5 \le x \le 5$

32. Which of the following is equal to $\sqrt[3]{\dfrac{x^3+x^3y^3}{y^3z^4}}$?

a) $\dfrac{x+xy}{yz\sqrt[3]{z}}$

b) $\dfrac{x}{yz}\sqrt[3]{\dfrac{2}{z}}$

c) $\dfrac{x(1+y)\sqrt[3]{z^2}}{yz^2}$

d) $\dfrac{x\sqrt[3]{z^2(1+y^3)}}{yz^2}$

e) $\sqrt[3]{\dfrac{x+xy}{yz^{4/3}}}$

33. A rectangular park is 480 meters wide and 550 meters long. Tasha must walk along two sides of the park to get home. If she walks through the park along a diagonal, how much shorter will her trip home be?

a) 322 meters b) 300 meters c) 281 meters d) 730 meters e) 515 meters

34. The least common multiple of $x^2 - 4$, $x^2 + 4$, and $(x-2)^2$ has degree

a) 3 or less b) 4 c) 5 d) 6 e) 7 or more

35. The value of the determinant $\begin{vmatrix} a & b & c \\ d & e & c \\ f & g & c \end{vmatrix}$ is 7. What is the value of $\begin{vmatrix} 2a & 1 & 3b \\ 2d & 1 & 3e \\ 2f & 1 & 3g \end{vmatrix}$?

a) $\dfrac{42}{c}$ b) $\dfrac{7}{6c}$ c) $\dfrac{-42}{c}$ d) $\dfrac{-7}{6c}$ e) 13

36. If the function $f(x) = 2x^2 + 15x + 27$ can be used to calculate the area of a rectangle whose length is 3 units more than twice its width, then the variable x represents

a) the length of the rectangle

b) the width of the rectangle

c) half of the length of the rectangle

d) three units more than the width of the rectangle

e) three units less than the width of the rectangle

37. A string of 33 pearls is worth \$65,000. The middle pearl is the most expensive. The pearls are arranged so that, from the left end, each successive pearl is worth \$100 more than the preceding pearl, up to and including the middle pearl. From the right end, each successive pearl is worth \$150 more than the preceding pearl, up to and including the middle pearl. How much is the middle pearl worth?

a) \$1700 b) \$2550 c) \$2800 d) \$3000 e) \$4500

38. The sum of the roots of the polynomial $x^3 - 5x^2 + bx + 25$, where b is a real number,

a) is 5.

b) is –5.

c) is 8.

d) is –8.

e) cannot be determined without knowing the value of b

39. Which complex number given below equals five times the conjugate of its reciprocal?

a) $1 + 2i$ b) $2 + 3i$ c) $2 - 3i$ d) $2 + 2i$ e) $1 + 3i$

40. Order $R = 2004^{2005}$, $S = 2005^{2004}$, and $T = 2005!$ from smallest to largest.

a) R, S, T b) S, R, T c) R, T, S d) T, R, S e) T, S, R