FORTY-SECOND ANNUAL MATHEMATICS CONTEST
Sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II
1998

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

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ALGEBRA II TEST 1998

1. The roots of the equation $x^2 + 5x = -6$ are:

a. -6 and -1 b. -6 and 1 c. 6 and -1 d. -3 and -2 e. 3 and 2

2. $27a^7b^2 + 64ab^{11} =$

a. $ab^2(3a^2 + 4b^3)(9a^4 - 12a^2b^3 + 16b^6)$ b. $ab^2(3a^2 - 4b^3)(9a^4 - 12a^2b^3 - 16b^6)$

c. $ab^2(3a^2 + 4b^3)(9a^4 - 24a^2b^3 + 16b^6)$ d. $ab^2(3a^2 + 4b^3)(9a^4 - 24a^2b^3 + 16b^6)$

e. $ab^2(3a^2 + 4b^3)(9a^4 + 24a^2b^3 + 16b^6)$

3. The graph of a polynomial with real coefficients has three distinct x-intercepts. The degree of the polynomial must be:

a. 3 b. 3 or 5 c. at least 3

d. any odd integer greater than or equal to 3

e. any positive integer multiple of 3

4. The zeros of a polynomial P(x) are 0 and 3. The zeros of $P(x^2 + 2x)$ are:

a. -4 and 2 b. -2 and 0 c. 0 and 3

d. -4, 0, 2 and 3 e. -3, -2, 0 and 1

5. If $x = 6$, which of the following expressions has a base ten equivalent of 4035_{six} ?

a. $5x^3 + 3x^2 + 4$ b. $4x^4 + 3x^2 + 5x$ c. $4x^2 + 3x^3 + 5x^4$

d. $4x^3 + 3x + 5$ e. $4x + 3x^3 + 5x^4$

6. If $a^3 + b^3 = 50$ and $(a + b)^3 = 20$ then $ab(a + b) =$

a. -30 b. -10 c. 5 d. 10 e. 30

7. The reciprocal of $2 - 3i$ is:

a. $2 + 3i$ b. $\frac{2}{13} + \frac{3}{13}i$ c. $\frac{2}{13} - \frac{3}{13}i$ d. $\frac{1}{2} - \frac{1}{3}i$ e. $\frac{1}{2} + \frac{1}{3}i$

8. Simplify $\frac{(2x^2 + 3)^2 - 1}{(x^2 + 1)/2}$

a. $8(x^2 + 2)$ b. $2(x^2 + 2)$ c. 4 d. $4x^2 + 18$ e. 48

9. At noon, a person begins to walk due north at the rate of four miles per hour. At 1:00 P.M., another person leaves from the same starting point as the first person and walks due east at the rate of five miles per hour. The distance (in miles) between the two people t hours after noon, where $t \geq 1$, is:

a. $\sqrt{41}\,t$ b. $9t - 5$ c. $\sqrt{41t^2 - 50t + 25}$ d. $\sqrt{41t^2 + 32t + 16}$ e. $9t + 5$

10. The solution set of the equation $x^4 - x^2 - 20 = 0$ is:

a. $\{5,-5,2i,-2i\}$ b. $\{2,-2,\sqrt{5},-\sqrt{5}\}$ c. $\{2i,-2i,i\sqrt{5},-i\sqrt{5}\}$

d. $\{4,-4,5,-5\}$ e. $\{\sqrt{5},-\sqrt{5},2i,-2i\}$

11. Let $f(n) = \sqrt[n]{(-1)^n} + (-1)^n - (-1)^{n+1}$ for $n = 1,2,3,4,\ldots$. The only possible values for $f(n)$ are:

a. -3 and 1 b. -3 and 3 c. -1,0 and 1 d. -1 and 1 e. -1 and 3

12. A circle C is described by the equation $(x + 1)^2 + (y - \sqrt{3})^2 = 12$. An equation of the line that contains the center of C and intersects C in the point $(2, 2\sqrt{3})$ is:

a. $x - \sqrt{3}y = -4$ b. $\sqrt{3}x - y = 0$ c. $x + \sqrt{3}y = 6$ d. $\sqrt{3}x + y = 4\sqrt{3}$

e. $x + \sqrt{3}y = 20$

13. The value of y that satisfies the system of equations is:

$$\begin{aligned} 5x - y + 4z &= 33 \\ 2x + 7y - 3z &= -20 \\ x + y + z &= 5 \end{aligned}$$

a. 2 b. 3 c. –2 d. 4 e. 0

14. Given that $1 + 4 + 9 + \ldots + 100 = 385$, find $6 + 21 + 46 + 81 + \ldots + 501$.

a. 1925 b. 1926 c. 1935 d. 1990 e. 2025

15. In our notation 11/15/84 stands for the date November 15, 1984. A date in the form a/b/c is called multiplicative if $a \cdot b = c$. How many multiplicative dates are there in 1998?

a. 0 b. 1 c. 2 d. 3 e. 4

16. The solution set for the inequality $\left|\frac{x}{2}\right| < \left|\frac{2}{x}\right|$ is:

a. $\{x \mid x < -2 \text{ or } x > 2\}$ b. $\{x \mid x > 2\}$ c. $\{x \mid -2 < x < 2\}$

d. $\{x \mid -\sqrt{2} < x < \sqrt{2}\}$ e. $\{x \mid -2 < x < 0 \text{ or } 0 < x < 2\}$

17. A polynomial with real coefficients that has $4 + i$ and 1 as roots is:

a. $x^3 + 7x^2 + 7x - 15$ b. $x^3 + 7x^2 + 9x - 17$ c. $x^3 - 9x^2 - 9x - 17$

d. $x^3 - 9x^2 - 25x - 17$ e. $x^3 - 9x^2 + 25x - 17$

18. How many numbers between 100 and 999 (inclusive) do not have a 2 in their base 10 representation?

a. 640 b. 648 c. 720 d. 721 e. 810

19. When 2^{1998} is divided by 5, the remainder is:

a. 0 b. 1 c. 2 d. 3 e. 4

20. A number plus its reciprocal is 3. One possibility for the number is:

a. $\frac{-3+\sqrt{5}}{2}$ b. $\frac{3-\sqrt{5}}{2}$ c. $\frac{-3+\sqrt{13}}{2}$ d. $\frac{3+\sqrt{13}}{2}$ e. $\frac{3\sqrt{13}}{2}$

21. If Mrs. Walker leaves home at 7:30 A.M. and drives at an average speed of 40 mph, she arrives at work one minute late. If she leaves at the same time but drives at an average speed of 45 mph, she arrives at work one minute early. How far does Mrs. Walker drive to work?

a. 6 miles b. 10.4 miles c. 12 miles d. 17 miles e. 18 miles

22. $(2x - 1)^4 - 4(2x - 1)^2 + 4 =$

a. $(4x^2 - 4x - 1)^2$ b. $(4x^2 + 4x - 1)^2$ c. $(4x^2 - 4x + 1)^2$ d. $(4x^2 + 4x + 1)^2$

e. $(4x^2 + 4x + 3)^2$

23. Assume that the absolute value of a is not equal to the absolute value of b. Under what condition can $(a + b)^2 = a^2 + b^2$?

a. the statement is never true b. if a and b have opposite signs

c. if either a or b is zero d. if $a = b$ e. the statement is always true

24. $\dfrac{2p}{(p-4)(2p+1)} - \dfrac{p^2}{(2p-1)(2p+1)} - \dfrac{3p}{p(p-4)(2p+1)} =$

a. $\dfrac{-p(p^3-8p^2+8p-3)}{p(p-4)(2p-1)(2p+1)}$ b. $\dfrac{-p(p^3+8p-3)}{p(p-4)(2p-1)(2p+1)}$ c. $\dfrac{-p(p^3+8p^2+8p+3)}{p(p-4)(2p-1)(2p+1)}$

d. $\dfrac{-p(p^3+8p+3)}{p(p-4)(2p-1)(2p+1)}$ e. $\dfrac{-p(p^3+8p^2-8p+3)}{p(p-4)(2p-1)(2p+1)}$

25. The x-coordinates of the foci of the ellipse that has $x^2 - 2x + 2y^2 = 3$ as its equation are:

a. $\pm\sqrt{2}$ b. $1 \pm \sqrt{2}$ c. $\pm\sqrt{6}$ d. $1 \pm \sqrt{6}$ e. $-1 \pm \sqrt{6}$

26. The graph of $y = 2(x + 1)^2 - 3$ is symmetric about:

a. the y-axis b. the line $x = -1$ c. the line $x = 3$

d. the x-axis e. the line $x = 1$

27. On a trip, Jim drove from A to B in a car which gets 20 miles per gallon. Returning from B to A, he drove a different car which gets 30 miles per gallon. What is his miles per gallon figure for the total trip?

a. 20 b. 24 c. 25 d. 26 e. 30

28. The equation $3x^2 + bx - 7 = 0$ has two distinct real solutions (for x) if and only if:

a. b is any real number b. $b^2 > 84$ c. $b^2 \geq 84$

d. $b^2 < 84$ e. $b^2 \leq 84$

29. Two integers differ by four while their squares differ by 200. What is the product of the two integers?

a. 50 b. 92 c. 196 d. 621 e. 800

30. Rewrite $\dfrac{\left(a^{5/3}b^{-1/2}c^{2/5}\right)^3}{\left(a^{-1/2}b^{1/3}c^{3/5}\right)^{-2}}$ without negative exponents.

a. $\dfrac{a^4}{b^{5/6}}$ b. $\dfrac{a^4c^{12/5}}{b^{5/6}}$ c. $\dfrac{a^4}{b^{13/6}}$ d. $\dfrac{a^4c^{12/5}}{b^{13/6}}$ e. $\dfrac{a^{65/6}}{b^{25/6}c}$

31. $a^3b^{-2}c^4$ divided by $a^{-2}b^{-2}c^5$ is:

a. $\dfrac{a}{c}$ b. $\dfrac{a}{b^4c}$ c. ac^9 d. $\dfrac{a^5}{c}$ e. $\dfrac{a^5}{b^4c}$

32. The height of a circular cone is twice its radius, r. The radius of the cone expressed as a function of its volume, V, is r =

a. $\sqrt[3]{\dfrac{V}{6\pi}}$ b. $\sqrt[3]{\dfrac{3V}{2\pi}}$ c. $\sqrt[3]{\dfrac{3\pi V}{2}}$ d. $\sqrt[3]{\dfrac{6V}{\pi}}$ e. $\sqrt[3]{6\pi V}$

33. Let $f(x) = \dfrac{1}{x+1}$ and let $g(x) = \dfrac{1}{x+2}$. The domain of $f \circ g$ (the composition of f and g) is all real numbers except:

a. –3 b. –2 c. –1 d. -1 and –2 e. -3 and -2

34. The set of all complex numbers that are equal to the negative of their reciprocals is:

a. {-1} b. {-1,i} c. {-1,i,-i} d. {i,-i} e. {i}

35. The system of equations:

$$a_1x + b_1y = c_1$$
$$a_2x + b_2y = c_2$$

(where x and y are the variables) has exactly two solutions

a. whenever $a_1b_2 - b_1a_2 = 0$ b. whenever $a_1b_2 - b_1a_2 \neq 0$

c. whenever $a_1a_2 - b_1b_2 = 0$ d. whenever $c_1 \neq c_2$

e. always

36. One value of k in the equation $x^2 + 4kx + 3k^2 = 0$ that makes the sum of the roots equal to two-thirds of the product of the roots is:

a. -1 b. 2 c. 3 d. -2 e. 1

37. Let $f(x) = \frac{1}{2x+3}$. Find $f(f(f(1)))$.

a. $\frac{6x+11}{22x+39}$ b. 1/125 c. 1/5 d. 17/61 e. 5/17

38. If $2^x = 7$ then $2^{3x+1} =$

a. 22 b. 42 c. 256 d. 344 e. 686

39. In interval notation, the solution to the inequality $|x + 1| < |x - 3|$ is:

a. $(-3,1)$ b. $(-\infty,1)$ c. $(-1,2)$ d. $(1,\infty)$ e. $\varnothing$

40. The area of the triangle with vertices $(0,0)$, $(1,2)$ and $(2,5)$ is

a. $\frac{\sqrt{17}}{2}$ b. $\frac{1}{2}$ c. $\frac{\sqrt{85}}{2}$ d. $\frac{\sqrt{130}}{2}$ e. $\frac{\sqrt{442}}{2}$