FORTY-SIXTH ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 2002

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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1. Find the equation of the line that passes through (1,-2) and is perpendicular to the line $x + 3y = 6$

   a. $y = \frac{1}{3}x - \frac{7}{3}$
   b. $y = 3x - 5$
   c. $\frac{1}{3}x + y = -\frac{5}{3}$
   d. $3x + y = 1$
   e. $x - 3y = -7$

2. Find the center and radius for the circle $x^2 + y^2 - x + 2y + 1 = 0$

   a. $c(1, -1)$ $r = \frac{1}{2}$
   b. $c(-\frac{1}{2}, 1)$ $r = \frac{1}{4}$
   c. $c(\frac{1}{2}, -1)$ $r = \frac{1}{4}$
   d. $c(\frac{1}{2}, -1)$ $r = \frac{1}{2}$
   e. $c(1, -1)$ $r = 0$

3. Find the solution set of the inequality $|2x - 3| > k$, if $k < 0$.

   a. $(-\infty, k) \cup (k, \infty)$
   b. $(-k, k)$
   c. $\emptyset$
   d. $(-\infty, \infty)$
   e. $[k, \infty)$

4. Solve: $\frac{3x}{x^2+5x+6} - \frac{5x}{x^2+2x-3} = \frac{-2}{x^2+x-2}$

a. $\{\frac{1}{2}, -6\}$

b. $\{2, -\frac{1}{6}\}$

c. $\{-\frac{1}{2}, 6\}$

d. $\{-6\}$

e. $\{\frac{1}{2}\}$

5. How many pounds of an alloy containing 45% silver must be melted with an alloy containing 60% silver to obtain 40 pounds of a 48% silver alloy?

a. 8 pounds
b. 20 pounds
c. 32 pounds
d. 30 pounds
e. 22 pounds

6. If k is a negative number, then which of the following equations must have 2 imaginary solutions?

a. $x^2 + kx + 3k^2 = 0$
b. $k + (x - 5)^2 = 0$
c. $-(x+1)^2 + k^2 = 0$
d. $-x^2 - kx - 2k = 0$
e. $6x^2 + 7k = 0$

7. Identify the third term of the expansion $(3x + 4)^8$

a. $65{,}536x^6$
b. $48x^6$
c. $11{,}664x^6$
d. $569{,}592x^6$
e. $326{,}592x^6$

8. Solve $t = 25qx^2$ for x.

a. $x = \pm \frac{\sqrt{t}}{5q}$
b. $x = \pm \frac{\sqrt{tq}}{5q}$
c. $x = \pm 5q\sqrt{tq}$
d. $x = \pm \frac{1}{5}\sqrt{tq}$
e. $x = \pm 5q\sqrt{5t}$

9. Evaluate the determinate: $\begin{vmatrix} -3 & a & -3 \\ 4 & 4 & 1 \\ 2 & -3 & 2 \end{vmatrix}$

a. $27 + 6a$
b. $45 - 6a$
c. $27 - 6a$
d. $45 + 6a$
e. $-21 - 6a$

10. Solve: $| 3x - 5 | - 3 = 4$

a. $\{4, -\frac{2}{3}\}$

b. $\{-4, \frac{2}{3}\}$

c. $\{\frac{4}{3}, -\frac{4}{3}\}$

d. $\{4, \frac{4}{3}\}$

e. no solution

11. What is the remainder when $x^3 - x^2 - 10x + 4$ is divided by $x + 3$?

a. 10
b. – 2
c. – 8
d. 2
e. 0

12. For $f(x) = x^3 - 2x$ and $g(x) = 2x - 3$, find $(f \circ g)(-1)$

a. - 123
b. 135
c. - 5
d. 0
e. - 115

13. Solve: $\frac{2-2a}{3}$ - 2a = 14

a. $\{-\frac{14}{3}\}$
b. {– 6}
c. {10}
d. {– 5}
e. $\{\frac{10}{3}\}$

14. Factor completely: 5x (x – 4) – 7x + 10

a. (x – 5)(5x – 2)
b. (x – 4)(10 – 2x)
c. (x – 5)(5x + 2)
d. 2(x – 4)(x + 5)
e. prime

15. Find the vertex of the parabola $y = 2x^2 - 12x + a$

a. (3, a – 18)
b. (- 3, a – 18)
c. (3, a – 9)
d. (- 3, a – 9)
e. (6, a – 36)

16. Solve: $4^{2x-1} = 2^{-x}$

a. $\{\frac{1}{3}\}$

b. $\{\frac{2}{3}\}$

c. $\{-\frac{2}{5}\}$

d. $\{\frac{5}{2}\}$

e. $\{\frac{2}{5}\}$

17. The area of a triangle is 600 square feet. If the height is three times the base, what is the height of the triangle?

a. $10\sqrt{2}$ feet

b. 20 feet

c. 60 feet

d. $30\sqrt{2}$ feet

e. $\frac{10\sqrt{6}}{3}$ feet

18. Solve: $72 = (3p - 1)^2 + (3p - 1)$

a. $\{9, 8\}$

b. $\{-\frac{8}{3}, 3\}$

c. $\{9, -8\}$

d. $\{-\frac{7}{3}, \frac{10}{3}\}$

e. $\{3, \frac{8}{3}\}$

19. Simplify: $\dfrac{x+2+\dfrac{3}{2x-3}}{x-5-\dfrac{39}{2x-3}}$

a. $\dfrac{x+1}{x+8}$

b. $\dfrac{2x-3}{x-5}$

c. $\dfrac{x-1}{x-8}$

d. $\dfrac{2x-1}{x+8}$

e. $\dfrac{2x-3}{x+5}$

20. Solve the system:
$$\frac{1}{3}x + \frac{1}{6}y - \frac{2}{3}z = -1$$
$$\frac{3}{4}x + \frac{1}{3}y + \frac{1}{4}z = -3$$
$$\frac{1}{2}x + \frac{3}{2}y + \frac{3}{4}z = 21$$

a. $\{(-12, 18, 0)\}$

b. $\{(-\frac{486}{217}, \frac{129}{31}, \frac{37}{217})\}$

c. $\{(12, -18, 0)\}$

d. $\{(\frac{486}{217}, -\frac{129}{31}, -\frac{37}{217})\}$

e. $\{(2, 2, 3)\}$

21. $\frac{(r^3)^{-2}(s^2)^4}{r^0(s^{-3})^{-2}}$

a. $\frac{s^2}{r^7}$

b. $\frac{s^2}{r^6}$

c. $\frac{s^{14}}{r^6}$

d. $\frac{r^6}{s^2}$

e. $\frac{r^5}{s^2}$

22. Solve: $\sqrt{7-3x}$ - x = 1

a. {– 6, 1}
b. {6, – 1}
c. {1}
d. {– 1}
e. $\{\frac{-3 \pm \sqrt{33}}{2}\}$

23. Simplify: $\dfrac{3\sqrt[3]{x^6} - x\sqrt[3]{x^3}}{\sqrt[3]{x}}$

a. $3\sqrt[3]{x^5} - x\sqrt[3]{x^2}$
b. $2x^2\sqrt[3]{x}$
c. $2x\sqrt[3]{x^2}$
d. $3x\sqrt[3]{x^2} - x\sqrt[3]{x^2}$
e. $3x\sqrt[3]{x^2} - x^2$

24. A product will break even or produce a profit only if the revenue R from selling the product is at least the cost C of producing it. Suppose that the cost C to produce x units of cloth is $C = 30x + 3500$, while the revenue is $R = 40x$. For what values of x is R at least equal to C?

a. $[50, \infty)$
b. $[350, \infty)$
c. $(350, \infty)$
d. $(50, 350)$
e. none of these

25. Find the real solutions for $\frac{1}{2}x^2 = \sqrt{2}x + 1$

a. $x = \frac{1}{2} \pm \sqrt{2}$
b. $x = 1 \pm \sqrt{2}$
c. $x = -\sqrt{2}$
d. $x = \sqrt{2} \pm 2$
e. $x = \sqrt{2} \pm 4$

26. Give the solution set for the compound inequality:
$2k < 5(k + 3) - 3$ and $5k + 1 < 21$

a. $(-\infty, -4) \cap (4, \infty)$
b. $(-\infty, -4) \cup (4, \infty)$
c. $(-4, 4)$
d. $(-4, \infty)$
e. all real numbers

27. What values of x could not be considered as possible solutions of the equation $\frac{2}{x^2-4} - \frac{x}{x+2} = -\frac{1}{2}$?

a. $\{-2, 2\}$
b. $\{0\}$
c. $\{0, 4\}$
d. $\{0, -2, 2\}$
e. $\{-2, 4\}$

28. Simplify: $\frac{8-i}{3-2i}$

a. $2 + i$
b. $\frac{26+13i}{5}$
c. $-2 - i$
d. $\frac{22-19i}{5}$
e. $22 + i$

29. Solve the equation $\log_{1/5} x = -2$

a. $\{-5\}$

b. $\{\frac{1}{25}\}$

c. $\{5\}$

d. $\{25\}$

e. $\{-25\}$

30. If r varies directly as t and r = 10 when t = 2, find r when t = 9.

a. 45

b. 5

c. 20/9

d. 17

e. 90

31. Find the 51st term in the sequence 2, 5, 10, 17, . . .

a. 2501

b. 2602

c. 101

d. 2705

e. 103

32. The license plates for a certain state consist of 2 letters followed by a 4-digit number such that the first digit of the number is not zero. How many plates would not have any digits repeated in the number part of the plate?

a. 4,212,000
b. 3,407,040
c. 4,380,480
d. 3,276,000
e. 3,066,336

33. Simplify $(5 + 2\sqrt{3})(2 - 4\sqrt{3})$

a. $10 - 19\sqrt{3}$
b. $10 - 8\sqrt{3}$
c. -14
d. $-62 - 16\sqrt{3}$
e. $-14 - 16\sqrt{3}$

34. A student can type a 60-page term paper in four hours. With a friend's assistance, the paper can be typed in three hours. How long would it take the friend working alone to type the paper?

a. 12 hours
b. 6 hours
c. 2 hours
d. 10 hours
e. 8 hours

35. Find three consecutive odd integers such that five times the middle integer is three less than the sum of the first and third. The sum of the three integers is

a. 3

b. - 15

c. - 3

d. - 10

e. 9

36. Simplify: $\frac{1}{i^{37}} - \frac{1}{i^2} + \frac{1}{i^3} - \frac{1}{i^4}$

a. i

b. 0

c. – 1

d. 1

e. $- i$

37. A positive integer N with three digits in its decimal representation is chosen at random (that is, each such number is equally likely to be chosen). What is the probablility that $\log_2 N$ is an integer?

a. 0

b. 3/899

c. 1/300

d. 1/225

e. 1/450

38. A rectangular garden measures 8 feet wide and 16 feet long. The gardener wants to put a strip of gravel of uniform width around the garden. There is enough gravel for 112 square feet. How wide will the strip be?

a. 2 feet

b. 4 feet

c. 1 foot

d. 3 feet

e. 6 inches

39. Find the slopes of the asymptotes of a hyperbola with the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

a. $\frac{a}{b}$ and $-\frac{a}{b}$

b. a and - a

c. b and - b

d. $\frac{b}{a}$ and $-\frac{b}{a}$

e. $\frac{1}{a}$ and $\frac{1}{b}$

40. The point $\left(\frac{16}{5},\frac{24}{5}\right)$ is 3/5 of the way along the line segment from (2,3) towards (a, b). The sum of a and b is

a. 8

b. 10

c. – 25

d. $\frac{3\sqrt{13}}{5}$

e. $\sqrt{13}$