FORTY-THIRD ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1999

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Scoring formula: $4R - W + 40$

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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Algebra II

1. Two fair coins are tossed. The probability that they are both heads is:

   a) 1/3 b) 1/2 c) 1/4 d) 1 e) 0

2. If s varies directly with r^2 and s = 12 when r = 2, find s when r = 7.

   a) 147 b) 21 c) 196 d) 49/3 e) 7/3

3. Find all values of k such the $k^2x^2 - 8x + 4 = 0$ has imaginary roots.

   a) $k > 2$ b) $k < -2$ c) $-2 < k < 2$ d) $|k| > 2$ e) $|k| < 2$

4. Determine k so that $y = kx^2 + 7$ passes through (2, 5).

   a) 2 b) -0.2 c) –0.5 d) 3 e) 0.5

5. Find a polynomial equation with roots -1, 3, 3, 1+2i, 1-2i.

   a) $(x-1)(x+3)(x-2)(x+2) = 0$ b) $(x+1)(x-3)^2(x^2-2x+5) = 0$ c) $(x-1)(x+3)^2(x-2i)(x+2i) = 0$

   d) $(x+1)(x-3)^2(1-4x) = 0$ e) $(x+1)(x-3)^2(x-1)^2(x-4i^2) = 0$

6. Given $f(x) = x^2 + 3$ and $g(x) = 7 - x$, find f(g(3))

   a) 19 b) -5 c) 48 d) 16 e) -2

7. Given $f(x) = \sqrt{x+1}$, find $\dfrac{f(x+h)-f(x)}{h}$.

   a) $\dfrac{\sqrt{x+1}+h-\sqrt{x+1}}{h}$ b) $\dfrac{\sqrt{h(x+1)}-\sqrt{x+1}}{h}$ c) $\dfrac{\sqrt{(x+1+h)-(x+1)}}{h}$

   d) $\dfrac{h}{\sqrt{x+h+1}+\sqrt{x+1}}$ e) $\dfrac{1}{\sqrt{x+h+1}+\sqrt{x+1}}$

8. Find the largest possible domain of $\dfrac{5}{\sqrt{x^2-9}}$.

   a) $(-\infty, -3] \cup [\, 3, \infty)$ b) $(-3, 3)$ c) $(-\infty, -9) \cup (9, \infty)$ d) $[\, -3, 3\,]$ e) $(-\infty, -3) \cup (3, \infty)$

9. Tickets to an attraction cost \$29 for three adults and one child. They cost \$23 for one adult and three children. What is the cost a child's ticket?

   a) \$8 b) \$4 c) \$3.50 d) \$5 e) \$2

10. Solve $|2m+3| = 5$ for m.

a) {1, -4} b) {-1, 4} c) {-2/3, -1} d) { 1, -3/2} e) { -1, -4}

11. Factor completely, $16a^2b - 25b^3$.

a) b(4a-5b)(4a+5b) b) -b(4a-5b)(4a+5b) c) $16(a^2b - \frac{25}{16}b^3)$

d) $16b(a^2 - \frac{25}{16}b^2)$ e) $(4ab-5b^3)(4ab+5b^3)$

12. Find the remainder of $2x^3 + 5x + 1$ divided by x - 3.

a) 40 b) $2x^2 + 6x + 23$ c) $2x^2 - 6x - 13$ d) 70 e) -68

13. Choose the solution of $| 2x + 1 | > 3$.

a) $x > 1$ b) $-2 < x < 1$ c) $x < -2$ or $x > 1$ d) $x > 0.5$ e) $x > 2$

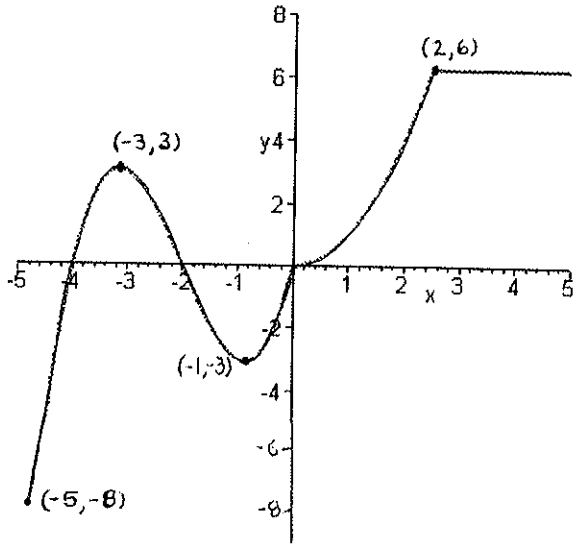
14. The sides of a right triangle have lengths in inches of three consecutive multiples of 4. Find the area of the triangle.

a) 48 sq inches b) 16 sq inches c) 96 sq inches d) 24 sq inches e) 160 sq inches

15. A poster has a uniform two inch border around 512 square inches of text. If the height of the text is twice its width, what are the dimensions of the poster?

a) 16 in. by 32 in. b) 18 in. by 34 in. c) 26 in. by 52 in. d) 20 in. by 36 in. e) 28 in. by 54 in.

16. Given the following graph, specify all intervals where the rate of change is positive.



a) $-2 < x < 0$ b) $-4 < x < -2$ or $0 < x < 5$

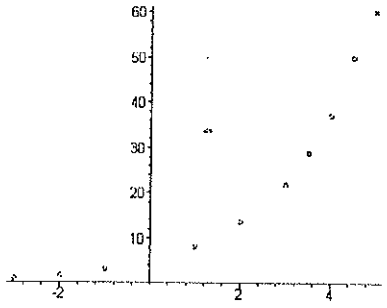
c) $-3 < x < -1$ d) $-5 < x < -3$ or $-1 < x < 2$ e) $0 < x < 5$

17. In 1980, aerospace industry profits in the United States were \$2.59 billion whereas in 1990 profits were \$4.45 billion. Find the average annual rate of change from 1980 to 1990.

a) \$3.52 billion/yr b) \$0.704 billion/yr c) \$0.93 billion/yr

d) \$0.186 billion/yr e) \$18.6 million/yr

18. The data depicted in the graph can best be modeled by which type of function?



a) linear b) quadratic c) square root d) exponential e) logarithmic

19. Find the equation of the line perpendicular to $y = 5x - 3$ which contains the point (3, 2).

a) $y = -5x + 1.2$ b) $y = -0.2x + 1.2$ c) $y = -2.6x - 5$

d) $y = -2.6x + 1.2$ e) $y = -0.2x + 2.6$

20. Find the equation of the horizontal line which contains the point (-2, 50).

a) $x = 50$ b) $y = -2$ c) $x = -2$ d) $y = 50 - 2x$ e) $y = 50$

21. The y-axis, the x-axis, the line $x = 6$, and the line $y = 12$ determine the four sides of a 6 by 12 rectangle in the first quadrant of the xy plane. Imagine that this rectangle is a pool table. There are pockets at the four corners and at the points (0, 6) and (6, 6) in the middle of the longer sides. Your pool ball is at (2, 8). You hit it toward the y-axis, along the line with slope 2. Where does it hit the y-axis?

a) (0, 12) b) (0, 8) c) (4, 0) d) (8, 0) e) (0, 4)

22. Simplify $\dfrac{3}{x^2 - 5x + 6} + \dfrac{2}{x^2 - 4}$

a) $\dfrac{5x(x-2)}{(x^2 - 5x + 6)(x^2 - 4)}$ b) $\dfrac{5}{(x^2 - 5x + 6)(x^2 - 4)}$ c) $\dfrac{5x}{(x^2 - 4)(x - 3)}$

d) $\dfrac{6}{(x-2)^2(x-3)}$ e) $\dfrac{6}{(x^2 - 5x + 6)(x^2 - 4)}$

23. The graph of y = |x| is flipped over the x-axis, moved 3 units up and 5 units to the right. What is the equation of the resulting graph?

a) y = -|x-5| + 3 b) y = |x-5| + 3 c) y = - |x+5|+3 d) y = -|x+3|-5 e) y = -|x+5|+3

24. Simplify $\frac{3x^2-6x}{x^2-6x+9}\cdot\frac{x^2-x-6}{x^2-4}$.

a) $\frac{3x(x-6)(x+1)}{(x-3)^2(x+2)}$ b) $\frac{3x(x-3)}{(x+3)^2}$ c) $\frac{3x(x-3)}{(x-9)(x+1)}$ d) $\frac{3x}{x-3}$ e) $\frac{3x(x+2)}{(x-2)(x-3)}$

25. Simplify $\left(\frac{x^2y^{-1}}{z}\right)^{-2}\left(\frac{x^2y^2}{z^{-3}}\right)$.

a) $\frac{y^4z^2}{x^6}$ b) $\frac{y^4z^5}{x^2}$ c) $\frac{x^4y^0}{z^{-2}}$ d) $\frac{x^4y}{z^{-2}}$ e) $\frac{z^5}{x^2}$

26. Simplify $\frac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}}$.

a) $\frac{x-y}{x+y}$ b) 1 c) $\frac{y^2-x^2}{y^2-2xy+x^2}$ d) $\frac{y-x}{x+y}$ e) $\frac{x^2+2xy+y^2}{x^2-y^2}$

27. Simplify by performing the indicated matrix operations. $\begin{bmatrix}-1 & 3 & 1\\ 2 & 1 & 3\end{bmatrix}\cdot\begin{bmatrix}1 & 3\\ 1 & 1\\ 2 & 1\end{bmatrix}+\begin{bmatrix}1 & 2\\ 3 & 2\end{bmatrix}$

a) $\begin{bmatrix}5 & 11\\ 4 & 12\end{bmatrix}$ b) $\begin{bmatrix}5 & 3\\ 1 & 1\end{bmatrix}$ c) $\begin{bmatrix}3 & 2\\ 3 & 6\\ 8 & 4\end{bmatrix}$ d) $\begin{bmatrix}5 & 3\\ 12 & 12\end{bmatrix}$ e) not possible

28. What system of equations corresponds to the matrix equation $\begin{bmatrix}2 & 1\\ -1 & 4\end{bmatrix}\cdot\begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}1\\ 6\end{bmatrix}$

a) $\begin{matrix}2x-3x=1\\ y+4y=6\end{matrix}$ b) $\begin{matrix}2x-3y=1\\ x+4y=6\end{matrix}$ c) $\begin{matrix}2x+\ y=1\\ -3x+4y=6\end{matrix}$ d) $\begin{matrix}2x+y=1\\ -x+4y=6\end{matrix}$ e) $\begin{matrix}2y-3x=1\\ y+4x=6\end{matrix}$

29. Given the matrix equation, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} X = \begin{bmatrix} -3 & 2 \\ 2 & -1 \end{bmatrix}$, X =

a) $\begin{bmatrix} 8 & -5 \\ -5.5 & 3.5 \end{bmatrix}$ b) $\begin{bmatrix} -2 & 0 \\ 1 & 3 \end{bmatrix}$ c) $\begin{bmatrix} -2 & 1 \\ 1.5 & -.5 \end{bmatrix}$ d) $\begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ e) $\begin{bmatrix} 9 & -4 \\ -5.5 & 2.5 \end{bmatrix}$

30. The mode and median respectively of the 2, 6, 2, 7, 7, 3, 6, 1, 7, 5 are:

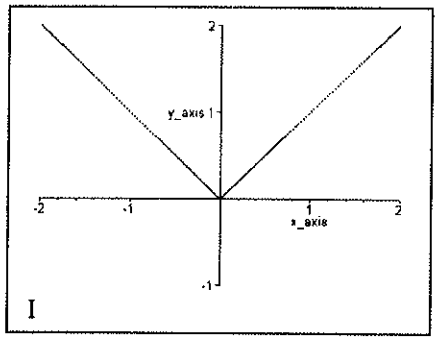
a) 7, 4.6 b) 7, 5.5 c) 2, 5 d) 4.6, 5.5 e) 7, 5

31. Suppose that the life of a light bulb has a normal distribution with a mean of 800 hours and a standard deviation of 40 hours. Approximately what percent of the bulbs last between 760 and 840 hours?

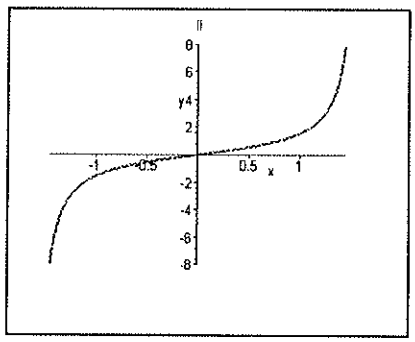
a) 20% b) 40% c) 68% d) 95% e) 100%

32. Which functions have inverses which are functions?

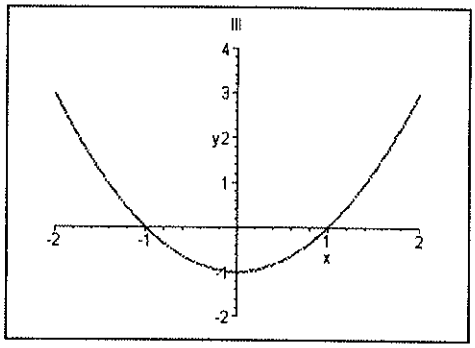
I



II



III



a) I b) II c) III d) I and III e) II and III

33. If $g(x) = x^3 + 2$, $g^{-1}(x) =$

a) $\sqrt[3]{x-2}$ b) $\sqrt[3]{x+2}$ c) $\frac{1}{x^3+2}$ d) $(x^3+2)^{-1}$ e) $x^{-3} - 2$

34. Which system has infinitely many solutions?

a) $2x - y = 8$, $x + y = 9$ b) $2x - y = 8$, $4x - 2y = 7$ c) $2x - y = 8$, $4x - 2y = 16$ d) $2x - y = 8$, $x - 2y = 8$ e) $2x - y = 8$, $4x - 2y = 8$

35. Solve $\log_2(x^2-9) = 4$ for x.

a) -5 b) 3 c) 5 d) a, b, and c e) a and c

36. Solve $e^{2x-1} = 3$ for x.

a) $\frac{\ln 3 + 1}{2}$ b) $e^{2(3)-1}$ c) $\sqrt[3]{2x-1}$ d) $\frac{\ln 3}{2} + 1$ e) $\frac{\ln 3}{2} - 1$

37. Solve $\log(x+1) + \log x = 1$ for x.

a) { -1, 0} b) $\left\{-\frac{1}{2} - \frac{\sqrt{41}}{2}, -\frac{1}{2} + \frac{\sqrt{41}}{2}\right\}$ c) {1, 9} d) {2, 5} e) $\left\{-\frac{1}{2} + \frac{\sqrt{41}}{2}\right\}$

38. Express as a single logarithm, $\frac{1}{2}\log_b r + \frac{1}{2}\log_b s - \log_b w$.

a) $\frac{1}{2}\left(\frac{(\log_b r)(\log_b s)}{\log_b w}\right)$ b) $\frac{\sqrt{(\log_b r)(\log_b s)}}{\log_b w}$ c) $\frac{\sqrt{\log_b rs}}{\log w}$ d) $\frac{\log_b \sqrt{rs}}{\log_b w}$ e) $\log_b \frac{\sqrt{rs}}{w}$

39. $\frac{(7.5 \times 10^6)(5.0 \times 10^{-1})}{1.5 \times 10^8} =$

a) 25 x 10^{13} b) 2.5 x 10^{-3} c) 250 d) 5.0 x 10^{-1} e) 2.5 x 10^{-2}

40. $f(x) = \frac{1 - 3x^2}{x^2 + 1}$ has a horizontal asymptote of :

a) x = -1 b) y = -3 c) y = -1 d) x = -3 e) y = 3