FORTY-FIRST ANNUAL MATHEMATICS CONTEST
sponsored by
THE TENNESSEE MATHEMATICS TEACHERS' ASSOCIATION

Algebra II 1997

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Scoring formula: 4R - W + 40

DIRECTIONS:

Do not open this booklet until you are told to do so.

This is a test of your competence in high school mathematics. For each problem, determine the best answer and indicate your choice by making a heavy black mark in the proper place on the separate answer sheet provided. You must use a pencil with a soft head (No. 2 lead or softer).

This test has been constructed so that most of you are not expected to answer all of the questions. Do your best on the questions you feel you know how to work. You will be penalized for incorrect answers, so wild guesses are not advisable.

If you change your mind about an answer, be sure to erase completely. Do not mark more than one answer for any problem. Make no stray marks of any kind on the answer sheet. The answer sheets will not be returned to you. If you wish a record of your performance, mark your answers in this booklet also. You will keep the booklet after the test is completed.

When told to do so, open your test booklet and begin. You will have exactly 80 minutes to work.

Contributors to TMTA for the Annual Mathematics Contest:

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TRW Commercial Steering Division, Lebanon, Tennessee
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1. The degree measure of each angle of a regular polygon with 20 sides is:

(a) 18° (b) 90° (c) 108° (d) 135° (e) 162°

2. Solve: $x^2 - 4x \geq 0$

(a) $x \leq 0$ or $x \geq 4$ (b) $0 \leq x \leq 4$ (c) $\{0,4\}$ (d) $x \geq 0$ (e) $x \geq 0$ or $x \leq 4$

3. Find the horizontal asymptote(s) for $f(x) = \dfrac{x^2}{x^2 + 9}$

(a) $x = \pm 3$ (b) $x = -9$ (c) $y = 1$ (d) $y = 0$ (e) $x = 0$

4. Find x, where $x > 0$, such that the distance from $(x, -5)$ to $(-1, 3)$ is $4\sqrt{5}$ units.

(a) 5 (b) 0 (c) 7 (d) 9 (e) 3

5. The path of a dive is given by $y = -\dfrac{4}{9}x^2 + \dfrac{24}{9}x + 10$, where y is the height in feet and x is the horizontal distance from the end of the diving board in feet. What is the maximum height of the dive above the board given that the board is 10 feet high?

(a) 8.6 feet (b) 4 feet (c) 3 feet (d) 10 feet (e) 14 feet

6. Given $f(x) = 1 - 3x$ and $g(x) = x + 2$, find $(f^{-1} \circ g^{-1})(1)$.

(a) $\dfrac{2}{3}$ (b) -6 (c) -11 (d) $-\dfrac{1}{3}$ (e) undefined

7. In how many ways can 3 novels, 2 mathematics books, and 1 chemistry book be arranged on a bookshelf if the novels must be together but the other books can be arranged in any order?

(a) 720 (b) 72 (c) 7 (d) 144 (e) 5040

8. The domain of the function $f(x) = \ln(5 - 4x)$ is

(a) $\left(-\infty, \dfrac{4}{5}\right)$ (b) $\left(-\infty, \dfrac{5}{4}\right)$ (c) $\left(\dfrac{5}{4}, \infty\right)$ (d) $(0, \infty)$ (e) $(-\infty, \infty)$

9. Simplify the expression: $\dfrac{8^{\frac{1}{6}} - 9^{\frac{1}{4}}}{\sqrt{3} + \sqrt{2}}$

(a) $2\sqrt{6}$ (b) $\dfrac{2\sqrt{6} - 5}{5}$ (c) $2\sqrt{6} - 5$ (d) $-\dfrac{1}{5}$ (e) $2\sqrt{6} + 5$

10. Find the values of A and B if: $\dfrac{A}{x+2} + \dfrac{B}{2x-3} = \dfrac{5x - 11}{2x^2 + x - 6}$

(a) $A = -3 \quad B = 1$ (b) $A = 5 \quad B = 1$ (c) $A = \dfrac{1}{7} \quad B = 4\dfrac{5}{7}$

(d) $A = -\dfrac{1}{7} \quad B = -4\dfrac{5}{7}$ (e) $A = 3 \quad B = -1$

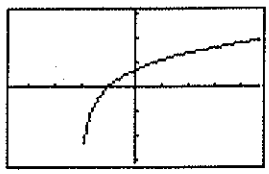
11. Supply the value of x so that the matrix $A = \begin{bmatrix} -4 & 8 \\ 3 & x \end{bmatrix}$ does <u>not</u> have an inverse.

(a) 6 (b) 0 (c) 3 (d) −3 (e) −6

12. Find the sum of all zeros of: $f(x) = x^3 + 6x^2 + 21x + 26$

(a) 6 (b) −6 (c) −2 (d) 0 (e) 2

13. The following graph corresponds to which function?



(a) $y = \ln x$ b) $y = \ln(x - 2)$ (c) $y = \ln(x + 2)$ (d) $y = e^x + 2$ (e) $y = e^x - 2$

14. If $g(x) = x^2 - 4x + 5$, $x \geq 2$ write a formula for g^{-1}

(a) $g^{-1}(x) = (x - 2)^2 + 1$ (b) $g^{-1}(x) = \sqrt{(x - 2)^2 + 1}$ (c) $g^{-1}(x) = 2 + \sqrt{x - 1}$

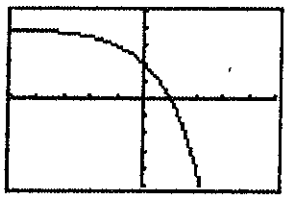
(d) $g^{-1}(x) = \sqrt{x^2 - 4x + 5}$ (e) $g^{-1}(x) = 2 - \sqrt{x - 1}$

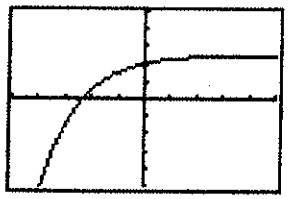
15. Solve the equation $x^3 - 27 = 0$ over complex numbers.

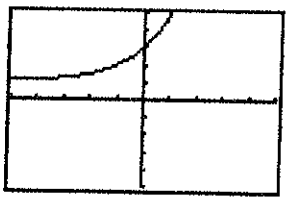
(a) $3, \frac{-3 \pm 3i\sqrt{3}}{2}$ (b) $-3, \frac{3 \pm 3i\sqrt{3}}{2}$ (c) 3 (d) $3, -3$

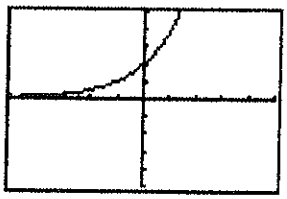
(e) $-3, \frac{-3 \pm 3i\sqrt{3}}{2}$

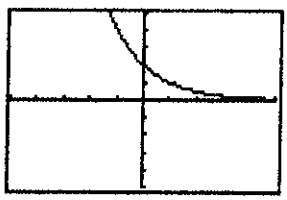
16. Graph: $f(x) = 2^{x+1}$

(a) 

(b) 

(c) 

(d) 

(e) 

17. Solve: $|x + 5| = |x - 4|$

(a) $-\frac{1}{2}$ (b) 1 (c) -1 (d) $\frac{1}{2}$ (e) $-\frac{9}{2}$

18. The multiplicative inverse of the matrix $A = \begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}$ is

(a) $\begin{bmatrix} -8 & -5 \\ -3 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & -5 \\ -3 & 8 \end{bmatrix}$ (c) $\begin{bmatrix} \frac{1}{8} & \frac{1}{5} \\ \frac{1}{3} & \frac{1}{2} \end{bmatrix}$ (d) $\frac{1}{\begin{bmatrix} 8 & 5 \\ 3 & 2 \end{bmatrix}}$

(e) $\begin{bmatrix} -8 & -5 \\ 3 & 2 \end{bmatrix}$

19. Find all the solutions of $4x + 5 = \ln e^{x^2}$

(a) $5, -1$ (b) $-5, 1$ (c) $-\frac{5}{4}$ (d) $\frac{5}{4}$ (e) $1, 5$

20. Which of the following functions can <u>not</u> have the following zeros: 0, 2, −3?

(a) $f(x) = 3x^3 + 3x^2 - 18x$ (b) $g(x) = x^2(x + 3)(x - 2)$ (c) $h(x) = 5x(x + 3)^2(x - 2)$

(d) $j(x) = x^7(x + 3)(x - 2)^4$ (e) $k(x) = 2x^3 + 8x^2 - 6x - 36$

21. Find the center of the circle $4x^2 + 4y^2 + 20x - 16y + 37 = 0$

(a) $\left(\frac{25}{4}, -4\right)$ (b) $\left(-\frac{5}{2}, -2\right)$ (c) $\left(\frac{5}{2}, 2\right)$ (d) $\left(-\frac{5}{2}, 2\right)$ (e) $\left(\frac{5}{2}, -2\right)$

22. A company's profit function based on advertising dollars spent is $P(x) = 200 + 27x - 0.3x^2$, where x is the number of dollars spent on advertising, and $P(x)$ is the profit in dollars.

The company's maximum possible profit is:

(a) \$200.00 (b) \$418.70 (c) \$807.50 (d) \$862.50 (e) \$2,660.00

23. Let f(x) = $\frac{sin^2x + cotxsinx + cos^2x - 1}{sin^2x + cos^2x + tan^2x}$, then f(x) =

(a) $\frac{cosx}{sin^2x}$ (b) $1 + sin^2x$ (c) $\frac{1 - sinx}{cosx}$ (d) $\cos^3 x$ (e) $sinx\ tanx$

24. What is the end behavior of the following polynomial, $f(x) = -3x^3 + 2x - 1$?

(a) The left-hand tail rises and the right-hand tail falls

(b) The left-hand and right-hand tails both rise

(c) The left-hand and right-hand tails both fall

(d) The left-hand tail falls and the right-hand tail rises

(e) The left-hand tail falls and the right-hand tail terminates

25. If two "fair" dice are rolled, what is the probability that the sum of the upturned faces equals 6?

(a) $\frac{2}{12}$ (b) $\frac{5}{36}$ (c) $\frac{3}{12}$ (d) $\frac{3}{36}$ (e) $\frac{1}{6}$

26. A ladder 15 feet long leans against a house and makes an angle of $60°$ with the ground. Find the distance from the house to the foot of the ladder.

(a) 13 feet (b) 7.5 feet (c) 8 feet (d) 10 feet (e) 8.2 feet

27. Suppose a properly inflated basketball is designed to bounce back up to one-half of the height from which it is dropped. If such a ball is initially dropped from a height of 10 feet, find the total vertical distance it has traveled when it hits the floor the 5th time.

(a) 50 feet (b) 19.375 feet (c) 28.75 feet (d) 30 feet (e) 38.75 feet

28. Find all the roots of $8x^3 - 27 = 0$

(a) $\frac{3}{2}$, no other solutions (b) $\frac{3}{2}, -\frac{3}{4} \pm \frac{3i\sqrt{3}}{4}$ (c) $\frac{3}{2}, \frac{-3 \pm 3\sqrt{3}}{4}$

(d) $\frac{3}{2}, \frac{-6 \pm \sqrt{216}}{8}$ (e) $\frac{3}{2}, -12 \pm i\sqrt{3}$

29. Solve for $\left(\frac{1}{3}\right)^{2x-1} = 81$

(a) $\frac{3}{2}$ (b) $-\frac{3}{2}$ (c) $\frac{5}{2}$ (d) $-\frac{5}{2}$ (e) 2

30. Find $f(x + \Delta x) - f(x)$ for $f(x) = 2x^2 + 3x + 1$

(a) $2(\Delta x)^2 + 3(\Delta x)$ (b) $4x(\Delta x) + 2(\Delta x)^2 + 3(\Delta x)$

(c) $2x^2(\Delta x)^2 + 3x(\Delta x) + 1$ (d) $2(\Delta x)^3 + 3(\Delta x)^2 + (\Delta x)$

(e) $2x^2 + 4x(\Delta x) + 2(\Delta x)^3 + (3(\Delta x) + 1$

31. Identify the graph of $4x^2 + y^2 - 8x + 6y + 9 = 0$

(a) circle (b) hyperbola (c) parabola (d) ellipse (e) logarithm

32. Find the y-intercept for $g(x) = \dfrac{1}{x + 2} + 2$

(a) $\left(-\frac{5}{2}, 0\right)$ (b) $\left(0, -\frac{5}{2}\right)$ (c) $(-2, 2)$ (d) $(0, 2)$ (e) $\left(0, \frac{5}{2}\right)$

33. An airplane flies 300 miles from the airport in the direction 110° and then travels 400 miles in the direction 200° for 400 miles. How far is the airplane from the airport?

(a) 250,000 mi. (b) 10,000 mi. (c) 500 mi. (d) 50 mi. (e) 800 mi.

34. Find a polynomial with real coefficients that has zeros: 0, 2, −2, 2i, −2i

(a) $f(x) = x^5 + 16x$ (b) $f(x) = x^5 - 16x$ (c) $f(x) = x^5 - 16$ (d) $f(x) = x^4 + 16$

(e) $f(x) = x^4 - 16$

35. Find the value of b In the quadratic equation, $y = ax^2 + bx + c$, if its graph passes through the points (−1, 4), (1, −2), and (2, −2).

(a) 2 (b) −3 (c) −2 (d) −1 (e) 3

36. Let x be a real number and in the domain of the function

$f(x) = arc\ cos(3x^2)$, then $sin(arc\ cos[3x^2]) =$

(a) $\sqrt{1-9x^4}$ (b) $3x^2 - 1$ (c) $\pm\frac{\sqrt{3}}{3}$ (d) $\pm\sqrt{1-9x^4}$ (e) $1 - 3x^2$

37. Simplify: $\dfrac{a^{-1} + b^{-1}}{\left(\dfrac{a^2 - b^2}{ab}\right)}$

(a) $\frac{1}{a} - \frac{1}{b}$ (b) $\frac{a+b}{a^2 - b^2}$ (c) $a - b$ (d) $a + b$ (e) $\frac{1}{a-b}$

38. Find $log_8 \frac{1}{64}$

(a) 2 (b) −4 (c) $-\frac{1}{3}$ (d) $-\frac{1}{4}$ (e) −2

39. The path of a particular bottle rocket is modeled by the function $h(t) = -16t^2 + 117t + 5$ where t represents the number of seconds that have elapsed and $h(t)$ represents the height of the rocket in feet. According to the model, approximately how long will the bottle rocket stay in the air?

(a) 5 seconds (b) 7 seconds (c) 20 seconds (d) 117 seconds

(e) 2 seconds

40. Solve $A = B^2 - 4B$ *for* B

(a) $B = 3A^2 - A$ (b) $B = 4 \pm \frac{\sqrt{19A}}{2}$ (c) $B = 2 \pm \sqrt{4 + A}$ (d) $B = 2 \pm \sqrt{5A}$

(e) $2A \pm \sqrt{4A^2 - 1}$